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THE MASSES OF THE STARS

WITH A GENERAL CATALOGUE OF
DYNAMICAL PARALLAXES

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THE MASSES OF THE STARS

WITH A GENERAL CATALOGUE OF DYNAMICAL PARALLAXES

By

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and

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PREFACE

This monograph has grown out of a lecture on the masses of the stars presented by the senior author in connection with the Harvard Tercentenary. Shortly afterward it became evident that an adequate treatment demanded a new discussion of the relation between trigonometric and spectroscopic parallaxes. This proved to be time consuming, as did also the theoretical discussion of the various sources of error. It is hoped that the results here given include substantially all the direct evidence regarding the masses of the stars which is at present available—i.e., that derived by gravitational motions from binary systems.

The determination of the masses of such systems is, in principle, very simple. In practice, it is seriously complicated by the great distances of the stars. Reliable individual values can be obtained only for a couple of dozen visual binaries of large parallax, and for about as many eclipsing variables, which have been spectroscopically observed. For the great mass of the others, recourse must be had to average values for properly selected groups. The utilization of pairs whose motion is so slow that no orbits can be computed for centuries to come, the elimination of sources of systematic error, and the diminution and evaluation of the effects of accidental error present problems of considerable statistical interest and some difficulty, the discussion of which occupies a large part of this volume.

The remarkably close correlation between mass and luminosity, which has been fully confirmed by the present investigation, relieves the authors from the difficult task of attempting to deduce the properties of the average star in a given volume of space from those of the observable binaries, which are subject to complicated observational selection. This relation can be represented over the whole observable range by a very simple empirical formula, with the luminosity (corrected for surface temperature according to Eddington) varying as the 10/3 power of the mass. No complete theoretical

explanation of the close approximation of this formula over so wide a range is yet available, but it should evidently be fully adequate for the computation of dynamical parallaxes.

Individual deviations from the mean relation undoubtedly exist; but the well-established cases can still be counted on the fingers. There are also groups of stars (for which only rough data are yet available) which probably deviate seriously—notably the "Trumpler stars."

Chapter i deals with visual binaries and contains most of the statistical discussions. Chapter ii deals with spectroscopic and eclipsing binaries, while chapter iii treats of several topics which were deferred to it to avoid breaking the thread of the preceding discussions.

Chapter iv presents a new treatment of the determination of dynamical parallaxes in which various refinements, not previously applied, have been introduced and ends with tables and precepts for the calculation of such parallaxes from double-star data. Chapter v contains a general catalogue of the dynamical parallaxes of 2529 stars, calculated anew in a strictly homogeneous manner by the methods of chapter iv, and summarizes also various other observational data important to anyone who desires to make a critical reexamination of the work.

It may be added that the senior author is responsible for the general plan of the work and the theoretical discussions, while the junior author has made by far the greater part of the heavy computations and has scrutinized both the theoretical and numerical results with respect to accuracy.

It is a pleasure to acknowledge our obligation to Messrs. W. S. Adams and A. H. Joy for spectroscopic data from Mount Wilson; and to Miss Henrietta Young and Mrs. Jay Murray for the care and accuracy with which they have carried out the typing of the manuscript and tables.

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CHAPTER I

VISUAL BINARIES

A. GENERAL EQUATIONS

1. *Introduction.*—The only available way to determine the mass of an astronomical body is by means of its gravitational attraction. The body's attraction upon itself produces observable effects—such as the oblateness of a rotating body and the period of a pulsating mass of gas—but these effects depend on the internal distribution of density as well as the mean density, so that the latter cannot be precisely determined. This is a fortiori true of those indirect effects of gravity which result in the mass-luminosity relation among the stars and in the relations between pressure and density in stellar atmospheres. So many other parameters are involved that such considerations can, at best, give only an idea of the order of magnitude of the masses—unless empirical relations, such as the observed mass-luminosity correlation, can be invoked. The determination of the mass of an agglomeration of matter (such as a globular cluster or the Galaxy) from the relative velocities of its parts also involves assumptions regarding steady-state conditions, as well as a precision of observation often still unattained, and falls outside the present discussion.

The attraction between two stars is utterly insensible at ordinary interstellar distances, so that determinations of mass can be made only for binary systems.

All known multiple systems are composed of wide and close pairs, of periods so different that the perturbations of one by the other are small.[1] The elementary theory of motion in a Keplerian ellipse therefore suffices for the determination of the masses within the limits of precision set by the observations.[2] The resulting relations are summarized here.

[1] E. W. Brown, *M.N.*, 97, 56, 1936.

[2] An exception might be urged for pairs so close that the ellipticity of figure is considerable; but in this case the measurement of the radial velocities is also subject to comparable uncertainty, owing to the "reflection" effect.

2. *Formulae for Visual Binaries.*—For a visual binary the fundamental equation is

$$\text{M} = \text{M}_1 + \text{M}_2 = \frac{4\pi^2 a^3}{GP^2}, \tag{1}$$

where M_1 and M_2 are the masses, a the semi-major axis of the relative orbit, and P the period. This equation holds true in any system of units having the appropriate value of the gravitational constant G. With the sun's mass, the astronomical unit, and the year as units, $G = 4\pi^2$ and

$$\text{M} = \frac{a^3}{P^2}. \tag{2}$$

Observation gives the semi-major axis in seconds of arc, a''. If p is the parallax,

$$\text{M} = \frac{a''^3}{P^2 p^3}. \tag{3}$$

Methods of determining the orbit or the parallax lie outside the scope of this review, but an understanding of their limitations is essential to a sound interpretation of the results obtained by the formal application of equation (3).

The individual masses can be found only when the positions of one or both components have been observed relative to other stars. If x_1, y_1, x_2, y_2 are the rectangular co-ordinates of the components and $z = \text{M}_2/(\text{M}_1 + \text{M}_2)$, then

$$\left.\begin{aligned} x_1 &= x_0 + nt - z(x_2 - x_1)\,, \\ y_1 &= y_0 + nt - z(y_2 - y_1)\,. \end{aligned}\right\} \tag{4}$$

Observations of right ascension and declination have given results for many pairs,[3] but greater accuracy can be secured by micrometric observations of distant companions (when present) or by photography (as for stellar parallax). When the components are clearly resolved on the plates, this method should give precise results. Measures of the fainter component may give an independent determination. Settings on the unresolved image of an unequal pair,

[3] See Boss, *General Catalogue*, Carnegie Institution of Washington, 1937.

though often showing that the "center of light" does not coincide with the center of gravity, are unsuited for reliable determinations of the mass, since the relative brightness of the components of so close a pair cannot, at present, be accurately measured; and even if this were known, the position of the effective center of light cannot be precisely specified theoretically.

The separation of z from the other unknowns in (4) is practicable only when the relative motion departs conspicuously from linearity during the interval of observation. For some wide pairs, such as 61 Cygni, this condition will be satisfied long before a trustworthy orbit can be computed.

3. *Formulae for Physical Pairs.*—There are great numbers of visual pairs, shown to be binary by common proper motion, in which the relative motion, though observable, is too small to permit the calculation of an orbit. Let r be the distance of the components, and v their relative velocity, at any time. Then in all cases

$$rv^2 = G\text{M}\left(2 - \frac{r}{a}\right). \tag{5}$$

If s is the apparent distance, and w the apparent relative velocity, in seconds of arc per year, and if i and j are the angles between the vectors r and v and the line of sight, this may be written

$$\frac{sw^2}{4\pi^2 p^3} = \text{M} \sin i \sin^2 j \left(2 - \frac{r}{a}\right). \qquad (6)^4$$

The last three factors are unknown for individual pairs; but the mean value of their product, in a large number of cases, may be computed from principles of geometrical probability and is found[5] to be $\frac{1}{32}\pi(5 - 2\bar{e}^2)$, where $\bar{e}$ is the mean-square orbital eccentricity. If we adopt $\bar{e} = 0.561$ (the value found for binaries with well-determined orbits), this becomes 0.429.

[4] This formula was proposed independently by Russell (*Science*, N.S., 34, 524, 1911) and by Hertzsprung (*A.N.*, 190, 113, 1911).

[5] H. N. Russell, "On the Determination of Dynamical Parallaxes," *A.J.*, 38, No. 897, p. 89, 1928 (referred to hereinafter as "I").

4. *Formulae for Spectroscopic Binaries.*—When the spectra of both components have been observed, and orbits computed for each, giving $a_1 \sin i$ and $a_2 \sin i$, the best available equation is

$$(\mathrm{M}_1 + \mathrm{M}_2) \sin^3 i = \frac{C(a_1 + a_2)^3 \sin^3 i}{P^2}, \tag{7}$$

where $C = 1$ if a is given in astronomical units and P in years, but $C = 0.0400$ if a is in millions of kilometers and P in days. An alternative form is

$$(\mathrm{M}_1 + \mathrm{M}_2) \sin^3 i = 1.042 \times 10^{-7}(K_1 + K_2)^3(1 - e^2)^{-3/2} P, \tag{8}$$

where K_1 and K_2 are the semi-amplitudes in kilometers per second and P is in days.

The mass ratio is given by

$$\frac{\mathrm{M}_1}{\mathrm{M}_2} = \frac{a_2}{a_1} = \frac{K_2}{K_1}. \tag{9}$$

When only one spectrum has been observed, all that can be determined is the "mass-function"

$$f = \frac{Ca_1^3 \sin^3 i}{P^2} = \frac{\mathrm{M}_2^3 \sin^3 i}{(\mathrm{M}_1 + \mathrm{M}_2)^2}. \tag{10}$$

Individual values of $\sin i$ can be found only for eclipsing binaries. In other cases statistical averages must be used. If the orbit planes were distributed at random, the mean value of $\sin^3 i$ would be

$$\int_0^{\pi/2} \sin^4 i \, di = \frac{3\pi}{16} = 0.589.$$

This is the minimum permissible value, since binaries with small values of i and K certainly do not stand a better chance of being discovered. For stars with sharp lines and showing only one spectrum, it is probably a good value to adopt; but for those with diffuse lines, and especially with double lines, there must be a strong discrimination against small inclinations. Schlesinger[6] assumes that the

[6] Quoted by Aitken, *The Binary Stars*, p. 219, New York, 1935.

chance of discovery is proportional to sin i and so finds the mean value

$$\frac{\int_0^{\pi/2} \sin^5 x}{\int_0^{\pi/2} \sin^2 x} = \frac{32}{15\pi} = 0.679 \,.$$

The mean value appropriate to any particular group of binaries will depend upon the limits of observational selection in the particular case.

5. *Comparison with Observed Parallaxes.*—For visual pairs the simple equations (3) and (6) can be profitably employed only for a few stars with very large parallaxes. Otherwise the observational errors of the parallax make the calculation inaccurate, or even illusory (since the observed parallax may be negative).

To utilize most of the existing data, the equations must be transformed so that a function involving the mass and having the dimensions of a parallax may be compared with the observed parallaxes. Instead of (3) we may write

$$h_1 = a'' P^{-2/3} = M^{1/3} p \,. \tag{11}$$

Here h_1 is the "hypothetical parallax" for a mass equal to that of the sun, and may be directly calculated when the orbit is known. The observational errors affecting it, though considerable for poorly determined orbits, are not nearly so great as those of the parallax.

To diminish the latter, we must take suitably weighted means for selected groups (see below). We thus obtain the mean values

$$\overline{M^{1/3} p} = \overline{h_1} \,. \tag{12}$$

For slow-moving pairs equation (6) may be written

$$h_1 = 0.418 \sqrt[3]{sw^2} = M^{1/3} p (1 + z) \,, \tag{13}$$

where the constant 0.418 represents the mean effect of the factor

$$\left[4\pi^2 \sin i \sin^2 j \left(2 - \frac{r}{a}\right)\right]^{-1/3} ,$$

and z takes account of individual variations from this mean. The statistical distribution of z closely resembles a normal one, with probable error ± 0.20. (See § 54.)

We now have

$$\overline{\mathrm{M}^{1/3} p(1+z)} = \overline{h_1}\,. \tag{14}$$

If M, p, and z were free from correlation, both (12) and (14) would reduce to

$$\overline{\mathrm{M}^{1/3}} = \frac{\overline{h_1}}{\overline{p}} \tag{15}$$

since $\bar{z} = 0$. (The actual correlations are discussed on p. 18, sec. D.)

This gives a "cube-root mean" of the masses. If we assume that, for individual stars, $\mathrm{M}^{1/3} = \overline{\mathrm{M}^{1/3}}(1 + a)$, we have, in the mean, $\overline{\mathrm{M}} = (\overline{\mathrm{M}^{1/3}})^3(1 + 3\overline{a^2} + \overline{a^3})$. The last term will be small, and we have substantially

$$\overline{\mathrm{M}} = (\overline{\mathrm{M}^{1/3}})^3(1 + 3\overline{a^2})\,. \tag{16}$$

According to Eddington's mass-luminosity curve, a change of 1^{m} in the bolometric absolute magnitude corresponds to a change in log M ranging from 0.10 for the fainter dwarfs to 0.20 for absolute magnitude -3. The corresponding values of a are approximately 0.08 and 0.16. If the absolute magnitudes are distributed with a standard deviation of $\pm 1^{\mathrm{m}}$, the factor $1 + 3\overline{a^2}$ ranges from 1.02 for dwarfs to 1.08 for supergiants. The empirical relation found in the present work gives $\Delta \log \mathrm{M}/\Delta M = 0.12$, $a = 0.092$, and $1 + 3\overline{a^2} = 1.025$. This correction is therefore unimportant, except for very heterogeneous groups. In any case, the mean value of $\mathrm{M}^{1/3}$ is as well entitled to be considered typical of a group as that of M.

B. DATA

6. *Dynamical Parallaxes.*—The values used in the present discussion are derived from the lists of Russell and Moore,[7] Aitken and Moore,[8] and Finsen[9] and are statistically homogeneous, being based

[7] *A.J.*, 39, No. 930, p. 1, 1929.

[8] *Lick Obs. Bull.*, No. 451, 1933; No. 485, 1937.

[9] *Union Obs. Circ.*, No. 93, 139, 1935.

on the same formulae and tables, with deliberate avoidance of preferential selection of "moving" pairs. For a number of stars in the first list, and for some in the second, revised values have been used, based on additional measures. Many, though not all, of these revised values were available in time for inclusion in Schlesinger's catalogue.[10] The parallaxes of the first and second lists have been divided into three grades—good, fair, and poor—according to the extent and consistency of the observations. Following Schlesinger's suggestion (*op. cit.*, p. viii), Finsen's parallaxes of southern stars, which depend in general upon a much shorter interval of observation, have been graded "fair" in general, and "poor" when marked with a colon. A few misprints in Finsen's list have been rectified. A very few stars not in these lists (such as Kuiper's short-period binary) have been added from other sources.

7. *Trigonometric Parallaxes.*—These are all taken from Schlesinger's catalogue, in which both the parallaxes and the probable errors have been carefully reduced to a homogeneous system, which has been taken as definitive. When the catalogue gives parallaxes for two neighboring stars of common proper motion, obviously physically connected, the mean has been taken, with weights according to the published probable errors.

Parallaxes derived from moving clusters have been discussed along with the trigonometric determinations. They include those derived by Kapteyn for the Scorpius-Centaurus[11] and the Orion groups,[12] and by Wilson[13] for the Hyades. The probable errors given by Kapteyn have been adopted; those of Wilson's results were estimated by the writers. When both trigonometric and cluster parallaxes are available for the same star, they have been combined with weights inversely proportional to their probable errors (see Table 47, p. 154).

8. *Spectroscopic Parallaxes.*—These are based mainly on the list of 4179 stars in *Mount Wilson Contribution* No. 511.[14] For stars

[10] *General Catalogue of Stellar Parallaxes*, Yale University Observatory, ed. 1935.

[11] *Ap. J.*, 40, 43, 1914; *Mt. W. Contr.*, No. 82.

[12] *Ap. J.*, 47, 146, 1918; *Mt. W. Contr.*, No. 147.

[13] *A.J.*, 42, No. 978, p. 50, 1932.

[14] *Ap. J.*, 81, 187, 1935.

which appear in this work, the values in this list have been taken as final, partly for the sake of homogeneity and partly because, as Schlesinger points out (*op. cit.*, pp. vii and viii), determinations at different observatories are not really independent, being affected by the same spectral peculiarities. When two or more spectroscopic parallaxes are given for members of a physical system, their geometric mean has been taken (corresponding to the simple mean of the moduli, $m - M$). The two values are usually, but not always, consistent.

For a few other stars of the later spectral classes, parallaxes determined at other observatories have been taken from Schlesinger's catalogue, in which they have been reduced to the Mount Wilson system.

The spectroscopic parallaxes of stars of classes A and B are much less accurate. Two sources have been drawn upon: Schlesinger's catalogue, which often gives mean values for various observers, and the lists published in *Mount Wilson Contributions* Nos. 244[15] and 262.[16] These parallaxes, though not included in the catalogue, appear to be comparable in accuracy with those of other observers and have been added, or combined with the others, in the present list (see Table 49, p. 156).

9. *Corrections.*—The spectroscopic parallaxes of double stars are necessarily based on the individual magnitudes of the components whose spectra were observed. Reliable determinations of their magnitudes, especially for the closer pairs, are notoriously difficult. In the Mount Wilson list, the magnitude given in the *Henry Draper Catalogue* was assumed to refer to the combined light for pairs with less than 10″ separation and to the brighter component for wider pairs. The lists of magnitudes of components of double stars in *Harvard Annals*, Volumes 56 and 64, show that, even for some pairs as wide as 30″, the *Henry Draper Catalogue* gives the combined light; and we have assumed this for closer pairs, in default of better data. The correction from the combined light to that of the brighter star depends on the difference of magnitude of the components, for which only rough estimates are usually available. In some cases this cor-

[15] *Ap. J.*, 56, 242, 1922. [16] *Ap. J.*, 57, 294, 1923.

rection appears to have been omitted inadvertently in calculating the spectroscopic parallax; and in two cases (where the *Henry Draper Catalogue* gives the magnitudes of the separate components of a very close pair), to have been applied twice. We are very greatly indebted to Messrs. Adams and Joy, who have most courteously examined a list of suspected discrepancies and cleared up practically every case, so that little or no uncertainty now exists; and the required corrections to the spectroscopic parallaxes have been made (see Table 48, p. 155).

The additional correction to the magnitude of a visual component which is required when this is a spectroscopic binary has been taken at Mount Wilson as $0^m.7$, $0^m.5$, and $0^m.32$, corresponding to magnitude-differences of $0^m.0$, $0^m.5$, and $1^m.0$. When these values were not available, we have adopted a somewhat more summary process, assuming a correction of $0^m.6$ when two spectra are visible and of $0^m.3$ in other cases. The latter correction is doubtless sometimes too great, but, in the general mean, this will tend to compensate for undiscovered spectroscopic binaries.

For some stars, e.g., ζ Herculis, the variable radial velocity listed in Moore's catalogue[17] arises from the motion in the visual orbit, and no correction should be applied. Some cases of possible pulsation (τ Cygni) were also dropped from the spectroscopic binary list.

C. METHOD OF REDUCTION

10. *Elimination of the Distances.*—The chief difficulty in combining the data from different stars arises from the great range in their distances. To take brute means of the parallaxes as they stand would absurdly overweight the nearer stars. The greater part of this trouble may be avoided by dividing the stars into groups of similar spectral class, and therefore presumably similar absolute magnitude, and by reducing the data for each star of the group to the values which they would have if the star were brought to such a distance that it had a standard apparent magnitude, M_0. If the stars of the group were really of the same absolute magnitude, this would completely eliminate the effects of distance. As things are, it sub-

[17] *Pub. Lick Obs.*, 18, 1932.

stitutes for them a much smaller dispersion, depending on the differences of the actual absolute magnitudes from M_0. The "reduced" values of the quantities are:

$$\left.\begin{array}{ll} \text{True parallax} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & p' = fp \\ \text{Trigonometric parallax} \ldots\ldots\ldots\ldots & t' = ft \\ \text{Probable error of same} \ldots\ldots\ldots\ldots & r' = fr \\ \text{Spectroscopic parallax} \ldots\ldots\ldots\ldots & s' = fs \\ \text{Hypothetical parallax for unit mass} \ldots & h'_1 = fh_1 \\ \text{Proper motion} \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \mu' = f\mu \end{array}\right\} \qquad (17)$$

where

$$f = 10^{0.2(m_b - M_0)} \qquad (18)$$

and m_b is the apparent visual magnitude of the brighter component of the pair.[18]

Giants and main-sequence stars were separated, from class G0 onward, and also the few subgiants of classes G–K (identified by their Mount Wilson absolute magnitudes).

Two supergiants[19] were omitted. They are too few for statistical discussion, and the relative motions are not well determined. Their reduced parallaxes, etc., are so small that their inclusion among the ordinary giants would have had no sensible effect upon the ratio $\overline{h'_1}/\overline{t'}$, etc., though the inclusion of a single dwarf would be objectionable.

The one white dwarf which is the principal component of a binary (40 Eridani B) was kept separate.

For the standard magnitudes M_0, the mean visual absolute magnitudes of stars of the group considered were adopted. From class A6 onward, these were taken from the Mount Wilson spectroscopic data. We are indebted to Mr. Joy for a very convenient summary of these. For the earlier classes, a compromise was made between the results of Strömberg[20] and those of the Mount Wilson spectroscopic

[18] The correction to m_b, necessary when the brighter visual component is a spectroscopic binary, has been applied. See § 9.

[19] Polaris and Antares.

[20] *Ap. J.*, 75, 115, 1932; *Mt. W. Contr.*, No. 442.

observers.[21] The resulting system of assumed magnitudes is given in Table 1. It should be noted that, since the final masses depend on the ratio of the reduced mean values h_1' and p', an error in the assumed M_0 is completely eliminated. It is convenient, however, to have them nearly correct, for then the "reduced" values of the observed parallax should be close to 0″100 (barring accidental errors and real differences in luminosity), which affords a valuable check.

TABLE 1

STANDARD ABSOLUTE MAGNITUDES*

Main Sequence				Giants		Subgiants	
Sp.	M_0	Sp.	M_0	Sp.	M_0	Sp.	M_0
B0	−2.8	G5	+ 4.7	F5	(+0.7)	....	....
B5	−0.7	K0	+ 5.5	G0	+ .7	G0	(+2.8)
A0	+0.7	K5	+ 6.7	G5	+ .7	G5	+2.5
A5	+1.7	M0	+ 8.4	K0	+ .5	K0	+2.2
F0	+2.5	M2	+ 9.3	K5	+ .1	K5	+1.9
F5	+3.2	M5	+11.6	M0	− .2	....	....
G0	+3.9	....	....	M5	−0.4	....	....

* The same method of reduction, and substantially the same values of M_0, have been used in the discussion of spectroscopic and trigonometric parallaxes (*Ap. J.*, 87, 389, 1938; *Mt. W. Contr.*, No. 589) which was written later than the first draft of the present work. As the stars grouped together cover but a very narrow range of spectral type, the correction from visual to bolometric magnitude is substantially the same for all.

11. *Probable Errors.*—It is now necessary to estimate, as reliably as possible, the probable errors which attach to the reduced parallaxes.

For the trigonometric parallaxes, which are sometimes negative, the ratio of the probable error to the observed parallax itself is not a reliable criterion. But the ratio of the reduced probable error to the standard reduced parallax for the group—which with our system of reductions is approximately 0″10—gives a fair measure of the relative precision of the results. For an ideal method of eliminating the distance, the factor f should be 0″10/p. The adopted values of f are too great for stars brighter (absolutely) than M_0, so that r' is large and the resulting weight too small. The reverse is true for fainter stars. This will make the final mean values correspond to a some-

[21] *Ap. J.*, 56, 242, 1922; *Mt. W. Contr.*, No. 244; *Ap. J.*, 57, 294, 1923; *Mt. W. Contr.*, No. 262.

what smaller luminosity than in the ideal case but should not perceptibly influence the derived relation between mass and luminosity.

For the spectroscopic parallaxes Adams and his colleagues state[22] that the accidental error in the absolute magnitude corresponds to a probable error somewhat less than $\pm 0^m.4$, or one of ± 18 per cent in the parallax. The calibration is so adjusted that the arithmetical means of the spectroscopic parallaxes agree with the best available determinations of the true value.

A recent investigation[23] confirms this systematic agreement but shows that the method of calibration has unavoidably underestimated the accidental errors, which correspond to a mean error of ± 38 per cent, or a probable error of ± 26 per cent, for giants and main-sequence stars alike.

For the dynamical parallaxes evidence may be obtained from the revision of the older values, already mentioned. New observational data were included for a large number of stars. These were plotted on the cards of our working catalogue, and new values of the relative motion were determined whenever a change appeared desirable. The division of the data into three grades—good, fair, and poor—was also revised, these terms describing the character of the determination of the observed motion, upon which the parallax depends. The number of stars for which revised values were computed is approximately 26 per cent of the whole.

TABLE 2

Grade	Number	Mean d/d_0	Average Deviation
Good.........	237*	0.974	±0.13
Fair..........	166	.970	.19
Poor..........	52	0.989	±0.29

* The large proportion of "good" stars requiring revision arises from the fact that, in the earlier work, unpublished observations up to date were added only in the more doubtful cases.

The new dynamical parallaxes (d) compare with the old (d_0) as shown in Table 2. The new parallaxes average slightly smaller than

[22] *Ap. J.*, 81, 192, 1935; *Mt. W. Contr.*, No. 511, p. 6.

[23] Russell and Moore, *Ap. J.*, 87, 389, 1938; *Mt. W. Contr.*, No. 589 (referred to hereafter as "II").

the old, presumably because the inclusion of new material diminishes the influence of the errors of observation, which, on the average, tend to exaggerate the relative motion. The average deviation, regardless of sign, of the ratio d/d_0 from unity, naturally increases for the lower grades. It should be remembered that these are selected cases in which the new observations are more or less discordant with the old. For the average of all pairs, the change would be considerably less.

The inherent statistical probable error of the dynamical parallax of a slow-moving pair (arising from the use of average values of the projection factors) is ±25 per cent (p. 120, § 63). Except for the "poor" cases, the changes produced by the revision are therefore of no statistical importance.

There are 14 cases, out of more than 450, in which the revision changed the parallax by a factor of 2 or more in either direction. Two of these arose from misprints in the catalogues first used, and one from the inadvertent inclusion, in a mean, of a wildly discordant measure. For the remaining 11, the average revised parallax is 0."0030; and the average difference, regardless of sign, between the old and the new values, ±0."0034.

As a further test of the effect of observational errors on the dynamical parallax, least-squares solutions for the motions in angle and distance were made for 41 giant stars for which trigonometric parallaxes were available. The resulting dynamical parallaxes d' compare with the revised graphical solutions d as shown in Table 3.

TABLE 3

Grade	Number	Mean d'/d	r_0	r'
Good..........	20	1.06	±0.064	±0.051
Fair............	12	0.93	.17	.21
Poor...........	9	1.12	±0.18	±0.36

Here r_0 denotes the probable error of one of the quantities d'/d, derived from their individual deviations from the mean, while r' represents the mean-square probable error arising from that of the observed relative motion, w, as found in the least-squares solutions.

Calling the latter r_w, we have $r' = \frac{2}{3}r_w/w$ (since r' is a percentage error and d varies as $w^{2/3}$).

The graphical solutions differ from the analytical, on the average, by little more than the probable errors of the latter and by much less than the statistical error except for the "poor" group. Combining the statistical and observational errors, we have for the total probable error (in terms of the true parallax of the system): good, ± 28 per cent; fair, ± 35 per cent; poor, ± 46 per cent—so that their weights should be roughly as 3:2:1. The observed motions in these pairs are exceptionally slow, and the observational errors are more important than for most others.

For stars with "good" orbits the dynamical parallaxes are much more accurate. The statistical scatter of the masses around the values given by the mass-luminosity relation has but a small effect, corresponding to a probable error of ± 4 per cent in the parallaxes.[24] The uncertainties of the orbital elements are more serious but are difficult to evaluate numerically. To get an idea of them, the values of h_1, computed from the orbits finally adopted as the best, were compared with those derived from the earlier orbits entered on our cards, and we chose the most discordant value among these (except for a few cases where obvious errors existed). Dividing the orbits into three grades, and separating the larger and smaller parallaxes in the "good" group, we secured the results given in Table 4. Here "A.D." denotes the average deviation, regardless of sign, of an individual ratio from the mean; and "P.E.," the corresponding probable error.

TABLE 4

RATIOS OF OLD TO NEW VALUES OF h_1

Grade	h_1	No.	Mean Ratio	A.D.	P.E.
Good..........	$>0''.05$	25	0.98 ± 0.007	±0.043	±0.036
Good..........	<0.05	30	$1.03\pm\ .012$	.080	.068
Fair..........	All	24	$1.14\pm\ .04$	.21	.18
Poor..........	All	8	1.17 ± 0.14	±0.44	±0.40

For the poorer orbits the new parallaxes are smaller than the old—probably because the early distances were usually measured too great.

[24] I, p. 96; also § 23.

For the "good" orbits this does not happen; but the accidental error is larger for the small parallaxes, for obvious reasons.

The differences between the old and new orbits represent the effect of including about twenty years' additional observations. The probable errors of the parallaxes derived from the new orbits are presumably at least as large as those given in Table 4. A dynamical parallax derived from a "fair" orbit is therefore better than a spectroscopic parallax, and one from a "poor" orbit is little better than a "poor" dynamical parallax where no orbit is known.

12. *Combined Weights.*—A consistent system of weights, inversely proportional to the squares of their percentage probable errors, may now be assigned to the various data. A "good" dynamical parallax was assigned weight 1, corresponding to a probable error of ± 28 per cent. The "fair" and "poor" determinations then have weights $\frac{2}{3}$ and $\frac{1}{3}$. Dynamical parallaxes derived from "good," "fair," and "poor" orbits have been given weights 10, 2.5, and 0.67. The theoretical weights for some wide and well-observed systems, like Sirius and α Centauri, are still greater; but to adopt them would give these stars an overwhelming preponderance in the means, which is statistically undesirable.

The probable error which was originally assigned to the Mount Wilson spectroscopic parallaxes corresponds to the weight 2.5, and this has been used in the computations. For the spectroscopic parallaxes determined elsewhere for stars of class F0 and later, the weight 2.0 has been assumed, and 1.0 for all A and B stars. For the trigonometric parallaxes, the weight differs from star to star and is given by

$$p_t = \left(\frac{0''.028}{r'}\right)^2, \tag{19}$$

where r' is the reduced probable error. The same process has been applied to the cluster parallaxes, the error being estimated in a few cases where it is not published.

In the final means, all the data for each star must evidently be given the same weight, which corresponds to the accuracy with which the *ratio* of the dynamical and spectroscopic (or other) paral-

laxes can be determined. Since all our weights are assigned on a percentage basis, this weight p is given by the usual equation

$$p = \frac{p_h p_s}{p_h + p_s}, \tag{20}$$

where p_h and p_s are the weights of the dynamical (h_1) and spectroscopic determinations, respectively. For the trigonometric comparisons a different weight, involving p_t instead of p_s, must be used. In this case the weights vary from star to star and were computed individually; but for the spectroscopic parallaxes only a few values of p_h and p_s appear. The weights given by equation (20) are then as shown in Table 5. Since there is no real significance in giving weights

TABLE 5

WEIGHTS FOR SPECTROSCOPIC PARALLAXES

Dyn. Par.	p_h	$p_s=2.5$ p	$p_s=2.0$ p	Adopted p_a	$p_s=1.0$ $p=p_a$
Good	1	0.71	0.67	0.7	0.50
Fair	0.67	.53	.50	.5	.40
Poor	0.33	0.29	0.29	0.3	0.25

to a few per cent, the round numbers (headed p_a in Table 5) were used throughout, with considerable saving of labor.

13. *Corrections to the Weighting System.*—Long after the extensive tabulations based on this weighting system had been completed, it was found that the relative weights assigned to the trigonometric, spectroscopic, and dynamical parallaxes should have been different —though the weighting system for each separately is fairly good. It can be shown (§ 29) that, under these circumstances, the mean error of a parallax of each sort will, nevertheless, be correctly given by the equations, and that the only effect of a recomputation would be to alter to a small degree the relative weights assigned to "good," "fair," and "poor" dynamical parallaxes in deriving the final mean results, so that no revision is necessary.

A more serious complication arises as follows: The statistical errors of equation (13) are independent of the distance of the star, but

this is not true of the observational errors. These are proportionally larger for distant stars and tend also to introduce a systematic error. If the observed relative motions in angle and distance were determined by least squares—or, as was actually the case, by an impartial graphical process which leads to substantially the same results—positive and negative errors are equally likely in the two components or in the total rate of motion w; but the value of w^2, which appears in equation (13), will be systematically too great. If the dispersion in absolute magnitude is small, the modulus $m_b - M_0$ gives a good idea of the actual distance. When this is large, the reduction factor f is great, and errors of observation are much magnified. This does no harm for the trigonometric parallaxes, since the probable error of these is correspondingly multiplied, and the weight accordingly reduced.

In the comparison with spectroscopic parallaxes the weights of both s' and h'_1 were treated as independent of the distance. It would appear desirable to diminish the weight of h'_1 for distant stars; and this would be necessary if we had included all pairs which had been observed over a given interval, regardless of the number or the consistency of the observations.

In fact, however, only well-observed pairs have been given high weight, and badly observed pairs have been rejected. Great care was taken to make the criterion of "goodness" depend on the *accuracy* with which the motion w could be determined (as compared with pairs of similar type) and not on the *amount* of this motion. This policy should result automatically in the exclusion of most of the pairs in which the derived value of w would be due largely to errors of observation, and so should diminish the difficulty.

14. *Observational Selection.*—It emphasizes, however, a more serious difficulty arising from the well-known concentration of the interest of double-star observers, during most of the past century, upon "moving" pairs. There can be no doubt that the "fixed" pairs have been systematically neglected. It might be possible to remove much of the effect of this neglect by systematic reobservation (largely by photography) of all the stars which have been well observed at a distant date—notably all the Struve stars. But the computer, at the present time, must take what there is.

The effect of this type of selection, when seriously present, is strongly systematic, favoring large values of w, and thus of h_1. It should be met by rejecting the vitiated results (if possible) rather than by changes in the weights. This is discussed in § 35.

These difficulties appear only when the statistical process has to be used. Even a poor orbit usually corrects pretty well for the foreshortening factors, and the accidental errors of measurement are diminished in the determination of a, much as in any ordinary mean. Systematic errors in the distances may, however, still be present in pairs which are always very close.

D. SOURCES OF ERROR AND CORRELATIONS

15. *Statement of the Problem.*—Accuracy in an investigation such as the present one depends upon a full study of the various sources of error which are present in the data, and of the correlations between them.

In general, we may write

$$t = p(1 + x') \; ; \quad s = p(1 + y') \; ; \quad h_1 = \mathrm{M}^{1/3} p(1 + z') \, , \quad (21)$$

where x', y', and z' represent the combined influence of all errors—accidental, systematic, and statistical. If M_b is the absolute magnitude corresponding to the observed magnitude m_b and the true parallax p, $M_b = m_b + 5 + 5 \log p$, whence by (18)

$$f = \frac{1}{10p} \, 10^{0.2(M_b - M_0)} \, . \qquad (22)$$

Set

$$10^{0.2(M_b - M_0)} = 1 + w' \, . \qquad (23)$$

Let us also set

$$\mathrm{M}^{1/3} = \mathrm{M}_0^{1/3}(1 + u') \, , \qquad (24)$$

where M_0 is a mean value to be specified later. We then have, by (17), for every star

$$\left.\begin{aligned} t' &= 0.1(1 + x')(1 + w') \, , \\ s' &= 0.1(1 + y')(1 + w') \, , \\ h_1' &= 0.1\mathrm{M}_0^{1/3}(1 + u')(1 + z')(1 + w') \, . \end{aligned}\right\} \qquad (25)$$

We have now to express these equations in terms of variates between which the correlation is zero, or at least very small. This is already the case for x', z', and w'; the first arises from errors of parallax photographs, the second from those of double-star measures and from the foreshortening effect for slow-moving pairs, and the third from differences in absolute magnitude.

16. *Effects of Mass-Luminosity Relation.*—There is, however, a correlation between u' and w'. For single stars, u' may be resolved into two components, one arising from differences in mass which are in accordance with the mass-luminosity relation, and the other from the outstanding individual discordances.

The mass-luminosity relation is here taken as an *empirical* fact—which is explained in principle by Eddington's theory.[25] For those stars for which reliable data are available, the deviations from this semi-empirical relation are very small (except for white dwarfs), and their mean value does not differ significantly from zero (see § 23). Differences which are in accord with this relation will be completely correlated with w'. The empirical relation may be sufficiently represented by

$$\mathfrak{M} = CL^n\,, \tag{26}$$

where L is the luminosity and C and n are constants (p. 112, § 60). If L_0 and $\mathfrak{M}_0$ correspond to M_0, $L = L_0(1 + w')^{-2}$, giving

$$\mathfrak{M}^{1/3} = \mathfrak{M}_0^{1/3}(1 + w')^{-2n/3}\,. \tag{27}$$

We may then set

$$1 + u' = (1 + u)(1 + w')^{-2n/3}\,, \tag{28}$$

where u represents the discordance from the mass-luminosity relation—that is, the differences in mass among stars of the same absolute magnitude—arising presumably from differences in internal composition or constitution. These are quite uncorrelated with w', and doubtless also with x' and z'. We then have

$$h_1' = 0.1\mathfrak{M}_0^{1/3}(1 + u)(1 + z')(1 + w')^{1-2n/3}\,. \tag{29}$$

[25] *The Internal Constitution of the Stars*, pp. 137 and 153, Cambridge, 1926.

We may make the mean $\bar{u}$ vanish by defining $\text{M}_0^{1/3}$ as the mean of $\text{M}^{1/3}$ for all stars of our group which have the absolute magnitude M_0. In the present work we have to deal with groups of stars within very narrow ranges of spectral type. In such a group the visual and bolometric differences $M - M_0$ will be substantially the same; and the values of n may be derived from Eddington's equations,[26] which give $n = d \log \text{M}/d \log L = (20 - 15\beta)/(28 - 6\beta)$, and thence,

M bol.......	12.5	10	7.5	5	2.5	0	−2.5	−5
n...........	0.23	0.23	0.24	0.26	0.29	0.37	0.48	0.59

For main-sequence stars, M bol is +0.9 for A0, and +6.9 for M0, and n ranges from 0.34 to 0.24. The mean value 0.27 may be adopted from A5 to M5. For typical giants M bol is +0.4 for G2 and −2.7 for M2, giving $n = 0.36$ and 0.49. It suffices to adopt a mean value 0.42.

The theoretical values were adopted early in the present work. Its final results (p. 89, Eq. [94]) indicate that $n = 0.30$ gives a satisfactory representation for all binaries so far observed.

Equation (28) holds good for the brighter component of the pair. For the fainter, we may use the same value of w', which amounts to assuming that the standard absolute magnitude used for deriving f is $M_0 + \Delta m$, where Δm is the observed magnitude difference. The value of u is then definite and may not be the same as for the brighter component.

We now have

$$\text{M}_1 = \text{M}_{01}(1 + u_1)^3(1 + w')^{-2n}\,; \qquad \text{M}_2 = \text{M}_{02}(1 + u_2)^3(1 + w')^{-2n}\,,$$

whence

$$\text{M} = \text{M}_0(1 + u)^3(1 + w')^{-2n}\,,$$

where M_0 is the sum of the masses corresponding to the absolute magnitudes M_0 and $M_0 + \Delta M$ and where $(1 + u)^3$ is a mean of $(1 + u_1)^3$ and $(1 + u_2)^3$ weighted according to the masses.

Unless the fainter component is a white dwarf, u is likely to differ very little from u_1. Barring such exceptional cases, we should have

[26] *Ibid.*, p. 135.

$\bar{u} = 0$. It should be especially noted that the *bolometric* magnitudes (corrected for Eddington's temperature factor) are supposed to be employed in determining M_0, though visual magnitudes may perfectly well be used in calculating f.

17. *Correction of Spectroscopic Parallaxes.*—Spectroscopic parallaxes are not direct determinations but depend upon calibration of the spectral criteria with the aid of other data, such as trigonometric parallaxes and proper motions. The authors have shown[27] that the method of calibration which was used (and which was the reasonable one to use) gives correct values for the mean of the spectroscopic absolute magnitudes (or the closely related quantities s') for the stars of a given spectral subclass, but that it leads inevitably to a serious underestimate of the differences of these quantities from the mean.

The effects of this may be represented with sufficient accuracy by setting, in place of equation (25),

$$s' = 0.1\{1 + l(y' + w' + y'w')\}\,, \tag{30}$$

where l is a constant, considerably less than unity, which must be determined empirically by comparison of spectroscopic with trigonometric or other parallaxes.

Here y' represents the combined effect of the accidental errors of the spectroscopic estimates and of the real differences in absolute magnitude among stars whose spectra are similar in the criteria for absolute magnitude as well as those for spectral class. It is well known that identity of these criteria demands (to the first approximation theoretically) that the stars shall have the same surface brightness and surface gravity (whence M/L = constant). It tells us only that the point representing the star on a mass-luminosity diagram lies on a certain straight line. Spectroscopic determination of parallax is possible because the points are strongly clustered along a mean mass-luminosity curve. The intersection of the line with this curve gives the "spectroscopic luminosity." Let M_1, L_1 be a point on this curve. The approximate equation of the curve in its vicinity is

[27] II. This conclusion has been confirmed by van Rhijn, *Groningen Pub.*, No. 49, p. 30, 1939.

$M/M_1 = L^n/L_1^n$. Consider a star of luminosity L_1 but of mass $M_1(1+u)^3$. The spectral criteria give

$$\frac{M}{M_1(1+u)^3} = \frac{L}{L_1}.$$

The spectroscopic luminosity derived for this star will then be $L_2 = L_1(1+u)^{3/(n-1)}$; and the spectroscopic parallax, computed on the assumption that the light is L_2 when it is really L_1, will be multiplied by the factor $(L_1/L_2)^{1/2}$ or $(1+u)^{3/(2-2n)}$.

If y'' represents the accidental error of the spectroscopic estimates, we then have $1 + y' = (1 + y'')(1+u)^{n'}$, where

$$n' = \frac{3}{2-2n}, \tag{31}$$

or, to the first order,

$$y' = y'' + n'u. \tag{32}$$

There is no reason to suppose that y'' is sensibly correlated with u, x', z', or w', or, therefore, that y' is correlated with the last three.

The spectroscopic parallaxes s', as tabulated, give very nearly correct mean results for groups of stars selected according to the spectroscopic absolute magnitude. But when the selection has been made by criteria independent of these spectroscopic peculiarities (as is the case for the double stars here considered), a correction is necessary to reduce them to the system defined by the trigonometric parallaxes. This may be made by the equation[28]

$$s'' = s' + C(s' - s_1), \tag{33}$$

where $C = (1/l) - 1$. For giant stars, $C = 0.6$, $s_1 = 0''.094$, while for those of the main sequence $C = 1.6$ and s_1 has the following values:

Sp.	A5	F2	F7	G2	G8	K4	M2
s_1	0''.108	0''.107	0''.104	0''.104	0''.100	0''.104	0''.101

Equation (33) may be applied directly to the means, $\bar{s'}$.

[28] II, p. 418.

18. *Trigonometric Errors.*—We have now reached a system of variates x', y'', z', w', and u which should be free from sensible correlations, and we may consider their mean values and standard deviations.

The errors x arise solely from the measures of the parallax observations. The systematic errors of t have been very carefully investigated and corrected by Schlesinger. The parallaxes of his catalogue[29] form a homogeneous system, and later investigations have confirmed his conclusion: "I regard it as more probable than not that the correction needed to make this system absolutely true as a whole does not exceed 0."001." The probable errors given in the catalogue have also been carefully adjusted, and there is strong evidence[30] that they fairly represent the real values of the accidental errors.

In general, however, let us set

$$(1 + x') = T(1 + x) , \tag{34}$$

where $T = 1 + \overline{x'}$ and $\bar{x} = 0$. Here T is the mean ratio of the trigonometric to the true parallax. We will assume $T = 1$, which amounts to adopting Schlesinger's system as a standard; but it should be kept in the equation for the present.

19. *Errors of Double-Star Data.*—The values of z' arise from two sources—the errors of double-star observations, and the foreshortening effect in slow-moving pairs. The constants for these have been so adjusted that $\overline{z'} = 0$ for a random geometrical distribution—and for a certain assumed distribution of orbital eccentricities derived from visual binary orbits. Neglect of "fixed" pairs by observers may vitiate this assumption and produce a positive value. The mean value of the observational part of z' will vanish if there is no general systematic tendency of observers to measure distances too great, or too small, or to exaggerate slow motions. This last effect is doubtless present for the most distant stars and has been partly allowed for (§ 35).

[29] *General Catalogue of Stellar Parallaxes*, Yale University Observatory, ed. 1935.

[30] II, p. 422.

We may then set

$$1 + z' = H(1 + z) , \tag{35}$$

where H may be slightly greater than 1 but cannot be exactly estimated a priori, and $\bar{z} = 0$.

20. *Dispersion in Absolute Magnitude.*—The quantity w' depends on the real deviations in absolute magnitude from the standard value M_0 and on the observational errors of the apparent magnitude m_b. The dispersion of the former is of the order of $\pm 0^m.7$ even for stars of the same spectral subclass,[31] while the standard deviation of the latter is less than $\pm 0^m.1$ for photometric determinations and probably less than $\pm 0^m.3$ even for *Bonner Durchmusterung* magnitudes (with the Harvard corrections). For statistical purposes the observational errors of m_b may therefore be neglected.

Let us now set $1 + w' = C(1 + w)$, where $\bar{w} = 0$. By equation (23) $M_b - M_0 = 5 \log_{10} (1 + w') = 2.17\, ln(1 + w') = 2.17(lnC + w - \frac{1}{2}w^2 + \ldots .)$, whence, in the mean,

$$\overline{M_b} - M_0 = 2.17(lnC - \tfrac{1}{2}W^2 + \ldots .) ,$$

where W is the standard deviation of w. If we set

$$F = 10^{0.2(\overline{M_b} - M_0)} , \tag{36}$$

we have

$$1 + w' = F(1 + \tfrac{1}{2}W^2)(1 + w) . \tag{37}$$

We now have from (25), (34), and (37)

$$t' = 0.1\, TF(1 + \tfrac{1}{2}W^2)(1 + x)(1 + w) . \tag{38}$$

The mean is

$$\overline{t'} = 0.1\, TF(1 + \tfrac{1}{2}W^2) , \tag{39}$$

whence

$$t' = \overline{t'}(1 + x)(1 + w) . \tag{40}$$

[31] II, p. 413.

The spectroscopic parallaxes are reduced to the trigonometric system by the correction (33), so that, apart from individual accidental errors, $\overline{s'} = \overline{t'}$. Then by (33)

$$s'' = \overline{t'} + \frac{s' - \overline{s'}}{l} .$$

But by (30), (37), and (38)

$$s' = 0.1(1 - l) + \frac{l\overline{t'}(1 + w)(1 + y')}{T} .$$

Setting $T = 1$ and $y' = y + \overline{y'}$, we have

$$s'' = \overline{t'}(1 + w + y + wy + w\overline{y'} + \overline{wy}) .$$

The last two terms are small, and we may set

$$s'' = \overline{t'}(1 + y)(1 + w) , \tag{41}$$

where $\bar{y} = 0$.

21. *Final Equations.*—From (29) and (35)

$$h_1' = 0.1\ M_0^{1/3}H(1 + u)(1 + z)(1 + w')^{1-2n/3} . \tag{42}$$

Setting

$$1 - \tfrac{2}{3}n = k , \tag{43}$$

we find, without difficulty,

$$(1 + w')^k = F^k(1 + \tfrac{1}{2}k^2W^2)\{1 + kw + \tfrac{1}{2}k(k - 1)(w^2 - W^2)\} .$$

The last term in the braces vanishes in the mean and has the small factor $\frac{1}{2}(k - 1) = -\frac{1}{3}n$. It may be neglected.

Let M_c be the combined mass, corresponding, according to the mean mass-luminosity relation, to the absolute magnitudes $\overline{M}_b$ and $\overline{M}_b + \Delta M$. Then by (22), (27), and (36), $M_c^{1/3} = M_0^{1/3} F^{-2n/3}$. Substituting in (42),

$$h_1' = 0.1 M_c^{1/3}HF(1 + \tfrac{1}{2}k^2W^2)(1 + u)(1 + z)(1 + kw) . \tag{44}$$

This equation has here been derived on the assumption that the mass-luminosity relation is given by (26); but, if M_b had been

adopted as standard in the reductions instead of M_0, it would obviously have been true, with $F = 1$, whatever the form of this relation. The choice of M_0 simply multiplies every value of h' by the constant factor F given by (36). Hence, (44) is generally true. From star to star M_c is not the same, for ΔM differs though $\overline{M_b}$ is fixed. If M_b is the mass corresponding to $\overline{M_b}$, we may set $\mathrm{M}_c^{1/3} = \mathrm{M}_b^{1/3}(1 + o')$, where o' is a function of ΔM and $\overline{M_b}$, defined by the mass-luminosity law. If we set

$$\overline{\mathrm{M}^{1/3}} = \mathrm{M}_b^{1/3}(1 + \overline{o'}) , \tag{45}$$

we may write

$$\mathrm{M}_c^{1/3} = \overline{\mathrm{M}^{1/3}}(1 + o) ,$$

where $\bar{o} = 0$; whence

$$h_1 = 0.1\overline{\mathrm{M}^{1/3}}HF(1 + \tfrac{1}{2}k^2W^2)(1 + o)(1 + u)(1 + z)(1 + kw) . \tag{46}$$

If the peculiarities of the components in absolute magnitude are not correlated, unusual brightness in the principal component (increasing u or w, or both) will increase ΔM and decrease o. In practice, o is small, and this correlation may be neglected. We may then set

$$\overline{h_1'} = 0.1\overline{\mathrm{M}^{1/3}}HF(1 + \tfrac{1}{2}k^2W^2) , \tag{47}$$

$$h_1' = \overline{h_1'}(1 + o)(1 + u)(1 + z)(1 + kw) . \tag{48}$$

E. THE DISPERSION CONSTANTS

22. *Magnitude Differences.*—The mean-square values O, U, W, X, Y', and Z of the variates o, u, w, x, y', and z may be found in various ways.

The first depends only on the distribution of the magnitude differences ΔM. If $\mathrm{M}(M)$ denotes the mass corresponding, by the mass-luminosity relation, to absolute magnitude M,

$$(1 + o')^3 = 1 + \frac{\mathrm{M}(M + \Delta M)}{\mathrm{M}(M)} .$$

The value of o' is always 0.260 when $\Delta M = 0$, and 0.000 if $\Delta M = \infty$. For other values it depends slightly upon M. For $M = 0$ and $M = 5$

the maximum difference occurs when $\Delta M = 3.0$, the values of o' being 0.149 and 0.117.

There are 85 pairs in the list for which the visual Δm has been measured photometrically; and both spectra are known, so that the bolometric ΔM may be accurately determined. Taking the means of o' for $M = 0$ and $M = 5$, we find for these stars $\overline{o'} = 0.180$ with a standard deviation σ of ± 0.058 for the single case. The corresponding value of O is ± 0.049.

For 124 binaries for which orbits have been computed, using the visual Δm, it is found that $o' = 0.209$, $\sigma = \pm 0.051$, and $O = \pm 0.042$.

23. *Orbits; Deviations from Mass-Luminosity Relation.*—For reliable orbits the combined effect of z and u is very small. It has been determined anew from those binaries which have good orbits, and from trigonometric parallaxes whose probable error r_t is less than $0.1d$[32] (where d is the dynamical parallax derived as in I [selection according to r_t/t would have introduced a systematic error]). There are 24 such stars, but one—o_2 Eridani BC—is a white dwarf and must be excluded. Data for the remaining 23 are given in Table 6. The second column gives the "hypothetical" parallax for mass unity, h_1; the third, the trigonometric parallax and its probable error (all in units of 0".001). Next follow the bolometric absolute magnitude of the brighter component, M_b, corresponding to t, with bolometric correction from I, Table A, and the corresponding mass $\mathfrak{M}_b$ by Eddington's formula.[33] When the spectral type and bolometric correction are known for the fainter component, its mass can be similarly determined; but the mass ratio $\mathfrak{M}_f/\mathfrak{M}_b$ has been directly determined for all but four of the stars,[34] while there are 14 for which the spectral type of the companion would have to be estimated from the change with absolute magnitude along the main sequence. The mass of the companion has therefore been derived from the observed mass ratio whenever possible.[35] The estimated

[32] I, p. 96.

[33] *The Internal Constitution of the Stars*, p. 137, with correction $+0^m.50$ to the absolute magnitudes (I, p. 96).

[34] See Huffer, *Ap. J.*, 80, 269, 1934; Pitman, *A.J.*, 44, 65, 1935; Merrill, *Ap. J.*, 56, 40, 1922; *Mt. W. Contr.*, No. 240.

[35] This also avoids trouble with the white dwarf Sirius B.

values are in brackets. Both components of ξ Ursae Majoris are spectroscopic binaries—the only ones in the list. The mass ratios in the close pair Aa and the wide pair Aa, Bb are known, the latter

TABLE 6

VISUAL BINARIES WITH $r_t/d<0.10$

Name	h_1*	t*	Bol. M_b†	Eddington's Formula				Linear Formula	
				$\mathfrak{M}_b$	$\mathfrak{M}_f/\mathfrak{M}_b$	$h_1\mathfrak{M}^{-1/3}$	D	$h_1\mathfrak{M}^{-1/3}$	D
α CMa........	563	373±2	+ 1.52	2.65	0.39	365	−0.02	358	−0.04
α Cen........	949	756±7	+ 4.75	1.10	0.85	747	− .01	741	− .01
α CMi........	362	291±4	+ 3.04	1.70	0.35	274	− .06	268	− .08
α Aur........	123	63±1	− 0.21	4.77	0.79	60	− .05	61	− .03
Kr 60........	188	258±4	+10.2	0.33	0.59	234	− .09	255	− .01
70 Oph........	229	196±4	+ 5.47	0.93	0.82	192	− .02	192	− .02
η Cas........	195	182±5	+ 5.09	1.01	0.32	177	− .03	176	− .03
ξ Boo........	171	147±6	+ 5.64	0.89	0.92	143	− .03	143	− .03
β 416........	150	147±6	+ 6.29	0.77	0.75	136	− .08	138	− .06
ξ UMA........	166	138±6	+ 5.50‡	0.92	0.77	128	− .07	128	− .07
μ Her (BC)........	105	109±6	+ 8.9	0.43	1.00	110	+ .01	117	+ .07
ζ Her........	127	110±5	+ 3.35	1.58	0.43	97	− .12	95	− .14
δ Equ........	84	60±4	+ 4.24	1.25	[0.90]§	63	+ .05	62	+ .03
9 Pup........	84	60±4	+ 4.76	1.10	0.89	66	+ .10	65	+ .08
44 Boo........	102	79±5	+ 4.86	1.07	0.64	85	+ .08	84	+ .06
Σ 2173........	82	50±4	+ 4.46	1.18	[0.97]§	62	+ .24	61	+ .22
99 Her........	78	49±5	+ 3.88	1.37	0.43	62	+ .26	61	+ .24
β 648........	83	58±5	+ 4.17	1.27	0.54	66	+ .14	65	+ .12
85 Peg........	92	86±6	+ 5.62	0.89	0.82	78	− .09	78	− .09
γ Vir........	116	89±7	+ 3.71	1.43	1.00	82	− .08	80	− .10
ζ Cnc........	60	39±4	+ 3.68	1.45	[0.86]§	43	+ .10	42	+ .08
τ Cyg........	70	50±5	+ 2.72	1.86	0.67	48	− .04	47	− .06
β 395........	77	72±6	+ 5.62	0.90	[0.96]§	64	−0.11	64	−0.11

* Unit of parallaxes = 0.001.

† Bolometric absolute magnitude of brighter component corresponding to t.

‡ Spectroscopic binary correction applied.

§ Brackets indicate mass ratio calculated from Δm with estimated spectral class of fainter component.

being the tabular $\mathfrak{M}_f/\mathfrak{M}_b$. The entries M_b and $\mathfrak{M}_b$ refer to component A. The faint component of 44 Bootis is an eclipsing binary. The observed $\mathfrak{M}_f/\mathfrak{M}_b$ refers to the combined mass of the pair.

Having thus found the total mass, $\mathfrak{M}$, we compute $h_1\mathfrak{M}^{-1/3}$. This should equal t if the mass-luminosity relation were exact and there were no errors of observation. The proportional residuals D =

$(h_1\mathrm{M}^{-1/3} - t)/t$ are given in the eighth column. Their mean-square value is the observed coefficient of variability, v'. They are affected by the errors in t, both directly and indirectly through M, since

$$\frac{d\mathrm{M}}{\mathrm{M}} = n\frac{dL}{L} = -\ 2n\frac{dt}{t}\,,$$

so that

$$dD = (\tfrac{2}{3}n - 1)\,\frac{dt}{t}\,.$$

With the adopted value $n = 0.27$, $dD = -\ 0.82(dt/t)$. In the last two columns of the table are given $h_1\mathrm{M}^{-1/3}$ and D, computed with the empirical linear formula $\log \mathrm{M}_b^{1/3} = -\ 0.040\ (M_b - 5.20)$ (eq. [94], § 66), which has been finally adopted for the dynamical parallaxes of the present work. The differences are insignificant. The coefficient of variability, v_t, arising from the errors of the observed parallaxes is the mean-square value of $r_t/0.675t$. If $v^2 = v'^2 - (0.82v_t)^2$, v will include the errors of double-star observation (z) and the departures from the mass-luminosity law (u). We find the following:

r_t/d	No.	Eddington's Formula				Linear Formula		
		v'	v_t	v	$\bar{D}$	v'	v	$\bar{D}$
<0.05......	12	±0.060	±0.048	±0.045	−0.047	±0.061	±0.047	−0.037
0.05–0.10...	11	± .136	± .123	± .091	+ .060	± .126	± .078	+ .043
All.........	23	±0.103	±0.092	±0.070	+0.009	±0.098	±0.062	+0.001

The last values may be taken as representing the combined effect of z and u for good orbits. The departures from the mass-luminosity relation are very small. The algebraic means, $\bar{D}$, given in the sixth and ninth columns, indicate no certain correction to the zero-point. It is probable that U does not exceed 0.05. Inclusion of o, which has been eliminated in the foregoing calculation, would raise the mean-square error, resulting from z, u, and o together, for these stars to 0.086, taking O as 0.05.

These orbits are more accurate than the general run of those called "good," so that the combined effect for the average good orbit is probably 0.10 or more. But good orbits have received weight 10

on our system; hence, we may adopt provisionally $Z_1^2 = 0.10$ for unit weight. This includes the effects heretofore called z, u, and o.

24. *Physical Pairs; Statistical Errors.*—For slow-moving pairs, when the statistical process is used, z has a nearly normal distribution with a probable error of 20.4 per cent, and $v = \pm 0.30$ (I, p. 94). The error of u is negligible in comparison, but an addition should be made for observational error. From Table 3 we may estimate the corresponding coefficients of variability as ± 0.08 for cases graded "good," ± 0.31 for "fair," and ± 0.50 for "poor," making the resultant $v = \pm 0.31$, ± 0.43, and ± 0.55 in the three cases. With the weights previously assigned in the reductions (1, $\frac{2}{3}$, and $\frac{1}{3}$), this gives, for unit weight, $v = \pm 0.31$, ± 0.35, and ± 0.32. We may adopt $v = \pm 0.32$ and $Z_1^2 = 0.10$. The agreement with the value determined independently for the orbits shows that the original system of weights was a lucky guess.

For these stars, O^2 and U^2 are negligible in comparison with Z^2. Hence, in practice, the influences of z, u, and o may always be lumped under the heading "Z."

25. *Trigonometric Parallaxes; Absolute Magnitudes.*—There is good evidence[36] that the probable errors given in Schlesinger's catalogue correctly represent the accidental errors of the trigonometric parallaxes. The unit of weight adopted for p_t (eq. [19]) corresponds to a reduced probable error $r' = \pm 0''.028$, or a mean error $\pm 0''.0415$. Hence, by (40), we have for unit weight

$$X_1 = \frac{X_0}{10\overline{t}'} , \qquad X_0 = 0.415 . \tag{49}$$

The mean-square coefficient of variability for the adopted unit of weight is therefore $0.172/(10\overline{t}')^2$ for the trigonometric parallaxes and 0.100 for the dynamical. The weighting system which has been adopted gives too low a relative weight to the latter. This was derived from the probable error of ± 28 per cent, adopted for a good dynamical parallax in § 11; but this is not that of h_1 but of $d = n_0 A h_1$,[37] where the factor $n_0 A$, derived from the mass-luminosity law, is correlated with the statistical errors of h_1 and considerably increases them. To change the weights throughout the main mass of

[36] II, p. 422. [37] I, p. 98.

computations would be a waste of labor; for, as is shown in § 29, it would amount only to a slight change in the relative weighting of the various grades of dynamical parallaxes, which contains a considerable arbitrary element anyhow.

The value of W has been derived in the comparison of trigonometric and spectroscopic parallaxes. From 1140 stars of the main sequence the mean-square value of W was found to be ± 0.344[38] (corresponding to a standard deviation of $\pm 0^m.72$ in the absolute magnitude). The values for such groups by spectral class are all near this mean, with a slight tendency to diminish for the later types, which may here be disregarded. For 732 giants the mean-square W is ± 0.52 (and the standard deviation $\pm 1^m.07$). This larger value arises from the inclusion of a considerable number of supergiants and subgiants. The proportion of the former among the double stars is considerably smaller, so that this value of W should be too great for these. Values of W derived from the double stars themselves have therefore been preferred (§§ 33 and 37).

26. *Spectroscopic Parallaxes*.—The same investigation gave values of l and Y'. For the Mount Wilson spectroscopic parallaxes, all of which were given the same weight, the main-sequence stars gave $l = 0.38$ and $Y' = \pm 0.38$; and for the giants, $l = 0.63$ and $Y' = \pm 0.38$.

By (32) $Y''^2 + n'^2 U^2 = Y'^2$. Adopting $U = \pm 0.05$, we have for the main sequence $n' = 1.37$ and $Y'' = \pm 0.374$, and for the giants $n' = 1.73$ and $Y'' = \pm 0.371$. The correlation between h_1 and s_1'', introduced by the presence of u in both, is therefore practically negligible.

A check upon these values of Y may be obtained from the spectroscopic absolute magnitudes themselves. Counts of the Mount Wilson spectroscopic absolute magnitudes, S, communicated by Mr. Joy, give the distribution of $S - S_0$ for 1707 main-sequence stars of classes A6–K3 and for 1788 giants of classes G0–M7, shown in Table 7. Supergiants and subgiants are excluded. Both distributions are skewed, with the mode at about $+0.1$ and an excess of large negative residuals; but the assumption of a normal distribution would lead to no serious errors.

[38] II, p. 412.

TABLE 7

$S - S_0$	$0^m.0$	$0^m.1$	$0^m.2$	$0^m.3$	$0^m.4$	$0^m.5$	$0^m.6$	$0^m.7$	$0^m.8$	$0^m.9$	$1^m.0$	$1^m.1$	$1^m.2$	$1^m.3$	$1^m.4$	$1^m.5$	$1^m.6$	$1^m.7$	$1^m.8$	$1^m.9$
Giants																				
Pos.	231	249	207	180	123	65	39	14	9	8	6	2	1	...	...	...	...	...	...	...
Neg.	...	189	136	108	74	43	36	34	10	6	9	5	2	2	...	...	...	...	...	...
Main Sequence																				
Pos.	162	166	188	148	102	88	60	44	20	14	8	5	...	1	1	...	...	1	...	...
Neg.	...	142	126	110	83	58	40	37	21	21	18	7	7	11	4	5	4	3	1	1

From these data we have, setting $S - S_0 = \eta$,

Mean Value of	η	η^2	η^3	η^4
Main sequence	+0.001	+0.208	−0.055	+0.179
Giants	+0.029	+0.116	−0.005	+0.050

Now, if s'_0 corresponds to S_0,

$$s' - s'_0 = s'_0 \exp 0.46(S - S_0) \; ;$$

whence, if $a = 0.46$,

$$\begin{aligned} \overline{s'} &= s'_0(1 + a\bar{\eta} + \tfrac{1}{2}a^2\overline{\eta^2} + \dots) \\ &= 1.022 \text{ (main sequence) or } 1.026 \text{ (giants)} \, , \end{aligned}$$

$$\begin{aligned} \frac{\overline{(s' - \overline{s'})^2}}{s_0'^2} &= a^2\{\overline{\eta^2} - (\bar{\eta})^2\} + a^3\{\overline{\eta^3} - \overline{\eta^2}\cdot\bar{\eta}\} \\ &\quad + \tfrac{1}{12}a^4\{7\overline{\eta^4} - 4\overline{\eta^3}\cdot\bar{\eta} - 3(\overline{\eta^2})^2\} \, , \end{aligned}$$

giving 0.043 for the main sequence and 0.023 for the giants. The corresponding value of $l^2(Y^2 + W^2)$ for the main sequence is 0.041. With $l = 0.38$ and $W = \pm 0.34$, we find $Y' = \pm 0.41$. The agreement is close, as it should be, since less than half the data in the present group are independent.

For the giants we find $l^2(W^2 + Y'^2) = 0.022$. With the values previously found for a grouping including supergiants and subgiants, this quantity turns out to be 0.164.

If the regression-curves on which the spectroscopic calibration is based are correctly drawn (as appears to be substantially the case for the giants[39]), we should have $l = W^2/(W^2 + Y'^2)$. Introducing this, we have, with the new and old groupings,

$$W^4 - 0.022\, W^2 = 0.022\, Y'^2\,, \qquad W^4 - 0.164\, W^2 = 0.164\, Y'^2\,.$$

If we adopt the value $Y' = \pm 0.38$, given consistently by the dwarfs and giants, we find $W = \pm 0.51$ for the old grouping, and $W = \pm 0.26$ for the new—from which $l = 0.32$. These values are rather rough, but they illustrate the great effect of deliberate selection by absolute magnitude.

We may adopt for the spectroscopic parallaxes $Y' = \pm 0.38$ and $Y'^2 = 0.144$. The probable error of $\pm$ 18 per cent derived by Adams gives $Y'^2 = 0.071$; but this is too small, on account of the influence of the factor l.

If the unit of weight corresponds to a mean error of ± 0.172, as for the trigonometric parallaxes, we should have the weight of a Mount Wilson spectroscopic parallax 2.4 on the uncorrected data and 1.2 after correction. But, compared with the dynamical parallaxes for which unit weight corresponds to 0.100, the weight of a spectroscopic parallax, corrected as described in § 17, equation (33), should be 0.69.

27. *Equations for Trigonometric Parallaxes.*—The values of W and Y were originally obtained from solutions involving the double stars alone. We start with the equations

$$\left.\begin{aligned} h_1' &= \overline{h_1'}(1 + z)(1 + kw) = \overline{h_1'} + v\,, && \text{weight } p_h\,, \\ t' &= \overline{t'}(1 + x)(1 + w) = \overline{t'} + v'\,, && \text{weight } p_t\,. \end{aligned}\right\} \tag{50}$$

The entries for each star have a weight p, defined by

$$\frac{1}{p} = \frac{1}{p_h} + \frac{1}{p_t}\,. \tag{51}$$

[39] II, p. 414.

We then have, for the residuals for the N stars of our group,

$$\left.\begin{aligned}
\frac{N}{N-1}\frac{[pvv]}{\overline{h}_1'^2} &= [pz^2] + k^2[pw^2] &&= NE\,,\\
\frac{N}{N-1}\frac{[pv'v']}{\overline{t}'^2} &= [px^2] + [pw^2] &&= NE'\,,\\
\frac{N}{N-1}\frac{[pvv']}{\overline{h}_1'\,\overline{t}'} &= k[pw^2] &&= NF\,,
\end{aligned}\right\} \tag{52}$$

where the Gaussian brackets denote summations. Many terms vanish, since there is no correlation; and those of the fourth order are ignored.

The mean-square value of w^2 is always W; that of z^2 is Z_1^2/p_h; and that of x^2 is $X_1^2/p_t = (0.01/t'^2)(X_0^2/p_t)$. Then

$$\left.\begin{aligned}
NE &= \left[\frac{p}{p_h}\right]Z_1^2 + k^2[p]W^2 &&= NQZ_1^2 + NPk^2W^2\,,\\
NE' &= \left[\frac{p}{p_t}\right]X_1^2 + [p]W^2 &&= NQ''X_0^2 + NPW^2\,,\\
NF &= k[p]W^2 &&= NPkW^2\,,
\end{aligned}\right\} \tag{53}$$

where

$$NP = [p],\ NQ = \left[\frac{p}{p_h}\right],\ NQ'' = \left(\frac{0.1}{t'}\right)^2\left[\frac{p}{p_t}\right], \tag{54}$$

or by (51)

$$Q'' = \left(\frac{0.1}{\overline{t}'}\right)^2 Q'\,, \qquad Q' = 1 - Q\,. \tag{55}$$

The results for groups of stars of different spectral classes—and with different values of h_1, etc.—may be combined by adding the equations (53) and thus obtaining means of E, F, P, Q, etc., weighted according to the number of stars in a group, and, then, weighted mean values of the unknowns.

These equations do not suffice to determine the four unknowns X_0, Z, W, and k; but, if any one of them can be evaluated from other evidence, the rest can be found. For example, we may trust the probable errors of the trigonometric parallaxes and set $X_0 = \pm 0.415$;

or we may follow §§ 23 and 24 and set $Z_1^2 = 0.10$; or we may take $k = 1 - \frac{2}{3}n$ (§ 21).

28. *Equations for Spectroscopic Parallaxes.*—For (uncorrected) spectroscopic parallaxes we may set

$$s' = \overline{s'}(1 + ly + lw + lyw) = \overline{s'} + v' \tag{56}$$

and find, as above,

$$\left.\begin{aligned} NE &= \frac{N}{N-1}\frac{[pvv]}{\overline{h'_1}^2} = NQZ_1^2 + NPk^2W^2\,, \\ NE' &= \frac{N}{N-1}\frac{[pv'v']}{\overline{s'}^2} = NQ'l^2Y_1^2 + NPl^2W^2\,, \\ NF &= \frac{N}{N-1}\frac{[pvv']}{\overline{s'}\,\overline{h'_1}} = NPklW^2\,, \end{aligned}\right\} \tag{57}$$

where

$$NP = [p]\,, \quad NQ = \left[\frac{p}{p_h}\right], \quad NQ' = \left[\frac{p}{p_s}\right], \quad Q + Q' = 1\,. \tag{58}$$

(The use of the same symbols with similar, but different, meanings, in equations [53]-[55] and in equations [57]-[58] can lead to no confusion, as the two sets of equations can never appear in the same problem.)

To solve equations (57), two of the unknowns must be found from other sources. The constant l, which is a property of the calibration of the spectroscopic parallaxes, may be taken from II, and Y_1 may be derived from the same source.

29. *Effect of Change of Weights.*—The extensive computations in which the numerical coefficients of these equations were derived were made with erroneous values of the weights. It is therefore of importance to see to what extent this vitiates the derived constants.

In (57) let us set

$$\frac{P}{Q} = P_h\,, \qquad \frac{P}{Q'} = P_s\,. \tag{59}$$

Then $1/P_h = [p/p_h]/[p]$, so that $1/P_h$ is the mean of the individual values of $1/p_h$ with the weights p; and similarly for $1/P_s$. Now let

Z_m and Y_m be the standard deviations of values of h'_1 and s' with the weights P_h and P_s. Then $Z^2_m = Z^2_1/P_h = Z^2_1 Q/P$, etc., and (57) may be written

$$\left.\begin{aligned} Z^2_m + k^2 W^2 &= \frac{E}{P}\,, \\ l^2(Y^2_m + W^2) &= \frac{E'}{P}\,, \\ lkW^2 &= \frac{F}{P}\,. \end{aligned}\right\} \qquad (60)$$

Notice that, if p_s is constant, $P_s = p_s$, and Y_m is the standard deviation for a spectroscopic parallax.

These equations hold good formally, whatever arbitrary positive values are assigned to the weights p_h and p_s and their resultants p; and Z_m is always defined by

$$Z^2_m = \frac{[pz^2]}{[p]}\,.$$

If the *relative* weights p_h are correctly assigned, the assumption that, in the mean, $z^2 = Z^2_1/p_h$ will be true—and also that $Z^2_m = Z^2_1/P_h$—so that the true value of Z_1 will be found, even though the weights assigned to s may be quite wild. Correct relative values of p_s will lead to a correct value of Y_1; and correct relative values of both, though on different scales, to correct values of both Z_1 and Y_1, and hence of the standard deviations of observations of any assigned weights p_h and p_s.

For the trigonometric parallaxes the corresponding equations may best be written

$$\left.\begin{aligned} Z^2_m + k^2 W^2 &= \frac{E}{P}\,, \\ X^2_m + W^2 &= \frac{E'}{P}\,, \\ kW^2 &= \frac{F}{P}\,, \end{aligned}\right\} \qquad (61)$$

when by (55)

$$X^2_m = \frac{Q'' X^2_0}{P}\,.$$

An error in the relation of the weighting systems for parallaxes of two kinds does not, therefore, vitiate the determination of the dispersion constants, so long as each system is internally correct. The relative weights which are attributed to combinations of parallaxes of the two kinds will be altered, but this difference is not serious. For example, the weight p_s of a Mount Wilson parallax, relative to the system adopted for the dynamical parallaxes, should be changed from 2.5 to 0.69, which increases $1/p$ by 1.04.

TABLE 8

EFFECT OF CHANGES OF WEIGHT

Grade	Physical Pairs						Orbits		
	Trigonometric Parallaxes			Spectroscopic Parallaxes			Spectroscopic Parallaxes		
	Old p	New p	Adjusted	Old p	New p	Adjusted	Old p	New p	Adjusted
Good........	0.50	0.367	0.475	0.70	0.405	0.632	2.00	0.650	1.60
Fair........	.40	.310	.401	.50	.329	.513	1.25	.543	1.34
Poor........	0.25	0.212	0.274	0.30	0.228	0.355	0.50	0.329	0.81

The resulting changes are shown in Table 8 which also contains the changes for a trigonometric parallax of weight 1 on its own scale, for which the $1/p_t$ is changed from 1.00 to 1.72. In the third column of each section the new weights have been multiplied by such factors that their sum equals that of the old. The effect of the erroneous old weighting has been to give the good determinations a little too much weight, and the poor ones too little. The changes are obviously far too small to worry about.

30. *Formulae for Mean Error.*—The mean errors of the quantities $\overline{h'_1}$ and $\overline{t'}$ may be derived as follows. Let h_0 and t_0 represent the mean values for a very great number of observations similar to those employed. We have then, in place of equation (50),

$$h'_1 = h_0(1+z)(1+kw)\,, \qquad t' = t_0(1+x)(1+w)\,. \tag{62}$$

For the weighted mean of a group of N observations we will have

$$\overline{h'_1} = \frac{[ph'_1]}{[p]} = h_0\left\{1 + \frac{[pz]}{[p]} + k\frac{[pw]}{[p]} + \ldots\right\} ; \tag{63}$$

whence, for the average of a large number of such groups, in which the weights are identical,

$$\overline{(\overline{h'_1} - h_0)^2} = \frac{h_0^2}{[p]^2}\left\{\left[\frac{p^2}{p_h}\right] Z_1^2 + [p^2]k^2W^2\right\} = \frac{h_0^2}{[p]}\{RZ_1^2 + P_2k^2W^2\} ,$$

where

$$[p] \cdot R = \left[\frac{p^2}{p_h}\right] , \qquad [p] \cdot P_2 = [p^2] . \tag{64}$$

Similarly,

$$\overline{(\overline{t'} - t_0)^2} = \frac{t_0^2}{[p]}\{R'X_1^2 + P_2W^2\} ,$$

where

$$[p] \cdot R' = \left[\frac{p^2}{p_t}\right] . \tag{65}$$

By equation (51) $R + R' = 1$. Also, if we set $\overline{p^2} = (1 + A)(\overline{p})^2$,

$$P_2 = (1 + A)P . \tag{66}$$

The standard deviations of $\overline{h'_1}$ and $\overline{t'}$ are then given by

$$\{\sigma(\overline{h'_1})\}^2 = \frac{h_0^2}{N}\left\{\frac{R}{P} Z_1^2 + (1 + A)k^2W^2\right\} , \tag{67}$$

$$\{\sigma(\overline{t'})\}^2 = \frac{t_0^2}{N}\left\{\frac{R'}{P} X_1^2 + (1 + A)W^2\right\} . \tag{68}$$

We have also to the first order

$$\frac{\overline{h'_1}}{\overline{t'}} = \frac{h_0}{t_0}\left\{1 + \frac{[pz]}{[p]} - \frac{[px]}{[p]} - (1 - k)\frac{[pw]}{[p]} \ldots\right\} ;$$

whence

$$\left\{\sigma\left(\frac{\overline{h'_1}}{\overline{t'}}\right)\right\}^2 = \frac{1}{N}\frac{h_0^2}{t_0^2}\left\{\frac{R}{P} Z_1^2 + \frac{R'}{P} X_1^2 + (1 + A)(1 - k)^2W^2\right\} . \tag{69}$$

For the corrected spectroscopic parallaxes we have, similarly,

$$s'' = t_0(1 + y)(1 + w)\,,$$

and then

$$\{\sigma(\overline{s''})\}^2 = \frac{t_0^2}{N}\left\{\frac{R'}{P}\,Y_1^2 + (1 + A)W^2\right\}, \tag{70}$$

$$\left\{\sigma\left(\frac{\overline{h'_1}}{\overline{s''}}\right)\right\}^2 = \frac{1}{N}\left(\frac{h_0}{t_0}\right)^2\left\{\frac{R}{P}\,Z_1^2 + \frac{R'}{P}\,Y_1^2 + (1 + A)(1 - k)^2W^2\right\}, \tag{71}$$

where

$$[p]\cdot R' = \left[\frac{p^2}{p_s}\right], \qquad R = 1 - R'\,. \tag{72}$$

31. *Errors of Sampling.*—The values of E, E', and F, derived from a moderate number of observations, are all subject to considerable errors of sampling. For the average of a great number of observations they are defined (for spectroscopic parallaxes) by equation (57). For an individual group of N observations, let the values be $E + g$, $E' + g'$, and $F + h$.

If $e = (h'_1 - h_0)/h_0$ and $e' = (s' - s_0)/s_0$, then

$$Ng = [pee] - NE\,,$$

$$N^2g^2 = [pee]^2 - 2NE[pee] + N^2E^2\,.$$

The mean value for many cases will be

$$N^2\overline{g^2} = \overline{[pee]^2} - N^2E^2\,.$$

Now

$$[pee]^2 = \{[pz^2] + 2[pkzw] + [pk^2w^2]\}^2\,.$$

When we take the mean, many terms vanish, and we have

$$\overline{[pee]^2} = \overline{[pz^2]^2} + 4k^2\overline{[pzw]^2} + 2k^2\overline{[pz^2]}\,\overline{[pw^2]} + k^4\overline{[pw^2]^2}\,.$$

If the expression $[pz^2]^2$ is written out in full, we have to substitute $Z_1^4/p_h p_h'$ for any term of the form $z^2 z'^2$; but for z^4 we must set $(1 + c)(Z_1^4/p_h^2)$. For a normal distribution, $c = 2$. We then find

$$\overline{[pz^2]^2} = cZ_1^4\left[\frac{p^2}{p_h^2}\right] + Z_1^4\left[\frac{p}{p_h}\right]^2 = cNJ_1Z_1^4 + N^2Q^2Z_1^4\,,$$

$$\overline{[pw^2]^2} = cW^4[p^2] + W^4[p]^2 = cN(1 + A)P^2W^4 + N^2P^2W^4\,,$$

$$\overline{[pzw]^2} = Z_1^2W^2\left[\frac{p^2}{p_h}\right] = NPRZ_1^2W^2\,,$$

$$[pz^2] = Z_1^2\left[\frac{p}{p_h}\right] = NQZ_1^2\,,$$

$$[pw^2] = W^2[p] = NPW^2\,,$$

and also

$$\overline{[py^2]^2} = cY_1^4\left[\frac{p^2}{p_s^2}\right] + Y_1^4\left[\frac{p}{p_s}\right]^2 = cNJ_3Y_1^4 + N^2Q'^2Y_1^2\,,$$

$$\overline{[pzy]^2} = X_1^2Y_1^2\left[\frac{p^2}{p_h p_s}\right] = NJ_2X_1^2Y_1^2\,,$$

$$\overline{[pyw]^2} = Y_1^2W^2\left[\frac{p^2}{p_s}\right] = NPR'Y_1^2W^2\,,$$

$$[py^2] = Y_1^2\left[\frac{p}{p_s}\right] = NQ'Y_1^2\,.$$

Here

$$NJ_1 = \left[\frac{p^2}{p_h^2}\right], \qquad NJ_2 = \left[\frac{p^2}{p_h p_s}\right], \qquad NJ_3 = \left[\frac{p^2}{p_s^2}\right]; \tag{73}$$

whence

$$J_1 + 2J_2 + J_3 = 1\,.$$

It now follows easily (setting $c = 2$) that

$$\left.\begin{aligned}
N\overline{g^2} &= 2\{J_1Z_1^4 + 2PRk^2W^2Z_1^2 + (1 + A)P^2k^4W^4\} \\
&= 2E^2 + 2\{(J_1 - Q^2)Z_1^4 + 2(R - Q)Pk^2W^2Z_1^2 \\
&\qquad\qquad + AP^2k^4W^4\}\,, \\
N\overline{g'^2} &= 2l^4\{J_3Y_1^4 + 2PR'W^2Y_1^2 + (1 + A)P^2W^4\} \\
&= 2E'^2 + 2l^4\{(J_3 - Q'^2)Y_1^4 + 2(R' - Q')PW^2Y_1^2 \\
&\qquad\qquad + AP^2W^4\}\,, \\
N\overline{h^2} &= l^2\{J_2Z_1^2Y_1^2 + PW^2(R'k^2Y_1^2 + RZ_1^2) + 2(1 + A)P^2k^4W^4\} \\
&= F^2 + EE' + l^2\{(J^2 - QQ')Z_1^2Y_1^2 \\
&\qquad + P(R' - Q')W^2(k^2Y_1^2 - Z_1^2) + 2AP^2k^2W^4\}\,.
\end{aligned}\right\} \quad (74)$$

These expressions can be simplified. Since $R + R' = Q + Q' = 1$, we may set

$$R' - Q' = -(R - Q) = B\,. \quad (75)$$

Also, since $p/p_s = 1 - (p/p_h)$,

$$J_3 = \frac{1}{N}\left[1 - \frac{2p}{p_h} + \frac{p^2}{p_h^2}\right] = 1 - 2Q + J_1\,,$$

$$J_2 = \frac{1}{N}\left[\frac{p}{p_h} - \frac{p^2}{p_h^2}\right] = Q' - J_3\,;$$

whence we have

$$J_1 - Q^2 = QQ' - J_2 = J_3 - Q'^2 = C\,, \quad (76)$$

and our equation is

$$\left.\begin{aligned}
N\overline{g^2} &= 2E^2 + 2(CZ_1^4 - 2BPk^2W^2Z_1^2 + AP^2k^4W^4)\,, \\
N\overline{g'^2} &= 2E'^2 + 2l^4(CY_1^4 + 2BPW^2Y_1^2 + AP^2W^4)\,, \\
N\overline{h^2} &= F^2 + EE' + l^2(-CZ_1^2Y_1^2 + BPW^2(k^2Y_1^2 - Z_1^2) \\
&\qquad\qquad + 2AP^2k^2W^4)\,.
\end{aligned}\right\} \quad (77)$$

The equations for the trigonometric parallaxes may be found by substituting X_1^2 for Y_1^2 and 1 for l.

If p_s is constant, $R' = [p^2]/p_s[p] = (1 + A)Q'$ and $J_3 = (1 + A)Q'^2$, so that

$$B = AQ' , \qquad C = AQ'^2 , \tag{78}$$

and equation (77) becomes

$$\left.\begin{aligned} N\overline{g^2} &= 2\{E^2 + A(E - Z^2)^2\} , \\ N\overline{g'^2} &= 2(1 + A)E'^2 , \\ N\overline{h^2} &= F^2 + EE' + A\{F^2 + E'(E - Z_1^2)\} . \end{aligned}\right\} \tag{79}$$

We have now from equations (67)–(69), (53), and (75)

$$\left.\begin{aligned} \{\sigma(\overline{h_1'})\}^2 &= \frac{h_0^2}{[p]}(RZ_1^2 + (1 + A)Pk^2W^2) \\ &\qquad\qquad = \frac{h_0^2}{[p]}\{(1 + A)E - (B + AQ)Z_1^2\} , \\ \{\sigma(\overline{t'})\}^2 &= \frac{t_0^2}{[p]}\{(1 + A)E' + (B - AQ')X_1^2\} , \\ \left\{\sigma\left(\frac{\overline{h_1'}}{\overline{t'}}\right)\right\}^2 &= \frac{1}{[p]}\frac{h_0^2}{t_0^2}\{(1 + A)(E + E' - 2F) \\ &\qquad\qquad - (B + AQ)Z_1^2 + (B - AQ')X_1^2\} . \end{aligned}\right\} \tag{80}$$

For the spectroscopic parallaxes, when we use equations (70)–(72) for the corrected values but equations (57)–(58) for the uncorrected residuals, we have

$$\left.\begin{aligned} \{\sigma(\overline{h_1'})\}^2 &= \frac{h_0^2}{[p]}\{(1 + A)E - (B + AQ)Z_1^2\} , \\ \{\sigma(\overline{s''})\}^2 &= \frac{t_0^2}{[p]}\left\{(1 + A)\frac{E'}{l^2} + (B - AQ')Y_1^2\right\} , \\ \left\{\sigma\left(\frac{\overline{h_1'}}{\overline{s''}}\right)\right\}^2 &= \frac{1}{[p]}\frac{h_0^2}{t_0^2}\left\{(1 + A)\left(E + \frac{E'}{l^2} - \frac{2F}{l}\right)\right. \\ &\qquad\qquad \left. - (B + AQ)Z_1^2 + (B - AQ')Y_1^2\right\} . \end{aligned}\right\} \tag{81}$$

If p_s is constant, $B + AQ = A$ and $B - AQ' = 0$.

These equations are applicable when the dispersion constants Z, Y, and W are determined from the double-star observations themselves. It may be noted that k has been eliminated from them and that the value of l used in the correction formula (33) must be employed. When, however, mean values of the dispersion constants are adopted with the aid of other data, equations (67)–(69), etc., may be used; or the values of E, E', and F, computed for the adopted constants, may be introduced into equations (80) and (81).

F. DISCUSSION OF THE OBSERVATIONS

32. *Results for Pairs with Orbits.*—We are now, at last, in a position to discuss the results of observation and will begin with the pairs for which orbits have been computed, for which the dynamical parallaxes are the most reliable. Four stars have been separated from the rest—o^2 Eridani, a white dwarf; 36 Andromedae, a subgiant; α Aurigae, a giant; and γ Lupi, of class B. The last three have been discussed along with similar stars for which no orbits are available. The few dwarfs of class M have also been discussed separately (§ 42) since their parallaxes are so well determined that they may be handled individually.

The remaining stars fall on the main sequence from B8 to K8. A few dwarf stars whose spectra are given in the *Henry Draper Catalogue* as K2 were grouped with the Mount Wilson K5, and, similarly, HD K5 was grouped with Mt. W. K8—to allow for a systematic difference in classification in this region. They have been grouped according to spectral class and discussed as described above. The results of the comparison with trigonometric and cluster parallaxes are shown in Table 9. The notation is that of § 27. The weighted sums and products of the residuals $[pvv]$, etc., are multiplied by 10^4 to obtain whole numbers. The quantities E, E', and F are defined by equation (52). Their values for these small groups are much affected by errors of sampling. To diminish this, the general mean for all the stars has been taken (weighted according to N, as described in § 27). The mean value of Q'' must be found in the same way: the value derived by equation (55) from the separate means of Q and $\overline{l'}$ is about 5 per cent too small. The comparison with the uncorrected spectroscopic parallaxes is shown in Table 10, which is

similarly arranged, except that the group B8–A5, for which the spectroscopic parallaxes have not been investigated, is excluded from the general mean.

TABLE 9

ORBITS: TRIGONOMETRIC AND CLUSTER PARALLAXES

	SPECTRUM						
	B8–A5	A6–F2	F3–F6	F7–G3	G4–K2	K3–K8	All
N	21	19	17	25	15	8	105
$[p]$	30.2	20.6	36.0	66.7	49.2	20.3	223.0
P	1.44	1.08	2.12	2.67	3.28	2.54	2.12
Q	0.242	0.156	0.280	0.316	0.414	0.413	0.288
$\overline{h}_1^2$	0.″179	0.″160	0.″146	0.″148	0.″150	0.″103	....
$\overline{v}$	0.″100	0.″109	0.″109	0.″118	0.″122	0.″088	0.″110
$10^4\ [pvv]$	418	372	891	2128	1249	103	....
$10^4\ [pv'v']$	395	338	807	2137	966	169	....
$10^4\ [pvv']$	+162	+262	+582	+1947	+ 895	+ 46	....
Q''	0.76	0.71	0.60	0.49	0.39	0.76	0.609
E	0.065	0.089	0.253	0.404	0.410	0.142	0.231
E'	.197	.173	.412	.638	.471	.313	.378
F	+0.045	+0.091	+0.230	+0.463	+0.359	+0.074	+0.227
Z_1^2 adopted	0.141	0.141	0.141	0.141	0.141	0.141	0.141
k	.69	.74	.93	.77	.98	1.12	.840
X_0^2	.18	.07	.26	.08	.26	0.32	.178
W^2	0.05	0.11	0.12	0.22	0.11	0.02	0.128

We now have from equations (53) and (57)

$$\left.\begin{aligned} QZ_1^2 + Pk^2W^2 &= E\,, & QZ_1^2 + Pk^2W^2 &= E\,, \\ Q''X_0^2 + PW^2 &= E'\,, & Q'l^2Y_1^2 + Pl^2W^2 &= E'\,, \\ PkW^2 &= F\,, & PklW^2 &= F\,, \end{aligned}\right\} \tag{82}$$

for the trigonometric and spectroscopic parallaxes, respectively.

For the trigonometric parallaxes the value $X_0^2 = 0.172$ follows from the probable errors of Schlesinger's catalogue. If we introduce this into the equations for the general mean of the stars in Table 9, we find that $k = 0.83$, $W^2 = 0.129$, and $Z_1^2 = 0.149$. For the spectroscopic parallaxes we may adopt the values found in II from 1140

main-sequence stars—namely, $l = 0.383$ and $Y_m = \pm 0.378$. The last is the mean error corresponding to an average spectroscopic parallax of weight 2.5, so that, for the unit of weight, $Y_m^2 = 0.358$. With these values, for the stars A6–K8 in Table 10, we find that $k = 0.77$, $W^2 = 0.092$, and $Z_1^2 = 0.133$. The value of k to be anticipated is $1 - \frac{2}{3}n$, or 0.82 for the main sequence, while that of

TABLE 10

ORBITS: SPECTROSCOPIC PARALLAXES

	Spectrum						
	B8–A5	A6–F2	F3–F6	F7–G3	G4–K2	K3–K8	A6–K8
N	18	20	21	30	12	6	89
$[p]$	22.6	33.2	35.2	54.8	23.3	10.5	157.0
P	1.25	1.66	1.68	1.83	1.94	1.75	1.76
Q	0.268	0.335	0.328	0.270	0.225	0.30	0.294
$\overline{h'_1}$	0″.187	0″.163	0″.163	0″.160	0″.138	0″.098	....
$\overline{s'}$	0″.129	0″.105	0″.112	0″.109	0″.107	0″.091	0″.107
10^4 $[pvv]$	746	628	857	1142	169	42	....
10^4 $[pv'v']$	234	103	156	292	25	21	....
10^4 $[pvv']$	+204	+115	+203	+311	+30	+19	....
E	0.125	0.124	0.161	0.154	0.081	0.087	0.135
E'	.083	.049	.062	.085	.020	.050	.060
F	+0.050	+0.035	+0.056	+0.061	+0.019	+0.047	+0.047
Z_1^2 adopted	0.141	0.141	0.141	0.141	0.141	0.141	0.141
l adopted	.383	.383	.383	.383	.383	.383	.383
k	.66	.82	.77	.72	.98	.36	.76
Y_1^2	.51	.34	.35	.50	.11	.01	.352
W^2	0.16	0.07	0.11	0.12	0.03	0.20	0.093

W^2 for the 1140 stars was found to be 0.119. The agreement of both the new determinations is good.

The values of Z_1^2 are, however, greater than the anticipated value, 0.100. Their mean, 0.141, indicates that the standard deviation for an average "good" orbit of weight 10 should be ± 0.119 times h_1. For the selected group of exceptionally well-observed stars discussed in § 23 it was estimated as ± 0.086. The difference means only that the superiority of the best orbits over the good ones was not sufficiently appreciated.

Similar calculations may be made for the small separate groups. It is better to assume the mean value of Z_1^2 just obtained, and that of l, and derive the other unknowns, for if the calculation is made

TABLE 11

ORBITS: SAMPLING ERRORS OF E, E', AND F

TRIGONOMETRIC PARALLAXES

	Spectrum						
	B8–A5	A6–F2	F3–F6	F7–G3	G4–K2	K8–M	B8–K8
A	+2.90	+1.29	+1.35	+0.92	+0.90	+0.93	+1.43
B	−0.43	−0.13	−0.24	− .22	− .26	− .16	−0.28
C	+0.09	+0.02	+0.06	+0.07	+0.11	+0.06	+0.08
ΔE	−0.071	−0.009	+0.073	+0.171	+0.111	−0.095	+0.040
P.E.	±0.052	±0.031	±0.064	±0.060	±0.095	±0.104	±0.023
$\Delta E'$	−0.091	−0.068	+0.076	+0.260	+0.044	−0.099	+0.041
P.E.	±0.076	±0.059	±0.096	±0.087	±0.130	±0.160	±0.031
ΔF	−0.082	−0.004	+0.044	+0.228	+0.071	−0.149	+0.041
P.E.	±0.057	±0.037	±0.072	±0.067	±0.105	±0.117	±0.032

SPECTROSCOPIC PARALLAXES

	Spectrum						
	B8–A5	A6–F2	F3–F6	F7–G3	G4–K2	K8–M	A6–K8
A	+0.15	+0.05	+0.07	+0.04	+0.01	+0.02	+0.05
ΔE	−0.001	−0.040	−0.003	−0.013	−0.087	−0.078	−0.030
P.E.	±0.028	±0.035	±0.034	±0.029	±0.046	±0.064	±0.017
$\Delta E'$	+0.024	−0.012	0.000	+0.018	−0.052	−0.015	−0.005
P.E.	±0.014	±0.013	±0.013	±0.012	±0.020	±0.026	±0.007
ΔF	+0.008	−0.021	−0.001	0.000	−0.046	−0.012	−0.012
P.E.	±0.015	±0.018	±0.017	±0.015	±0.025	±0.033	±0.007

from assumed values of Y_1 or X_0, any errors are greatly multiplied as it proceeds—as appears from examination of the coefficients. The results of the assumption that $Z_1^2 = 0.141$ and $l = 0.383$, are given in the last lines of the tables. Except for the smallest group, they are fairly consistent.

33. *Errors of These Results.*—To test whether the discordances between the groups may be attributed to random sampling, the values of E, E', and F were calculated from equation (82) with $X_0^2 = 0.172$, $Y_1^2 = 0.358$, and $l = 0.383$, and with $Z_1^2 = 0.141$, $W^2 = 0.110$, and $k = 0.80$, the first three being taken from other sources, and the last three the means of the results from trigonometric and spectroscopic parallaxes. The differences, ΔE, etc., of the observed values of Tables 9 and 10 from the calculated values (in the sense O − C) are given in Table 11, which contains also the *probable* error due to sampling, which is to be anticipated for each value of E, E', and F, according to equations (77) and (79). The quantities A, B, and C, derived from the weights which appear in these equations, are also tabulated. The *computed* values of E, E', and F were used in this computation, so that the probable errors represent the sampling effect to be expected from a homogeneous population defined by the adopted values of X_0^2, etc.

The values of A are large for the trigonometric parallaxes, which vary greatly in weight (owing mainly to the reduction process, § 12), and small for the spectroscopic case (since the variations in p arise from the dynamical parallaxes and are small). The value of C is so small that it might be entirely neglected.

Taking the ratios Δ/r for the individual groups, we have, in all, thirty-six quantities from which three unknowns, Z_1, W_1, and k, have been derived. The distribution of these ratios, without regard to sign, is as follows:

Limits	0		1		2		3		4
Observed		18		14		2		2	
Normal distribution		18		12		4		2	

The mean-square value of Δ/r is ± 1.44. Its expected value is $\sqrt{33/36} \div 0.6745$, or 1.42. Almost all the variation must therefore arise from random sampling—though there are some "runs" of sign among the residuals that suggest that the differences may be partly real.

The dispersion constants for the stars for which orbits have been computed are remarkably similar to those found for main-sequence stars as a whole (§ 34). Indeed, the sample is not large enough to establish the reality of any differences—with one exception.

The mean values $\overline{t'}$ and $\overline{s'}$ for all these stars are, respectively, 0".110 and 0".107, as against 0".103 and 0".103 for main-sequence stars at large. If the difference for the spectroscopic parallaxes is divided by l, as it should be, the corrected spectroscopic mean is 0".113. The probable error of $\overline{t'}$ by equation (68) is $\pm$0".004, so that the difference is probably real. It has long been recognized that observational selection favors the nearer pairs for orbit computation. Orbital pairs, being main-sequence stars, are, of course, very much nearer than the general run of stars of the same spectral type and apparent magnitude. As compared with stars of the same spectral class in the main sequence, the difference is now shown to be perceptible but small.

It may be added that the small values of $[pv'v']$ and $[pvv']$ found in the comparison of dynamical and spectroscopic parallaxes afforded the first evidence of that peculiarity of the spectroscopic calibration which is fully discussed in II. The value $Z_1^2 = 0.141$, found above, signifies that the combined effect of all the errors of an average dynamical parallax for a star with a "good" orbit—that is, for a majority of those for which orbits have been computed—amounts to a mean error of $\pm$12 per cent, or a probable error of $\pm$8 per cent, of the dynamical parallax itself. This applies to the "hypothetical" parallax h_1; the transition to the final dynamical parallax d by the use of the mass-luminosity relation multiplies this result by $1/k$ and makes it $\pm$10 per cent. This includes the effects of deviations from the mass-luminosity relation (so far as these are revealed by a study of visual binaries) as well as the errors of observation. Except for a few of the nearer stars, they should therefore be regarded as determinations of high precision.

34. *Observational Selection in Slow-moving Pairs.*—The stars of the main sequence may next be considered. Here we meet a new problem—the effects of the systematic neglect of "fixed" pairs by past observers (§ 14). It is easy to see that the incidence of this difficulty will vary greatly with the type of star.

For dwarf stars of spectrum K5 and later, trouble need not be feared. All these stars which have so far been recorded as double are so near that their orbital motions are conspicuous, though small compared with the proper motion (Burnham's "61 Cygni type").

At the other end of the main sequence, stars of class B are practically all so remote that their motions are apparently slow. There is still an observational preference for bright and easily observed pairs, but not much selection according to motion. These stars were described by Burnham as "fixed." Much the same situation must exist among the giants in general.

Passing down the main sequence, we should at first expect this trouble to increase and to reach a maximum when the proportions of "fixed" and "moving" pairs to be found in an impartially selected group become comparable, and then to diminish.

The principal influence of this selection is to vitiate the assumptions regarding geometrical probability and to make the mean value of h_1 too great. It should, however, have some influence on the dispersion constants. Consider a group of main-sequence stars covering a small range both in spectral class and in visual magnitude—or, more generally, a group with a wider spectral range, for which the modulus $m_b - M_o$ is nearly the same. If these stars were really of the assumed absolute magnitude M_o (according to spectral class), their distances would be nearly the same; but they will actually differ considerably, owing to the dispersion in absolute magnitude. Stars brighter than the standard (for which w is negative) will be farther away and more likely to show slow apparent motions, after allowance for their greater masses, so that more of them would be neglected. The increase of the mean h_1' above an impartial mean will therefore be greater for stars with w negative than with w positive. It follows that the coefficient k, derived from the observed distribution, should come out too small.

The effect on the observed dispersion W is less simple. If the actual distribution of w was normal, and the probability that a star was included in the observed list was of the form Ae^{aw}, the mode of the distribution would be altered without changing the standard deviation. A fairly sharp cutoff within a moderate range of w would make W come out too small.

With increasing modulus the visual magnitude becomes fainter, and the observers' preference for "interesting" pairs is more effective, so that, near the limit, our list of well-observed stars should consist mainly of those with abnormally rapid motion.

35. *Rejection of Stars Affected by This Error.*—At what value of the modulus this effect becomes serious cannot be predicted a priori, but it may be investigated by dividing the stars in each of the spectral groups according to the value of the modulus and by taking means of h_1' and s' or t'. The effect of observational selection—or,

TABLE 12.

INVESTIGATION OF OBSERVATIONAL SELECTION
SPECTROSCOPIC PARALLAXES

Sp.	Mod.	N	$[p]$	$\overline{h_1'}$	$\overline{s'}$	$\overline{h_1'}/\overline{s'}$
B0–B7	<6.2 6.2–8.0	21 20	8.7 8.6	0″.268 0.335	0″.122 0.125	2.19 2.68
B8–A5	<5 5.0–6.5	61 46	32.8 21.0	0.157 0.184	0.110 0.130	1.43 1.42
A6–F2	<4 4–5 5.0–6.5	34 14 16	21.6 8.8 9.0	0.121 .170 0.261	0.100 .101 0.102	1.21 1.68 2.55
F3–F6	<4 4–5 5.0–6.5	52 16 7	32.0 10.2 3.5	0.143 .193 0.306	0.106 .110 0.103	1.35 1.75 2.97
F7–G3	<4 4–5 5.0–6.5	40 12 6	26.6 7.6 3.6	0.159 .226 0.269	0.109 .108 0.107	1.46 2.09 2.52
G4–K2	<3 3–4 4–5	24 11 4	15.2 6.9 2.6	0.126 .181 0.213	0.098 .105 0.103	1.29 1.72 2.07
K3–K8	<2 2–3	10 7	6.6 4.7	0.124 0.109	0.108 0.096	1.15 1.13
Giants (G0–M5)	<5 5.0–6.5	45 35	24.3 17.7	0.161 0.183	0.097 0.109	1.67 1.67

rather, neglect—will increase h_1' without much influence on the others—more precisely, will increase $\overline{h_1'}/\overline{s'}$. The spectroscopic parallaxes, whose percentage errors are independent of distance, should give a good standard of comparison—even in their uncorrected form.

The limits of modulus for the different groups have been varied to meet the conditions; for example, no dwarf later than K3 has a modulus greater than 3^m, and no star earlier than B8 one less than $3^m.9$. Table 12 indicates that the selection effect is small or absent

among the giants, in classes B and A and again beyond K3, but that it is very serious for most of the main sequence. Exclusion of all stars with modulus greater than 5 and with spectra later than A5 is obviously indispensable if systematic error is to be avoided; and the limit might well be raised to 4^m for F3–G3 and to 3^m for the G4–K2 group. It should be emphasized that this exclusion is for the purpose of eliminating a systematic error, whose existence and sign—though not its actual amount—can be determined by a priori reasoning. To fix the limits of exclusion by examination of the relative values of observed results for the same spectral class is then legitimate. We may then adopt as the limits of modulus for exclusion the values given in Table 13. By addition of M_0 we obtain the corresponding limits of visual magnitude. These run smoothly and indicate that slow-moving pairs have been badly neglected below the eighth magnitude. A systematic reobservation (preferably by photography) of all the double stars for which good early observations exist would go far toward removing this neglect. Such a program, combined with determinations of spectroscopic parallax, is by far the most promising way to increase our knowledge of the masses of visual binaries.

TABLE 13

	Spectrum						
	B0–B7	B8–A5	A6–F2	F3–F6	F7–G3	G4–K2	Giants
Modulus	8.0	6.5	5.0	4.0	4.0	3.0	6.5
M_0	−1.5	+1.1	+2.4	+3.2	+4.0	+5.2	+0.5
m_b	6.5	7.6	7.4	7.2	8.0	8.2	7.0

A similar grouping of the comparisons of dynamical with trigonometric and cluster parallaxes is given in Table 14. Here the low percentage accuracy of trigonometric determinations for distant stars deprives the results of much significance. The stars of large modulus have such low weight that it matters little whether they are excluded or not. The data for the B stars depend essentially on Kapteyn's cluster parallaxes. In this case the stars of different visual magnitude are at roughly the same distance, and the differences in the

TABLE 14

INVESTIGATION OF OBSERVATIONAL SELECTION
TRIGONOMETRIC AND CLUSTER PARALLAXES

Sp.	Mod.	N	$[p]$	$\overline{h'_1}$	$\overline{t'}$	$\overline{h'_1}/\overline{t'}$
O9–B6	<6.4	22	10.11	0.″294	0.″123	2.39
	6.4–8.0	12	4.45	0.298	0.196	1.52
B8–A5	<5	51	18.96	0.144	0.097	1.48
	5.0–6.5	21	2.39	0.173	0.115	1.54
A6–F2	<4	19	9.89	0.132	0.087	1.52
	4–5	3	0.27	.206	.216	0.95
	5.0–6.5	5	0.41	0.225	0.144	1.56
F3–F6	<4	26	13.50	0.145	0.093	1.56
	4–5	8	1.10	.176	.096	1.84
	5.0–6.5	5	0.17	0.147	0.290	0.51
F7–G3	<4	30	14.23	0.159	0.115	1.38
	4–5	9	1.08	.241	.152	1.59
	5.0–6.5	4	0.24	0.235	0.104	2.26
G4–K2	<3	19	11.47	0.109	0.088	1.24
	3–4	9	2.84	.182	.149	1.22
	4–5	6	0.71	0.214	0.182	1.18
K3–K8	<1.5	8	6.69	0.124	0.116	1.08
	1.5–3.0	4	2.91	0.119	0.115	1.03
Giants (G0–M5)	<5.0	39	11.98	0.146	0.060	2.43
	5.0–6.5	16	1.63	0.187	0.089	2.10

"modulus" represent mainly differences in real brightness, so that it is not surprising that $\overline{h'_1}/\overline{t'}$ is large for the brighter group (though the difference may be partly accidental).

36. *Distribution Constants; Main Sequence; Physical Pairs.*—The discussion of the distribution constants for stars for which the dynamical parallaxes were found by the statistical method will be complicated by observational selection. For the trigonometric parallaxes, its effects should be much diminished by the low weighting of the remoter stars; for the spectroscopic parallaxes, they should appear in full.

Table 15, which is similar to Table 9, gives the data for all stars with modulus less than 5.0 (8.0 for O9–B7). Similar data for the spectroscopic parallaxes are given in Table 16.

TABLE 15

MAIN SEQUENCE: TRIGONOMETRIC AND CLUSTER PARALLAXES

	O9–B7	B8–A5	A6–F2	F3–F6	F7–G3	G4–K2	K3–K8	B8–K8
N	34	51	22	34	39	34	12	192
$[p]$	14.56	18.96	10.16	14.60	15.31	15.02	9.60	83.65
P	0.428	0.372	0.462	0.430	0.392	0.441	0.800	0.436
Q	0.560	0.417	0.494	0.528	0.440	0.531	0.851	0.497
$\overline{h'_1}$	0".294	0".144	0".135	0".147	0".165	0".128	0".122	....
$\overline{t'}$	0".146	0".094	0".090	0".094	0".118	0".103	0".115	0".101
$10^4\,[pvv]$	1690	916	228	573	713	485	126	....
$10^4\,[pv'v']$	727	712	301	483	538	538	41	....
$10^4\,[pvv']$	+ 144	+201	+ 84	+ 81	+236	+231	− 4	....
Q''	0.206	0.583	0.625	0.535	0.402	0.442	0.113	0.509
E	0.059	0.088	0.059	0.080	0.069	0.090	0.084	0.079
E'	.104	.161	.177	.166	.101	.154	.031	.142
F	+0.010	+0.030	+0.032	+0.018	+0.032	+0.053	−0.003	+0.030
A	+0.45	+0.43	+0.28	+0.35	+0.45	+0.37	+0.04	+0.38
B	− .15	− .15	− .12	− .12	− .16	− .15	− .02	− .15
C	+0.07	+0.06	+0.06	+0.06	+0.07	+0.09	+0.02	+0.08

TABLE 16

MAIN SEQUENCE: SPECTROSCOPIC PARALLAXES

	B0–B7	B8–A5	A6–F2	F3–F6	F7–G3	G4–K2	K3–K8	A6–K8
N	41	61	48	68	52	39	17	224
$[p]$	17.3	32.8	30.4	42.2	34.2	24.7	11.3	142.8
P	0.423	0.538	0.633	0.620	0.658	0.633	0.665	0.637
Q	0.578	0.617	0.733	0.741	0.727	0.738	0.726	0.734
$\overline{h'_1}$	0".300	0".157	0".135	0".155	0".174	0".150	0".118	....
$\overline{s'}$	0".123	0".110	0".100	0".107	0".110	0".100	0".103	0".105
$10^4\,[pvv]$	2861	1623	1363	1930	1744	1026	128	....
$10^4\,[pv'v']$	313	345	178	172	94	102	29	....
$10^4\,[pvv']$	+ 500	+ 115	− 24	+ 95	+ 37	+ 102	+ 33	....
E	0.079	0.110	0.159	0.120	0.113	0.120	0.057	0.122
E'	.052	.048	.038	.022	.015	.027	.017	.0244
F	+0.034	+0.011	−0.004	+0.008	+0.004	+0.018	+0.017	+0.0071
A	+0.04	+0.06	+0.04	+0.04	+0.02	+0.04	+0.01	+0.04

For the general means—trigonometric B8–K8 and spectroscopic A6–K8 (those on the system of *Mount Wilson Contribution* No. 511)—the equations (82) become, giving l the well-determined value 0.383,

$$\left.\begin{array}{rlr} 0.497Z_1^2 + 0.436k^2W^2 &= 0.079 \pm 0.006 & -0.004 \\ 0.509X_0^2 + 0.436W^2 &= 0.142 \pm 0.010 & +0.007 \\ 0.436kW^2 &= 0.030 \pm 0.005 & +0.007 \end{array}\right\} \quad (83)$$

(O − C in the last column)

$$\left.\begin{array}{rlr} 0.734Z_1^2 + 0.637k^2W^2 &= 0.122 \pm 0.008 & +0.002 \\ 0.266Y_1^2 + 0.637W^2 &= 0.166 \pm 0.011 & +0.001 \\ 0.637kW^2 &= 0.019 \pm 0.007 & -0.014 \end{array}\right\} \quad (84)$$

Setting $X_0^2 = 0.172$ in equations (83), we find $Z_1^2 = 0.127$, $W^2 = 0.125$, and $k = 0.56$; and with $Y_1^2 = 0.358$ in equations (84), $Z_1^2 = 0.159$, $W^2 = 0.112$, and $k = 0.27$. The values of Z_1^2 are greater than might have been expected. The errors of the statistical process lead to $Z_1^2 = 0.090$, and a reasonable allowance for the errors of a "good" set of observations is not likely to raise this above 0.100 (§§ 11 and 24).

If we set this in equations (83), we find $X_0^2 = 0.218$, $W^2 = 0.072$, and $k = 0.97$; while equations (84) give the ridiculous values $Y_1^2 = 0.60$, $W^2 = 0.011$, and $k = 2.6$. This makes it probable that Z_1^2 is actually greater. If we adopt the mean $Z_1^2 = 0.143$ and take $W^2 = 0.110$, as in § 33, a tolerable representation of all the data may be found by taking $k = 0.47$, which gives the residuals O − C as shown above. The small value of k may be attributed to the effect of observational selection. It may be smaller still for the spectroscopic parallaxes, but this would demand an increase either of Z_1 or of Y_1.

37. *Distribution Constants; Giant Stars.*—Among the giants, an orbit is available only for α Aurigae. This star has so high a weight, compared with any other, that it has been kept separate. The data for the rest are not numerous or precise enough to justify the forma-

tion of small groups, and all spectral classes from G to M have been combined in a general mean. In forming h_1', t', s', and their means, the individual values of M_0 given in Table 1 were used, so that variation of absolute magnitude with spectral class is substantially eliminated. Two supergiants—Polaris and Antares—were omitted. The

TABLE 17

GIANT STARS: ALL SPECTRA G0–M5

	Trigonometric Parallaxes		Spectroscopic Parallaxes	
	Mod. < 5.0	Mod. < 6.5	Mod. < 5.0	Mod. < 6.5
N	39	55	45	80
$[p]$	11.98	13.62	24.3	42.0
P	0.308	0.248	0.540	0.525
Q	0.416	0.342	0.773	0.781
$\overline{h_1'}$	$0''.146$	$0''.151$	$0''.161$	$0''.170$
$\overline{t'}$, $\overline{s'}$	$0''.060$	$0''.063$	$0''.097$	$0''.102$
$10^4\,[pvv]$	749	816	1527	2336
$10^4\,[pv'v']$	364	500	93	211
$10^4\,[pvv']$	+245	+287	+ 200	+ 468
E	0.092	0.066	0.134	0.104
E'	.266	.230	.022	.026
F	+0.074	+0.057	+0.029	+0.034
Q''	1.62	1.60	....	....
A	+0.47	+0.67	+0.087	+0.104
B	− .12	− .16	+0.020	+0.022
C	+0.05	+0.05	....	....

motions in both systems are still poorly determined. Seven subgiants were also excluded, since the values of h_1', etc., resulting from the assumption of normal giant absolute magnitude are so large as to vitiate the means. These are discussed in § 41.

The data for these systems are given in Table 17. The groups with modulus < 6.5 include the whole of the others. Almost all the stars with trigonometric parallaxes have spectroscopic determinations also.

The equations (82) are, in this case:

a) Modulus < 5

		O − C
$0.416Z_1^2 + 0.308k^2W^2$	$= 0.092 \pm 0.015$	$+0.009$
$1.62X_0^2 + 0.308W^2$	$= 0.266 \pm 0.041$	-0.058
$0.308kW^2$	$= 0.074 \pm 0.020$	$+0.040$
$0.773Z_1^2 + 0.540k^2W^2$	$= 0.134 \pm 0.019$	-0.018
$0.090Y_1^2 + 0.215W^2$	$= 0.022 \pm 0.003$	-0.043
$0.340kW^2$	$= 0.029 \pm 0.006$	$+0.008$

b) Modulus < 6.5

		O − C
$0.342Z_1^2 + 0.248k^2W^2$	$= 0.066 \pm 0.009$	$+0.023$
$1.60X_0^2 + 0.248W^2$	$= 0.230 \pm 0.030$	-0.082
$0.248kW^2$	$= 0.057 \pm 0.014$	$+0.031$
$0.774Z_1^2 + 0.536k^2W^2$	$= 0.104 \pm 0.011$	-0.050
$0.090Y_1^2 + 0.213W^2$	$= 0.026 \pm 0.003$	-0.037
$0.338kW^2$	$= 0.034 \pm 0.005$	-0.002

The value $l = 0.63$, found in II for 732 giant stars, has been introduced into the second group of equations. The probable errors are those to be anticipated from sampling.

If in the first two equations of each set we introduce the values already found, $Z_1^2 = 0.141$ and $X_0^2 = 0.172$, and in the third, $k = 1 - \frac{2}{3}n = 0.72$, we find, respectively, from (*a*) $W^2 = 0.27 \pm 0.10$; -0.04 ± 0.13; 0.33 ± 0.09, and from (*b*) $W^2 = 0.14 \pm 0.07$; -0.18 ± 0.12; 0.32 ± 0.08, where the probable errors represent the effects of sampling alone. Giving the second value in each case half-weight, the general mean W^2 is 0.19. The probable error of this mean, derived from the residuals, is ± 0.050; from the probable errors of the components it should be ± 0.038. The larger part, though probably not all, of the discordances is therefore to be attributed to accidents of sampling.

The value of W^2 found in II for giants, supergiants, and subgiants,

together, was 0.27, so that this value is not unreasonable—though far from accurate.

For the spectroscopic parallaxes we may set $k = 0.72$, $Z_1^2 = 0.141$, and $Y_1^2 = 0.358$. We then find, as above,

$$a) \quad W^2 = 0.09 \pm 0.06; \quad -0.05 \pm 0.014; \quad 0.12 \pm 0.03\,,$$

$$b) \quad W^2 = -0.02 \pm 0.05; \quad -0.03 \pm 0.013; \quad 0.14 \pm 0.02\,.$$

The brute mean for W^2 is 0.04, which is inconsistent with the last result, which includes more than two-thirds of the same stars.

It may be noted that the discussion of giant stars in II gave $l = 0.63 \pm 0.06$. This probable error represents only the effects of sampling. For the main sequence it was found that the probable error derived from the agreement of different groups was 1.7 times as great. It is therefore possible that l may really be as low as 0.50, which would raise the mean value of W^2 to 0.08.

It is not worth while, however, to spend much time on this, for the only purpose for which we require the statistical constants in the main discussion is to compute the mean errors of $\overline{h_1'}/\overline{t'}$, etc. (cf. eq. [69]), and certain small corrections to the mean values (cf. eqs. [39] and [47]). We will therefore adopt $W^2 = 0.15$ for the giants and take the other constants as above. The resulting residuals are given after the equations. They are too large to be due to chance, and the worst of them are negative, suggesting that the dispersion constants may have been estimated too great; but it is hard to see, by comparison with better-observed cases, how they can be much smaller. The most serious residuals, for E' spectroscopic, may be due to a difference in the value of l. The only risk involved in using these values is that the probable errors of the final results for the giants may be slightly overestimated—and this is to err on the right side.

38. *Comparison of Dynamical and Other Parallaxes.*—Collecting our data, we have the following values of the dispersion constants, which will be adopted in subsequent discussions.

	X_0^2	Y_1^2	Z_1^2	W^2	k	l
Orbits and main sequence..........	0.172	0.358	0.141	0.110	0.82	0.383
Giants............	0.172	0.358	0.141	0.150	0.72	0.63

The values of X_0^2, Y_1^2, and l are taken from the discussion of spectroscopic and trigonometric parallaxes in II, and those of W^2 are nearly the same. Those of k are derived from theory (§§ 16 and 21), leaving only Z_1^2 to be determined empirically from the double stars.

TABLE 18

RESULTS OF COMPARISON OF DYNAMICAL AND OTHER PARALLAXES

Sp.	Mod.	Parallaxes		N	$[p]$	$\overline{h_1'}$	$\overline{t'}, \overline{s'}, \overline{s''}$	Ratio
O9–B7....	<8.0	Phys	Tr, Cl	34	14.6	0".296±0".020	0".146±0".008	2.03±0.12
	<8.0	Phys	Sp	41	17.3	0.300±0.016	(0.123±0.008)	(2.44±0.19)
B8–A5....	All	Orb	Tr	21	30.2	0.179±0.016	0.100±0.010	1.79±0.09
	<6.5	Phys	Tr, Cl	72	21.4	.147± .007	.099± .005	1.48± .09
	All	Orb	Sp	18	22.6	.187± .010	(.129± .012)	(1.45± .12)
	<6.5	Phys	Sp	107	53.8	0.167±0.006	(0.118±0.005)	(1.42±0.06)
A6–F2.....	All	Orb	Tr	19	20.6	0.160±0.011	0.109±0.010	1.47±0.09
	<5.0	Phys	Tr	22	10.2	.135± .010	.090± .007	1.50± .13
	All	Orb	Sp	20	33.2	.163± .008	.100± .008	1.63± .10
	<5.0	Phys	Sp	48	30.4	0.135±0.006	0.087±0.004	1.55±0.09
F3–F6.....	All	Orb	Tr	17	36.0	0.146±0.011	0.109±0.010	1.34±0.06
	<4.0	Phys	Tr	26	13.5	.145± .010	.093± .006	1.56± .11
	All	Orb	Sp	21	35.2	.163± .008	.122± .009	1.34± .08
	<4.0	Phys	Sp	52	32.0	0.143±0.007	0.106±0.005	1.35±0.07
F7–G3....	All	Orb	Tr	25	66.7	0.148±0.008	0.118±0.008	1.25±0.04
	<4.0	Phys	Tr	30	14.2	.159± .011	.115± .007	1.38± .09
	All	Orb	Sp	30	54.8	.160± .006	.117± .007	1.37± .07
	<4.0	Phys	Sp	40	26.6	0.159±0.008	0.117±0.006	1.36±0.08
G4–K2....	All	Orb	Tr	15	49.2	0.150±0.011	0.122±0.010	1.23±0.05
	<3.0	Phys	Tr	19	11.5	.109± .009	.088± .006	1.24± .10
	All	Orb	Sp	12	23.3	.138± .008	.117± .012	1.18± .09
	<3.0	Phys	Sp	24	15.2	0.126±0.008	0.093±0.007	1.35±0.11
K3–K8....	All	Orb	Tr	8	20.3	0.103±0.010	0.088±0.011	1.17±0.08
	<3.0	Phys	Tr	12	9.6	.122± .011	.115± .008	1.06± .09
	All	Orb	Sp	6	10.5	.098± .008	.070± .010	1.40± .16
	<3.0	Phys	Sp	17	11.3	0.118±0.009	0.101±0.009	1.17±0.11
Giants (G0–M5)	<6.5	Phys	Tr	55	13.6	0.151±0.009	0.063±0.006	2.40±0.24
	.....	Phys	Sp	80	42.0	0.170±0.007	0.107±0.005	1.59±0.07
Subgiants..	.....	.....	Tr	5	2.6	0.136.......	0.091........	1.50.......
	.....	.....	Sp	10	5.4	0.166.......	0.105........	1.58.......

We may now derive corrected mean spectroscopic parallaxes $\overline{s''}$ by equation (33) and may compute the probable errors of $\overline{h_1'}$, $\overline{t'}$, $\overline{s''}$, and their ratios by equations (67)–(71). The auxiliary constants required may all be derived from the preceding tables, so that only the

results need be presented (Table 18). The second column gives the limiting modulus beyond which stars were excluded, to diminish the effect of the systematic neglect of "fixed" pairs by past observers (§§ 34 and 35). In the third column "Orb" and "Phys" denote dynamical parallaxes derived from orbits (§ 2) and physical pairs (§ 3), while "Sp," "Tr," and "Cl" denote spectroscopic, trigonometric, and cluster parallaxes. The spectroscopic parallaxes of B and A stars have not been compared with the trigonometric system, so that only the uncorrected mean parallaxes $\overline{s'}$ can be given. These are put in parentheses, along with the ratios derived from them. The data for the subgiants (§ 41) are given at the bottom, without probable errors.

The results from the orbits and physical pairs are strictly independent; but those from the trigonometric and spectroscopic parallaxes for the same group deal mainly with the same stars and differ only in the errors of the parallaxes.

There appears, at first sight, to be a tendency to smaller values of $\overline{t'}$ or $\overline{s''}$ for the physical pairs than from the orbits; but the mean-square value of the ratio of the difference (Δ = Phys − Orb) to its probable error r is 1.68 for the trigonometric parallaxes, and 1.49 for the spectroscopic, as against 1.48 to be expected; so that the real differences, if any, must be small. A small difference in the observed sense might perhaps be expected, for the stars for which orbits can be calculated are likely to be nearer than the average and somewhat fainter than the average absolute magnitude for the whole group.

For the ratios $\overline{h'_1}/\overline{t'}$ and $\overline{h'_1}/\overline{s''}$, $\sigma(\Delta/r)$ is 1.58 for the trigonometric group, and only 0.80 for the spectroscopic.

All these differences appear to be accidental. Weighting the ratios in the last column according to their probable errors, we obtain means as follows:

	Spectrum				
	A6–F2	F3–F6	F7–G3	G4–K2	K3–K8
Trig.	1.48	1.39	1.27	1.23	1.12
Spec.	1.59	1.34	1.36	1.28	1.24
Diff. = Δ	+0.11	−0.05	+0.09	+0.05	+0.12
P.E. = r	±0.08	±0.06	±0.05	±0.06	±0.07

The probable errors here given are smaller than would be calculated from Table 18 by the ordinary rules, because the majority of the stars are common to the trigonometric and spectroscopic lists, and hence the deviations z and w have substantially the same effect on both means, leaving their difference influenced by x and y. The corresponding correction factor for r ranges from 0.76 for the first group to 0.67 for the last. With this correction $\sigma(\Delta^2/r^2) = 1.43$, leaving nothing for systematic differences. The simple mean of the Δ's is $+0.06 \pm 0.028$. Hence, there is here again no evidence of significant systematic difference.

Comparison of the uncorrected spectroscopic parallaxes for the B and A stars with the trigonometric indicates only that the former are not seriously in error.

39. *Discordance of Trigonometric and Spectroscopic Results for the Giants.*—For the giants, however, there is a very serious difference between the mean reduced trigonometric and spectroscopic parallaxes. This arises from the parallaxes themselves. There are fifty-two stars for which both trigonometric and spectroscopic parallaxes have been determined. With the weights already used in the two cases, $\overline{t'} = 0''.063$ and $\overline{s'} = 0.102$, making $\overline{s''} = 0.106$—substantially the same as for the larger group. The weights used for $\overline{s'}$ are, however, not the same as for $\overline{t'}$ (which favor the nearer stars). To eliminate this, new means of s' were taken, with the same weighting used in computing $\overline{t'}$. The results are given in Table 19. The discrepancy

TABLE 19

	Spectrum			
	G1–G6	G7–K1	K2–M5	All
N	17	20	15	52
$\overline{t'}$	$0''.054$	$0''.079$	$0''.043$	$0''.063$
$\overline{s'}$	$0''.095$	$0''.097$	$0''.091$	$0''.095$
$\overline{s''}$	$0''.096$	$0''.099$	$0''.089$	$0''.096$

remains and is shown by all three spectral subdivisions—though most prominent in the M's. Its origin is very puzzling, for the sys-

tems of trigonometric and spectroscopic parallaxes are, as a whole, in excellent agreement for these stars. A comparison, made in connection with II but not published there, in which supergiants and subgiants were rejected, gives the values shown in Table 20. The weighting system here employed favors the nearer stars, in substantially the same manner as in the preceding table. There is nothing in the spectroscopic parallaxes of the double stars to indicate that they are unusual—though those of large modulus have somewhat greater values of $\overline{s}$ (Table 12). Nor is it easy to see how the trigonometric determinations can have been influenced by the duplicity of these stars, which are, for the most part, wide and unequal pairs. Only 10 stars of the 52 have at the same time $\Delta m < 3^m$ and $s < 3''$. It is very improbable that the presence of the companions should have exerted any sensible systematic influence on the mean parallax for the whole group.

TABLE 20

	Spectrum				
	G0–G7	G8–K1	K2–K5	M0–M6	All
N	145	166	201	129	641
$\overline{t}$	0.″091	0.″105	0.″100	0.″099	0.″0988
$\overline{s}$	0.″096	0.″101	0.″098	0.″100	0.″0985

40. *Appeal to Proper Motions.*—An independent test can be made with the aid of the "reduced" proper motions $\mu' = f\mu$ and the radial velocities V. The former have been taken from various sources, and the latter from Moore's catalogue.[40] Means of both have been derived, using for the double stars the same weighting system as in Table 19, and for "all stars" the same system as in Table 20. A few stars for which radial velocities were not available were omitted in calculating $|\overline{V}|$, the mean regardless of sign.

If the angles between the velocity vectors of the stars and the lines of sight are distributed at random, the average transverse

[40] *Pub. Lick Obs.*, 18, 1932.

TABLE 21

GIANT STARS: PARALLAXES FROM PROPER MOTION AND RADIAL VELOCITIES

Sp.	G0–G6			G7–K1			K2–M5			All		
Mod.	<5	5.0–6.5	All	<5	5.0–6.5	All	<5	5.0–6.5	All	<5	5.0–6.5	All
	Double Stars											
N	13	4	17	16	4	20	8	7	15	37	15	52
$[p]$	4.32	0.29	4.61	5.48	0.47	5.95	1.90	0.74	2.64	11.70	1.50	13.20
$\overline{s}$	0".094	0".107	0".095	0".094	0".133	0".097	0".082	0".114	0".091	0".092	0".118	0".095
$\overline{t}$	0".057	0".004	0".054	0".073	0".149	0".079	0".022	0".095	0".043	0".059	0".094	0".063
$\overline{\mu'}$	0".32	0".56	0".34	0".44	1".51	0".53	0".34	1".72	0".72	0".38	1".42	0".50
$\|\overline{V}\|$	14	21	14	16	19	16	28	25	28	17	22	18
$\frac{3.02\overline{\mu'}}{\|\overline{V}\|}$	0".069	0".081	0".073	0".083	0".240	0".100	0".037	0".208	0".078	0".067	0".195	0".084
	All Stars											
N	93	18	111	145	51	196	148	171	319	386	240	626
$[p]$	39.4	1.86	41.31	62.8	5.7	68.5	52.9	16.5	69.3	155.1	24.0	179.1
$\overline{s}$	0".098	0".103	0".098	0".099	0".101	0".099	0".096	0".102	0".098	0".098	0".102	0".098
$\overline{t}$	0".095	0".077	0".094	0".102	0".114	0".102	0".092	0".117	0".098	0".096	0".113	0".099
$\overline{\mu'}$	0".53	1".56	0".58	0".70	0".90	0".72	0".58	0".69	0".61	0".615	0".808	0".640
$\|\overline{V}\|$	15.0	20.9	15.2	19.0	22.8	19.3	22.7	20.7	22.3	19.2	21.2	19.5
$\frac{3.02\overline{\mu'}}{\|\overline{V}\|}$	0".107	0".226	0".115	0".111	0".119	0".112	0".077	0".101	0".082	0".097	0".115	0".099

velocity V_t will be $\pi/2$ times the average radial velocity V_r (regardless of sign). In astronomical units per year

$$V_t = \frac{\mu'}{p'} = \frac{\mu}{p}, \qquad V_r = \frac{V}{4.74}.$$

Now, by (17), (22), and (23),

$$p' = 0.1(1 + w'), \qquad \mu' = 0.1 V_t(1 + w').$$

If V_t and w' are not correlated, we will have

$$\overline{\mu'} = \overline{V_t} \cdot \overline{p'};$$

and hence

$$\overline{p'} = \frac{\overline{\mu'}}{1.57|\overline{V_r}|} = \frac{3.02\overline{\mu'}}{|\overline{V}|}. \tag{85}$$

Correlation between V_t and w' will be produced by a systematic change in velocity with absolute magnitude. This effect is probably insignificant in comparison with the errors of sampling for these small groups. The results are given in Table 21.

For the large groups of stars at the end of the lower part of the table, $\overline{p'}$ and $\overline{t'}$ agree better than might be expected. For the separate spectral classes the agreement is poorer—and bad for one group of very low weight.

Sp.	G7–K1		K2–M5		All	
Mod.	5.0–6.5	All	5.0–6.5	All	5.0–6.5	All
N	3	19	6	14	13	50
$[p]$	0.26	5.74	0.49	2.39	1.04	12.74
$\overline{s'}$	0."140	0."096	0."110	0."088	0."117	0."094
$\overline{t'}$	0."135	0."076	0."125	0."043	0."094	0."062
$\overline{\mu'}$	0."53	0."45	0."72	0."42	0."62	0."40
$\|\overline{V}\|$	12	16	14	26	15	17
$\frac{3.02\overline{\mu'}}{\|\overline{V}\|}$	0."133	0."085	0."155	0."049	0."125	0."071

The double stars are too few for this method to give more than rough values. For the brighter stars (modulus <5, all spectra), the results favor the trigonometric parallaxes; for the fainter stars, $\overline{p}'$ is wild, owing to a couple of very large proper motions, which affect also the general mean. The two stars in question are subgiants (§ 41). Their rejection would make the means shown in the table on page 63. This should improve the mean parallaxes derived from proper motions. The results thus obtained for all stars of a given spectral class lie between the spectroscopic and trigonometric values and, on the average, are nearer the latter. They do not settle decisively which is right, and the question must be left open.

41. *Subgiants.*—Seven stars with spectroscopic absolute magnitudes between $+2$ and $+3$ were omitted from the discussion of the giants, as probable "subgiants." There is no doubt that such faint giants exist. Whether they form a separate sequence parallel to the main bulk of the giants (as indicated by the values of M_0 adopted tentatively in Table 1) or whether they represent the statistical "fringe" of the ordinary giants is a question on which we do not wish to pass a final opinion here. It suffices to note that, if such stars are included in the list of ordinary giants, with the standard "giant" values of M_0 the factors f will be very large for them, and they will receive abnormally high weights in the averages (at least for spectroscopic parallaxes; the trigonometric weighting system automatically makes an almost complete correction). It was decided, therefore, to keep these stars separate, for fear of overloading the means. Only rough results can be obtained by discussing them separately—first, because they are so few; and, second, because the spectroscopic criteria for distinguishing between these stars and the common run of giants are still incompletely known. From a letter to us from Dr. Adams (February 28, 1938) we quote: "I am hoping that we shall be able to obtain spectra of some subgiant stars in the ultraviolet which will give us better criteria for the subgiant classification. I feel sure that such criteria exist, and that it is just a question of locating them."

These seven stars are listed in Table 50, Part IV, followed by three others, which, in the course of our discussion, were strongly sus-

pected of being subgiants. Two of these—95 Ceti (ADS 2459) and 84 Virginis (9000)—are mentioned in § 40 (p. 64). The third, γ Delphini (14279), has similar characteristics.

Dr. Adams, at our request, re-examined the spectra of these stars and writes that the first is clearly a subgiant, the revised spectroscopic absolute magnitude being 2.4. For the other two, the revised values are 2.0 and 1.8, and they should probably be included as subgiants.

These three stars had already been included in the general discussion of the giants, on the basis of their earlier spectroscopic absolute magnitudes (0.9, 0.7, and 0.7). To remove them would have involved much more labor than was worth while, especially in view of the general uncertainties (§ 40); but they have been added here in order to strengthen the scanty data for the subgiants.

It should be noted that, in this section of Table 50, the moduli and values of h_1' and t' have been computed for these three stars with the subgiant values of M_0 (as for the other seven) and that Adams' new values of the spectroscopic absolute magnitude have been used.

No correction has been applied to the spectroscopic parallaxes, as it is uncertain whether the conclusions derived from the correlations which hold good for the giants in general are applicable in this case.

The results obtained for the subgiants are included in Tables 18 and 23.

42. *Dwarfs of Class M.*—The dwarf stars of class M have been deliberately omitted from the foregoing statistical treatment, since their parallaxes are all large and the errors of the trigonometric values relatively small, so that these may safely be put in the denominator. We may then take the mean of the individual values h_1/t and obtain directly a mean value of $\mathrm{M}^{1/3}$. The data are given in Table 22.

The weights attached to h_1/t and h_1/s have been determined according to the ordinary rules. They are low for the first orbit because it has been graded as "poor."

The weighted mean of h_1/t for the four orbits is 0.85 ± 0.05 and for the seven physical pairs is 0.96 ± 0.10. These probable errors

are derived from the residuals for the individual values and should include all sources of error. The probable error of unit weight for the physical pairs is ±0.245—agreeing, as well as might be expected, with the statistical prediction. The spectroscopic parallaxes, as they stand, give h_1/s = 1.02 ± 0.07 for the orbits, and 1.17 ± 0.12 for

TABLE 22

DWARF STARS OF CLASS M

ADS	m_b	Δm	Sp.	h_1	t	s	h_1/t	p	h_1/s	p
	Orbits									
(Finsen)*	9.5	0.1	Ma	0″.060	0″.051±0.007	......	1.18	0.6	.....	.....
15972†	9.64	1.7	M3	.188	.258± .004	0″.158	0.73	10	1.19	2.0
(Kuiper)‡	9.86	0.2	M4e	.132	.147± .004	.138	0.90	2.5	0.96	1.25
10786§	10.21	0.5	M4	0.105	0.109±0.006	0.118	0.96	7	0.89	2.0
	Physical Pairs									
48	9.28	0.2	Mo	0″.113	0″.089±0.005	0″.079	1.27	0.97	1.43	0.7
7251	7.90	0.1	Mo	.215	.162± .003	.151	1.33	.99	1.43	.7
8887	9.52	0.2	Mo	.023	.067± .011	.045	0.34	.86	0.51	.7
9352	9.9	0.5	Mo	.034	....	.029	.....	.....	1.17	.7
7067	9.30	0.1	M1	.090	.096± .004	.066	0.94	.98	1.36	.7
246	7.80	3.0	M3	.230	.284± .005	.243	0.81	.97	0.95	.7
433	10.37	3.0	M3	.139	.103± .007	.069	1.35	.64	2.01	.5
11632	8.8	0.9	M4	0.201	0.282±0.004	0.282	0.71	0.98	0.71	0.7

* 21h39m.5 −58°8′ (1900).
† Kr 60.
‡ 16h50m.1 −8°9′ (1900).
§ μ Herc.

the physical pairs. It is well known that, for this one group of stars, the trigonometric parallaxes are much more reliable.

43. *Determination of Mass of Brighter Component.*—To pass from the mass M of the system to the mass M_b of the brighter component, we must divide by $1 + o'$, where $(1 + o')^3 = 1 + M_f/M_b$. The mass ratio is known only for a moderate number of binaries, and, in general, we must be content with estimating it from the difference in magnitude with the aid of the mass-luminosity relation. According to this, o' is a function of M and ΔM (both bolometric). From Ed-

dington's table (with zero-point shifted by $0^m.5$, as in I) values were computed the course of which is as follows:

M	ΔM 0	1	2	3	4	6	8	10
− 5..........	0.26	0.17	0.10	0.07	0.05	0.02	0.01	0.01
0..........	.26	.20	.15	.12	.09	.06	.04	.02
5..........	.26	.22	.18	.15	.12	.08	.06	0.04
10..........	0.26	0.22	0.18	0.15	0.13	0.09	0.06	....

A diagram giving contours for o' with M and ΔM as co-ordinates permitted its value to be read off. When one or both components of the double is a spectroscopic binary, the value of o' is greater. When the bright component is double, we have assumed (following the practice of § 9) that the second component of the brighter pair has a mass 0.9 that of the primary, if both spectra are visible, and 0.6 if only one spectrum appears. Contour diagrams were prepared for these cases and used when required. In the first diagram, o' ranges from 0.43 ($\Delta M = 0$) to 0.26 ($\Delta M = 10$); in the second, from 0.38 to 0.19). The values of M_b and ΔM must, of course, be corrected to refer to the brighter component of the spectroscopic pair (§ 9).

When only the fainter visual component is a spectroscopic binary, the observed ΔM must be increased by the appropriate amount, and the assumed mass of the secondary multiplied by 1.9 or 1.6. It was found that the results might be reproduced by multiplying the values of o', found when both components are single, by the factors 1.68 and 1.48. The outstanding errors never exceed 0.01.

When both components are spectroscopic binaries with both spectra visible, the value of $(1 + o')$ for single stars must be multiplied by $\sqrt[3]{1.9}$, or 1.24; when both show one spectrum, by 1.17. It is obviously sufficient to use the factor 1.20 when one shows one spectrum and the other two.

The absolute bolometric magnitudes M used in this discussion were obtained by adding to the standard visual absolute magnitudes M_0 for each spectral class the bolometric corrections given in I,

Table A (which include Eddington's correction for the influence of effective temperature on luminosity). The dependence of o' upon M is so small that individual differences from M_0 may be neglected.

For main-sequence stars it was assumed that the companion was also on the main sequence. The correction $C' = M_{\text{bol}} - M_{\text{vis}}$ was plotted against M_0 for the various spectral classes; and a contour diagram was derived from this, which gave $\Delta C'$ as a function of the spectral class and visual Δm.

For the giants it was assumed that, when Δm (vis) exceeded 1^m, the companion was of early type and had $C' = 0$, so that ΔM (bol) $= \Delta m$ (vis) $- C'_b$. When Δm was less than 1^m, it was assumed that ΔM (bol) $= \Delta m$ (vis). This summary process should give results not far from the truth.

It is well known that most observers tend to estimate the visual magnitude-difference too great, especially for close pairs.[41] Öpik[42] has given tables for correction of the estimated values. These corrections were applied for 50 giant stars for which no photometric measures of Δm exist. The resulting value of $\overline{o'}$ was 0.14, and would have been 0.13 if this correction had been neglected. It was therefore judged unnecessary to apply it.

For any group of stars we should then have

$$\overline{\mathfrak{M}^{1/3}} = \overline{\mathfrak{M}_b^{1/3}}(1 + \overline{o'}) \,. \tag{86}$$

G. COLLECTED RESULTS

44. *Review.*—At this point it may be well to review the significance of these results. The quantities $\overline{h'_1}$ in Table 18 represent means of the dynamical parallaxes for mass unity of the system, "reduced" by multiplying by factors f (eq. [18]), corresponding to a change of the visual magnitude m_b of the brighter component to a standard value M_0 substantially equal to the mean absolute magnitude of stars of the given spectral subclass (Table 1), dwarfs and giants being separated and subgiants and supergiants excluded. The values of $\overline{t'}$ represent means of the trigonometric and cluster parallaxes, similarly reduced; and $\overline{s''}$, similar means of the spectroscopic parallaxes, corrected to the trigonometric system by the

[41] See Kuiper, *Pub. A.S.P.*, 47, 24, 1935. [42] *Tartu Pub.*, 25, 6, 1924.

formulae found by the authors.[43] For the B and A stars these corrections are not available. Differences in the distances of the stars are thus eliminated; but new differences are introduced, depending upon the dispersion in absolute magnitude. For the trigonometric parallaxes, for example, we have

$$t' = \overline{t'}(1 + x)(1 + w) , \tag{40}$$

where $\overline{t'}$ is the mean for a very large group, x represents the errors of the parallax measures, and w the error of the assumption that the stars have the absolute magnitude M_0. For the corrected spectroscopic parallaxes,

$$\overline{s''} = \overline{t'}(1 + y)(1 + w) ,$$

where y represents the spectroscopic accidental errors. The mean values of x and w are zero, and they are not correlated with each other. We found, also, that

$$\overline{t'} = 0''.1 \, TF(1 + \tfrac{1}{2}W^2) , \tag{39}$$

where $T = 1$ if Schlesinger's system of trigonometric parallaxes is taken as standard (as is done here), and that

$$F = 10^{0.2(\overline{M_b} - M_0)} , \tag{36}$$

where $\overline{M_b}$ is the actual mean absolute magnitude of the brighter components and W is the mean-square value of w.

Similarly, we have

$$h_1' = \overline{h_1'}(1 + o)(1 + u)(1 + z)(1 + kw) , \tag{48}$$

where z arises from errors of the double-star measures and of the statistical averaging process adopted for slow-moving pairs, u from the deviations of individual stars from the mean mass-luminosity relation, and o from the differences in the mass ratio in individual pairs. The mean values of all these are zero, and they are not

[43] Eq. (33); see discussion in II.

sensibly correlated with each other or with w or x. If the mass-luminosity relation is expressible (over a given range) by

$$\mathfrak{M} = CL^n , \tag{26}$$

then

$$k = 1 - \tfrac{2}{3}n . \tag{43}$$

For the mean value we have

$$\overline{h'_1} = 0.1\overline{\mathfrak{M}^{1/3}}\, HF(1 + \tfrac{1}{2}k^2W^2) . \tag{47}$$

Here H takes account of the residual effects of the neglect of "uninteresting" pairs by double-star observers, the main part of which has been eliminated by confining the discussion to the brighter pairs (§§ 34 and 35). We shall see later (p. 118) that we may set $H = 1$. Finally,

$$\overline{\mathfrak{M}^{1/3}} = \overline{\mathfrak{M}_b^{1/3}}(1 + \overline{\sigma'}) , \tag{86}$$

where $\overline{\mathfrak{M}_b}$ is the mean mass of a star of absolute magnitude $\overline{M_b}$, and $1 + \overline{\sigma'}$ the mean value of $(1 + \mathfrak{M}_f/\mathfrak{M}_b)^{1/3}$. The ratio $\mathfrak{M}_f/\mathfrak{M}_b$ must be calculated from the difference of bolometric magnitude, using the mass-luminosity relation (§ 43). This relation need, however, be only roughly known to permit the calculation of $\overline{\sigma'}$—and also of k in equation (47)—with all needful accuracy. The deduced mean values of $\mathfrak{M}_b$ and $\overline{M_b}$ may be regarded substantially as observational data. The probable errors of $\overline{h'}$, $\overline{t'}$, and $\overline{\mathfrak{M}_b^{1/3}}$ depend upon the mean-square values W, X, Y, and Z of w, x, y, and z (§ 30). The determination of these quantities and its statistical uncertainty is discussed in §§ 22–29 and 31–37. From equations (36) and (39) we find

$$\left.\begin{aligned}\overline{M_b} - M_0 &= 5 \log_{10} \{10\overline{t'}(1 - \tfrac{1}{2}W^2)\} \\ &= 5 \log_{10} (10\overline{t'}) - 1.09W^2 .\end{aligned}\right\} \tag{87}$$

The correction to bolometric values is discussed in § 43. From equations (39), (86), and (47)

$$\overline{\mathfrak{M}^{1/3}} = \overline{\mathfrak{M}_b^{1/3}}(1 + \overline{\sigma'}) = \frac{\overline{h'_1}}{\overline{t'}}\frac{T}{H}\{1 + \tfrac{1}{2}(1 - k^2)W^2\} . \tag{88}$$

We may set $H = T = 1$.

For main-sequence stars, from B8 on, we have (§ 38) $k = 0.82$ and $W^2 = 0.110$; for giants, $k = 0.72$ and $W^2 = 0.150$; whence

	Main Sequence	Giants	
$\overline{M_b} = M_0 + 5 \log_{10} (10\overline{t'})$	$-0^m.12$	$-0^m.16$	(89)
$\overline{M^{1/3}} =$	$1.018 \dfrac{\overline{h'_1}}{\overline{t'}}$	$1.036 \dfrac{\overline{h'_1}}{\overline{t'}}$	(90)

The latter values may be adopted for the B stars. For the dwarf M's the correction is different. If σ is the mean error corresponding to r_t/t, the mean value of $1/t$ will be increased by accidental error in the ratio $1 + \sigma^2$. The weighted mean value of $\sigma^2 = 0.016$ for the orbits and 0.12 for the physical pairs. Taking the mean,

$$\overline{M^{1/3}} = 0.986 \overline{\left(\frac{h_1}{t}\right)} . \tag{91}$$

The absolute magnitudes need no correction.

45. *Final Results for Masses of Visual Pairs.*—Applying these equations to the data of Tables 18 and 22, we have the results given in Table 23, which represent the final results for the masses of visual double stars. The first three columns give the spectral class and the type of dynamical parallax (orbits or physical pairs). Then follows the cube root of the combined mass, derived from equation (90); then the factor $1 + \overline{\sigma}$, discussed in § 43, and the cube root of the mass of the brighter component. The next column gives the absolute bolometric magnitude of this component derived from equation (89). This quantity was computed for each spectral subclass separately (using the mean value of t' or s'' for the group); and the weighted mean of the results was taken.

Since log $\overline{M_b^{1/3}}$ varies almost linearly with $\overline{M_b}$, its values are next given, followed by their probable errors and weights, p (unit weight corresponding to a probable error of ± 0.020 in the logarithm, or ± 4.6 per cent).

Capella, which is the only giant for which a good orbit is available, has been put by itself. The observations are so accurate that it deserves the high weight. The eclipsing binary μ_1 Scorpii, for which

TABLE 23

VALUES OF $\overline{M_b^{1/3}}$ FOR VISUAL BINARIES

Sp.	$\overline{Sp.}$		N	$\overline{M^{1/3}}$	$1+\overline{\sigma'}$	$\overline{M_b^{1/3}}$	$\overline{M_b}$	$\log \overline{M_b^{1/3}}$	p	O−C₁	O−C₂
	Trigonometric Parallaxes										
μ_1 Sco....	B3	...	1	.....	.....	2.28.......	−2.24	+0.358........	0.5	+0.023	+0.068
O9–B7...	B2.8	P	34	2.08	1.21	1.72±0.10	−1.46	+ .236±0.028	0.5	− .052	− .024
Giants...	G9.1	P	55	2.49	1.11	2.24± .21	−1.46	+ .350± .043	0.2	+ .062	+ .090
Capella...	*G1*	O	1	1.96	1.21	1.62± .02	−0.25	+ 210± .010	1.0	− .010	− .003
B8–A5...	A1.3	P	72	1.51	1.19	1.27± .07	+1.00	+ .104± .025	0.6	− .055	− .060
	A2.1	O	21	1.82	1.17	1.56± .08	+1.21	+ .194± .022	0.8	+ .045	+ .039
A6–F2...	F0.0	P	22	1.53	1.19	1.28± .11	+2.48	+ .107± .038	0.3	+ .014	+ .001
	A9.5	O	19	1.49	1.24	1.20± .07	+2.78	+ .079± .025	0.6	− .003	− .016
F3–F6....	F4.7	P	26	1.59	1.19	1.34± .10	+3.09	+ .127± .032	0.4	+ .058	+ .044
	F3.7	O	17	1.36	1.18	1.15± .05	+3.34	+ .061± .019	1.1	+ .001	− .011
F7–G3...	F9.9	P	30	1.41	1.21	1.16± .08	+4.22	+ .064± .029	0.5	+ .037	+ .026
	F9.8	O	25	1.28	1.21	1.06± .03	+4.25	+ .026± .014	2.0	000	− .011
G4–K2...	G9.5	P	19	1.26	1.22	1.03± .08	+4.64	+ .013± .035	0.3	000	− .009
	G7.4	O	15	1.25	1.23	1.02± .04	+5.24	+ .008± .017	1.4	+ .016	+ .010
K3–K8...	K5.1	O	8	1.19	1.23	0.97± .06	+5.69	− .013± .029	0.5	+ .010	+ .006
	K4.7	P	12	1.08	1.22	0.88± .07	+6.24	− .055± .036	0.3	− .014	− .014
M0–M4..	M1.5	P	7	0.94	1.24	0.76± .08	+7.93	− .118± .046	0.2	− .022	− .011
	M2.9	O	4	0.84	1.24	0.68±0.04	+9.18	− .170±0.026	0.6	− .035	− .015
Subgiants.	K0	...	5	1.54	1.12	1.37.......	+1.1	+0.138........	0.07	−0.014	−0.026
	Spectroscopic Parallaxes										
O9–B7...	B3.1	P	41	(2.52)	1.20	(2.10±0.16)	(−1.46)	(+0.322±0.034)	....	....	....
Giants...	G8.9	P	80	1.65	1.13	1.46± .06	−0.25	+ .165± .020	1.0	−0.055	−0.047
B8–A5...	A1.2	P	107	(1.44)	1.21	(1.19± .05)	(+1.40)	(+ .076± .019)	....	....	....
	A1.9	O	18	(1.48)	1.22	(1.21± .10)	(+1.73)	(+ .083± .030)	....	....	....
A6–F2...	A9.9	P	48	1.57	1.20	1.31± .07	+2.37	+ .118± .024	0.7	+ .021	+ .008
	A9.5	O	20	1.66	1.26	1.32± .08	+2.60	+ .120± .028	0.5	+ .032	+ .018
F3–F6....	F4.6	P	52	1.37	1.21	1.13± .06	+3.36	+ .054± .023	0.8	− .006	− .019
	F4.4	O	21	1.36	1.24	1.10± .07	+3.65	+ .040± .028	0.5	− .009	− .021
F7–G3...	F9.3	O	30	1.39	1.24	1.21± .05	+4.18	+ .050± .022	0.8	+ .022	+ .010
	F9.9	P	40	1.38	1.21	1.14± .07	+4.27	+ .057± .025	0.6	+ .032	+ .021
G4–K2...	G8.2	P	24	1.38	1.22	1.13± .08	+4.92	+ .054± .034	0.35	+ .051	+ .044
	G7.6	O	12	1.20	1.24	0.97± .07	+5.24	− .013± .033	0.35	− .006	− .012
K3–K8...	K5.2	O	6	1.42	1.24	1.15± .13	+5.17	+ .060± .050	0.16	+ .065	+ .059
	K5.0	P	17	1.19	1.23	0.97± .08	+5.98	− .013± .039	0.26	+ .019	+ .018
M0–M4..	.....	P	8	(1.15)	1.26	(0.92± .10)	(+7.65)	(− .036± .045)	....	....	....
	.....	O	3	(1.00)	1.24	(0.81±0.06)	(+9.10)	(− .092±0.030)	....	....	....
Subgiants.	G9	...	10	1.63	1.17	1.39	+1.6	+0.134	0.12	+0.004	−0.010

good value of the mass has been derived by Elvey and Rudnick,[44] while the parallax is given as $0''.0074 \pm 0''.0012$ in Kapteyn's paper,[45] has been included, the mean value for the two nearly equal components being taken.

The spectroscopic parallaxes of stars of classes B and A have not been calibrated like the rest, while for the dwarf M's the calibration is not very accurate. The results depending on these spectroscopic parallaxes are therefore inclosed in parentheses and have not been used in the discussion.

H. THE MASS-LUMINOSITY RELATION

46. *New Empirical Formula.*—If the mean values of log $\overline{\mathfrak{M}_b^{1/3}}$ are plotted against $\overline{M_b}$,[46] a conspicuous slope is shown, which represents the mass-luminosity relation. These data may be discussed in various ways. We may first test the applicability of Eddington's original mass-luminosity curve.[47] In I (p. 96) this curve was used with the zero-point shifted by $+0^{m}.5$ (making the brightness less for a given mass), in order to represent the mean of the best-determined binaries. The only disposable constant in a new discussion is this zero-point. If this is changed by x magnitudes, the equations of condition are $ax = n$, where $a = \frac{1}{3}d \log \mathfrak{M}/dM$ and has the following numerical values:

M.....	−2	0	2	4	6	8	10
a......	−0.064	−0.052	−0.043	−0.036	−0.033	−0.032	−0.031

The resulting normal equation is $0.0309x = +0.0048$, whence $x = +0^{m}.15$. The resulting residuals are given in Table 23 as $O - C_1$.[48] They give ± 0.016 for the probable error of unit weight, and $\pm 0^{m}.09$ for that of x. The correction to Eddington's original zero-point (derived from Capella alone) is $+0^{m}.65 \pm 0^{m}.09$.

If we assume a linear relation, of the form log $\mathfrak{M}_b^{1/3} = A + BM_b$, we find from the same data

$$\log \overline{\mathfrak{M}_b^{1/3}} = (-0.0391 \pm 0.0014)(\overline{M}_b - 5.20)\ . \qquad (92)$$

[44] *Ap. J.*, 87, 553, 1938.

[45] *Ap. J.*, 40, 120, 1914; *Mt. W. Contr.*, No. 82, p. 80.

[46] Cf. Fig. 1, p. 109. [47] *The Internal Constitution of the Stars*, p. 137.

[48] The subgiants were added later and were not used in this solution.

The probable error of unit weight is ± 0.0147, and the residuals are given as $O - C_2$ in Table 23. This solution gives

$$\log \mathfrak{M}_b = -0.117 M_b + \text{constant} = + 0.293 \log L_b + \text{constant} .$$

Hence, $n = 0.293 \pm 0.011$ and $L \propto \mathfrak{M}^{3.42 \pm 0.12}$. Within the error of determination, we may set $\mathfrak{M} \propto L^{2/7}$ and $L \propto \mathfrak{M}^{7/2}$.

Both solutions apparently agree with the observations too well, for the probable error of unit weight, based on the values derived from the dispersion constants previously adopted, was ± 0.020. These values, however, though applying to the tabular values of $\log \overline{\mathfrak{M}_b^{1/3}}$, are not appropriate to their differences from the computed values of the same quantity. The latter have been computed from the observed value of $\overline{t'}$ or $\overline{s''}$, and the effects of observational errors upon them are correlated with those upon the tabular values. The "observed" value of $\log \overline{\mathfrak{M}_b^{1/3}}$ is

$$O = \log \overline{h'_1} - \log \overline{t'} - \log (1 + \overline{o'}) - \log \{1 - \tfrac{1}{2}(1 - k^2)W^2\} ,$$

or, by (62),

$$O = \log h_0 - \log t_0 - \log (1 + \overline{o'}) + \log (1 + \bar{z}) - \log (1 + \bar{x}) \\ + \log \{1 - (1 - k)\bar{w}\} - \log \{1 - \tfrac{1}{2}(1 - k^2)W^2\} .$$

For the computed value, adopting the second solution above, we have

$$C = \text{constant} + \tfrac{1}{3}n \log L = \text{constant} - \tfrac{2}{3}n \log \overline{t'} \\ = \text{constant} - \tfrac{2}{3}n \{\log (1 + \bar{x}) + \log (1 + \bar{w})\} .$$

Since $k = 1 - \frac{2}{3}n$, we have to the first order

$$O - C = \text{constant} + \log (1 + \bar{z}) - k \log (1 + \bar{x}) .$$

From equation (69) we have

$$\{\sigma(O)\}^2 = \frac{1}{N} (\text{Mod})^2 \left\{\frac{R}{P} Z_1^2 + \frac{R'}{P} X_1^2 + (1 + A)(1 - k)^2 W^2 \right\} ,$$

while

$$\{\sigma(O - C)\}^2 = \frac{1}{N} (\text{Mod})^2 \left(\frac{R}{P} Z_1^2 + \frac{R'}{P} k^2 X_1^2 \right) .$$

Examination of the calculations upon which the probable errors of Table 18 are based showed that this change would reduce the squares of these probable errors, r, by factors ranging from 0.94 to 0.68. Multiplying the value of r^2 for each value of log $\overline{\text{M}_b^{1/3}}$ in Table 23 by the appropriate factor and taking a mean with the tabular weights, the probable error of unit weight now comes out ± 0.0179. The discrepancy with the values found from the solutions is much reduced. The latter values, being derived from 29 entries, should have a statistical probable error of $0.675/\sqrt{58}$, or 9 per cent of their own amounts, that is, of ± 0.0014. This leaves little to explain.

If the mass-luminosity "law" is regarded as a relation between log M and M, it is evident that the data of Table 23 suffice for a good determination of the zero-point and slope of the line representing this relation, but not of its curvature. All available material for visual binaries is included in this table; hence, information regarding the curvature can be obtained, if at all, from spectroscopic binaries of great mass (see chap. ii). It will be shown later (§ 66) that the giants might well have been excluded in the linear solution upon which equation (92) is based. When this is done, we find

$$\log \overline{\text{M}_b^{1/3}} = (-0.0407 \pm 0.0016)(\overline{M_b} - 5.19)\,, \tag{93}$$

instead of

$$(-0.0391 \pm 0.0014)(\overline{M_b} - 5.20)\,. \tag{92}$$

The difference is within the probable error and need not disturb us. We will, however, adopt as our final empirical formula

$$\log \overline{\text{M}_b^{1/3}} = -\,0.0400\,(\overline{M_b} - 5.20)\,. \tag{94}$$

For the calculation of dynamical parallaxes we regard this as definitely preferable to Eddington's old curve.

The latter is based on the assumed molecular weight 2.11, that is, on the assumption that there is very little hydrogen in the interior of the stars; and it is remarkable that it affords as good an approximation to the facts as it does.

Present knowledge of the relative abundance of hydrogen, helium, and heavy elements in the stars does not appear to justify an at-

tempt to construct an improved mass-luminosity curve—at least for the purpose of calibration of dynamical parallaxes.

It is interesting to find so little evidence of departures from the mean mass-luminosity relation for groups of stars of various spectral types. The particularly interesting question whether the red giant stars behave similarly to the rest is unfortunately obscured by the systematic difference between the trigonometric and spectroscopic parallaxes of these stars (§ 39). The former indicate a greater mass for the same luminosity than the mean curve, and the latter a smaller one, by about the same amount. For equal masses the difference in luminosity derived from the two sets of data (on the linear formula) is $3\overset{m}{.}7$. The fault lies definitely with the parallaxes. Further trigonometric observations and a review of the spectroscopic methods and data are much to be desired both for giants and for subgiants.

The results for the subgiants are of low accuracy and may be regarded only as an indication that they are in general agreement with the mass-luminosity relation.

47. *Detection of Departures from Mass-Luminosity Relation.*— The observational requirements for verifying departures from the mass-luminosity relation, in the case of a visual binary, are severe.

We have, in general,

$$\log \mathfrak{M} = \text{constant} + 3 \log h_1 - 3 \log t \,,$$

$$M = \text{constant} + 5 \log t \,.$$

We may write

$$\log \mathfrak{M} = \text{constant} - \tfrac{2}{5} n(M + x) \,,$$

where x represents the deviation in absolute magnitude from the assumed linear relation (92).

We then have

$$3 \log h_1 - (3 - 2n) \log t + \tfrac{2}{5} nx = \text{constant} \,.$$

Setting $n = 0.293$, we have

$$x - 20.6 \log t + 25.6 \log h_1 = \text{constant} \,.$$

A departure of 1^m in x therefore corresponds to changes of 0.048 in log t, or 0.039 in log h_1, or to alterations of 12 per cent in t or 10 per cent in h_1.

If, however, M can be determined independently, as in the case of an eclipsing variable, we have

$$\log \text{M} = \text{constant} - 2n \log t - \tfrac{3}{8} nx\,,$$

or, with $n = 0.293$,

$$x + 5 \log t + 8.5 \log \text{M} = \text{constant}\,.$$

As M always involves the cube of an observed quantity (for example, the velocity amplitude K of a spectroscopic binary), the demands for precision upon this quantity are as great as upon h_1. But an error in parallax is only one-fourth as serious for a spectroscopic binary.

CHAPTER II

SPECTROSCOPIC AND ECLIPSING BINARIES

A. SPECTROSCOPIC BINARIES

48. *Observational Data.*—Orbits are available for a great number of spectroscopic binaries, 375 being listed in Moore's catalogue.[1] For the majority of these but one spectrum has been observed; and only the mass function

$$f = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2}$$

can be calculated. To obtain the individual masses—or their sum—from this, it is necessary to know the mass ratio M_2/M_1.

This cannot be directly determined. If we may take the sum of the masses from other sources (such as spectroscopic binaries showing two spectra), the values of f give important information about the statistical distribution of the mass ratio.

This problem has recently been discussed by Plummer,[2] Colacevich,[3] and Kuiper.[4] Their investigations, by different methods, agree in concluding that all values of the ratio $M_2/(M_1 + M_2)$ are equally probable, and there is no need of repeating the discussion here.

When both spectra have been observed, the values of $M_b \sin^3 i$ and $M_f \sin^3 i$ are separately determinable. In accordance with the general line of the present discussion we may replace these by $M_b^{1/3} \sin i$ and $M_f^{1/3} \sin i$. Two cases then arise:

1. For eclipsing variables $\sin i$ is known—usually with high accuracy, since the light-curve provides a value of $\cos i$—and the individual masses may be found, free of all hypotheses. This provides one of the most important sources of knowledge of stellar masses, especially for the stars of high luminosity.

[1] *Lick Obs. Bull.*, 18, No. 483, p. 5, 1936.

[2] *M.N.*, 98, 137, 1937.

[3] *Mem. Soc. astr. ital.*, N.S., 11, 115, 1938.

[4] *Pub. A.S.P.*, 47, 130, 1935.

2. In the absence of eclipses we must estimate the mean value of sin i. If the orbit planes are distributed at random,

$$\overline{\sin i} = \int_0^{\pi/2} \sin^2 i \, di = \frac{\pi}{4} = 0.785 , \qquad \log_{10} \overline{\sin i} = -0.105 .$$

But pairs with small inclinations are much less likely to show resolved double lines. With Schlesinger's suggestion that the probability of inclusion in our lists is proportional to sin i, we have

$$\overline{\sin i} = \frac{4}{\pi}\int_0^{\pi/2} \sin^3 i \, di = \frac{8}{3\pi} = 0.849 , \qquad \log \overline{\sin i} = -0.070 .$$

The exaggerated assumption that the probability of inclusion varied as $\sin^2 i$ (so that reduction of the velocity range to 71 per cent would cause a loss of half the pairs) would give

$$\overline{\sin i} = \frac{9\pi}{32} = 0.884 , \qquad \log \overline{\sin i} = -0.054 .$$

We will adopt Schlesinger's assumption, recognizing that for individual groups it may be in error by as much as 5 per cent.

The data upon which the present discussion is based are summarized in Table 24. They are taken from Moore's catalogue, with the exception of 10 stars, for which the sources are given in notes and they are grouped according to spectral class.

The successive columns give: (1) the number in Moore's catalogue or, when the star is not in this, a reference to a footnote (eclipsing binaries are indicated by an asterisk); (2) the visual magnitude of the combined light of the system (the maximum brightness for an eclipsing variable); (3) the spectrum of the brighter component; (4) the period in days; (5) the orbital eccentricity; (6) and (7) the values of $\mathfrak{M}_b^{1/3} \sin i$ and $\mathfrak{M}_f^{1/3} \sin i$; and (8) the spectroscopic absolute magnitude of the brighter component, according to *Mount Wilson Contribution* No. 511, for those stars which appear in this list. The parallaxes of these stars are so small that the trigonometric determinations give little information; and the spectroscopic parallaxes for A and B stars have not been recalibrated. They are therefore omitted from the table.

49. *Means by Groups.*—On inspection of the table, a few stars of exceptionally great mass are conspicuous—notably No. 116 (Plaskett's

TABLE 24

DATA FOR SPECTROSCOPIC BINARIES

No.	Mag.	Sp.	P	e	$\mathfrak{M}_b^{1/3} \sin i$	$\mathfrak{M}_f^{1/3} \sin i$	Mb sp
121*	4.90	O7	4.393	0.16	3.18	2.89	....
6*	6.1	O8.5	3.523	.04	2.60	2.54	....
45*	7.3	O8	3.369	.10	2.66	2.09	....
116	6.06	O8	14.414	.04	4.23	3.98	....
255	5.71	Oe5	3.367	.00	2.40	2.40	....
322*	6.95	O9	2.996	.13	2.58	2.59	....
86*	3.7	B0	7.989	.02	2.24	2.20	....
306	7.7	B0	8.334	.26	2.40	2.34	....
360*	6.63	B0	1.775	.03	2.25	2.14	....
374	6.05	B0	13.419	.1	2.38	2.22	....
137*†	4.14	B1	1.454	.08	2.67	2.41	....
50	3.94	B2	4.419	.00	1.75	1.56	....
88	4.66	B2	2.526	.06	1.77	1.61	....
192	1.21	B2	4.014	.10	2.08	1.78	....
199‡	3.06	B2	8.024	.5	2.02	2.02	....
225	2.90	B2	6.828	.27	2.35	2.02	....
313	6.28	B2	2.985	.10	1.43	1.33	....
12	4.44	B3	143.61	.56	3.00	2.22	....
55*	6.33	B3	2.029	.05	1.69	1.62	....
67	6.89	B3	2.207	.08	1.91	1.55	....
78*	8.06	B3	1.333	.00	1.88	1.74	....
103	5.89	B3	7.827	.25	2.18	1.57	....
223	3.00	B3	1.571	.05	1.44	1.31	....
241*‡§	3.09	B3	1.446	.05	2.02	2.02	....
250*	4.60	B3	2.051	.05	1.89	1.38	....
296*	6.97	B3	2.454	.00	1.74	1.33	....
299*	4.99	B3	1.950	.00	1.74	1.64	....
323	4.68	B3	2.855	.13	1.24	1.21	....
357	6.20	B3	10.911	.25	1.82	1.57	....
364	6.28	B3	7.251	.38	1.69	1.43	....
176	5.28	B4	2.963	.06	2.02	1.64	....
18	4.42	B5	4.283	.0	1.14	1.03	....
113	6.58	B5	1.190	.04	1.84	1.60	....
213*	7.65	B5	3.452	.07	1.86	1.34	....
249*	5.67	B5	1.677	.00	1.74	1.66	....
355	4.66	B5	2.616	.02	0.95	0.89	....
344	5.54	B7	17.326	.22	2.75	2.39	....
20	4.13	B8	1.670	.14	1.41	1.26	....
49	4.32	B8	6.224	.20	1.42	1.40	....
123	5.61	B8	2.260	.08	1.62	1.32	....
221	5.07	B8	12.585	.03	2.37	2.36	....
286	6.68	B8	1.849	.03	1.92	1.64	....
4	7.08	cB9	55.904	.03	4.84	3.55	....
59	3.59	B9	5.011	.01	0.82	0.82	....
76	5.19	B9	5.522	.07	1.36	1.30	....
84*	6.4	B9	4.135	.0	1.35	1.32	....
164*‖	6.78	B9	3.063	0.15	1.44	0.96	....

* Eclipsing variable.

† Van Gent, *B.A.N.*, 8, No. 317, p. 319, 1939.

‡ Masses assumed to be equal.

§ Rudnick and Elvey, *Ap. J.*, 87, 553, 1938.

‖ O'Keefe, *Harvard Bull.*, No 908, p. 29, 1938.

TABLE 24—*Continued*

No.	Mag.	Sp.	P	e	$\mathfrak{M}_b^{1/3} \sin i$	$\mathfrak{M}_f^{1/3} \sin i$	M_b sp
293*	6.52	B9, G2	3.381	0.03	1.88	1.27	....
308	3.37	B9	17.12	.69	0.83	0.76	....
319*	8.23	B9	0.718	.0	0.99	0.94	....
7*	7.30	A0	1.813	.0	1.17	0.98	....
30	5.08	A0	15.294	.61	0.52	0.51	....
102	4.54	A0	5.969	.02	0.86	0.76	....
B2142¶	6.24	A0	18.772	.17	1.36	1.30	....
170	4.78	A0	2.5	.11	0.49	0.43	....
182	4.00	A0	71.9	.34	1.14	1.02	....
187	5.81	A0	3.287	.04	1.35	1.27	....
237	6.27	A0	10.56	.43	1.29	1.10	....
245	3.92	A0	4.024	.02	1.17	1.00	....
273*	7.20	A0	1.779	.00	1.28	1.23	....
275	5.08	A0	9.810	.02	0.49	0.46	....
281	5.37	A0	4.118	.01	0.98	0.97	....
294	6.19	A0	10.393	.52	1.06	0.94	....
330	6.03	A0	20.30	.44	1.23	1.03	....
339	5.48	A0	1.729	.03	0.98	0.98	....
346*	8.50	A0	1.677	.12	1.44	1.37	....
85	5.72	A1	2.152	.00	1.19	1.14	....
309	5.46	A1	9.316	.01	1.31	1.27	....
104*	2.07	A2	3.960	.00	1.30	1.30	1.8
152	5.70	A2	15.986	.50	1.14	1.08	....
95 Leo**	5.49	A2	6.625	.02	1.00	0.89	1.2
191	2.40	A2	20.536	.54	1.19	1.17	0.8
206	4.60	A2	206.9	.25	1.43	1.23	....
251*	8.33	A2	2.060	.0	1.27	1.21	....
347*	8.21	A2	1.605	.0	1.26	1.14	....
109	5.31	A3	28.28	.56	1.11	1.03	....
258	6.13	A3	3.894	.04	1.22	1.18	....
274	6.31	A3	14.345	.21	1.25	1.23	....
297	6.03	A3	7.39	.05	0.97	0.87	....
269	5.54	A4	5.515	.00	1.20	1.06	....
13	5.02	A5	1.964	.01	1.11	1.10	....
105	5.10	A5	9.355	.21	0.97	0.92	....
178	4.48	A5	24.483	.61	0.91	0.85	....
186	5.24	A5	38.3	.07	1.67	1.33	....
333	7.67	A6	8.446	.04	1.21	1.21	1.7
338	6.17	A6	6.370	.0	1.17	1.15	1.4
115*	5.61	A7	2.525	.00	1.30	1.24	....
155*	6.39	A8	0.648	.0	0.80	0.66	....
5	6.1	A9	0.842	.03	0.64	0.63	1.7
340	6.90	A9	3.749	.19	1.06	1.05	....
HD109510††	6.72	F0	7.337	.21	1.04	0.99	2.5
163	4.06	F0	10.211	.54	0.65	0.62	....
142	7.15	F1	12.912	.2	1.12	1.10	2.1
210	6.8	F1	12.822	.39	1.08	1.01	3.2
252	4.61	F1	26.274	.49	0.96	0.94	2.8
266	6.41	F1	2.048	.04	1.01	1.00	2.7
278	7.3	F1	3.765	0.00	1.14	1.14	2.3

¶ Harper, *Pub. Dom. Ap. Obs.*, 6, No. 18, p. 305, 1936.
** Struve and Morgan, *Ap. J.*, 66, 135, 1927.
†† Petrie, *Pub. Dom. Ap. Obs.*, 16, No. 21, p. 365, 1937.

TABLE 24—*Continued*

No.	Mag.	Sp.	P	e	$M_b^{1/3} \sin i$	$M_f^{1/3} \sin i$	M_b sp
33	6.7	F2	2.236	0.01	0.97	0.95	3.2
240	5.91	F2	2.308	.0	1.04	1.00	3.8
261*	7.10	F2	3 993	.00	1.14	1.09	....
290	6.46	F2	4.812	.07	1.08	1.08	....
24	6.26	F3	4.435	.11	1.05	1.02	3.2
133	6.00	F3	31.50	.21	1.15	1.10	....
196	5.91	F3	36.04	.49	1.33	1.24	3.3
335	6.35	F3	12.21	.32	0.99	0.92	....
375	5.69	F3	12.155	.28	1.19	1.19	3.1
52	5.47	F4	30.434	.61	1.00	0.96	2.9
83	6.9	F4	3.435	.0	1.12	1.10	3.5
125	5.27	F4	1.933	.00	1.02	0.95	3.1
190*	8.12	F4	4.798	.0	1.21	1.18	....
300	5.86	F4	7.638	.53	1.13	1.13	3.0
156	3.76	cF5	14.498	.02	1.09	1.04	−0.8
202	4.82	F5	9.604	.17	1.11	1.09	2.9
264	6.18	F5	10.522	.31	0.77	0.74	3.4
230	5.76	F6	7.974	.08	0.98	0.94	3.7
358	6.74	F6	21.700	.38	1.14	1.11	3.6
157*	8.3	F8	0.334	.00	0.87	0.78	....
37	4.99	F9	331.0	.67	0.90	0.86	3.8
WZ Oph*‡‡	9.0	G0	4 183	.10	1.12	1.10	....
312§§	3.25	G0	1374.1	.42	1.63	1.68	2.0
81	0.21	G1	104.02	.02	1.06	0.98	0.1
32	5.5	G4	14.732	.04	1.04	1.04	0.9
350	7.37	G5	1.152	.0	0.87	0.85	4.8
WW Dra*‖‖	8.5	gG5	4.630	.00	1.50	1.34	....
183	8.2	G6	5.415	.00	0.93	0.89	....
348*	9.06	G9	5.074	.0	1.24	1.00	....
353*	6.96	K0	1.983	.04	1.12	1.12	....
75*¶¶	4.9	cK4	973	.41	2.48	2.02	−2.5
372	6.55	dK5	7.753	.00	0.68	0.64	6.5
131*	9.0	dM1	0.814	0.0	0.86	0.83	8.8

‡‡ Sanford, *Ap. J.*, 86, 153, 1937; Gaposchkin, *Harvard Bull.*, No. 907, p. 1, 1938.

§§ Sanford, *Ap. J.*, 89, 333, 1939.

‖‖ L. Plaut, *Photographische photometrie der veranderlijke sterren CV Carinae en WW Draconis* (dissertation), p. 25, Leiden: Luctor et Emergo, 1939.

¶¶ Christie and Wilson, *Ap. J.*, 81, 426, 1935.

star $+6°1309$) with the values 75.6 and 63.3 for $M \sin^3 i$ and No. 4 ($+57°28$) with 113.2 and 44.9. It has been judged best to exclude these stars from the means for their spectral groups. The two c stars, ζ Aurigae (75) and o Leonis (156), have also been omitted. Grouping the remaining 126 stars by spectral type, we find the mean values given in Table 25. The successive columns give the mean spectral class, the number of stars and of eclipsing binaries among them, the mean value of $\overline{M_b^{1/3} \sin i}$, and that of log $\overline{M_b^{1/3}}$ derived on the assumption that the correction for the mean value of

$\sin i$ is $+0.070$. The separation of giants and dwarfs for spectra later than G0 was made with the aid of the spectroscopic absolute magnitudes and of eclipsing variable data, which show that WZ Ophiuchi and AR Lacertae (353) are dwarfs, while WW Draconis and RT Lacertae (348) are giants. Star (183) is a dwarf; its proper motion is $0\rlap{.}''49$.

Of these 126 stars, 35 (or 28 per cent) are known to be eclipsing binaries. The percentage for pairs of classes O and B is 40; for class A, 20; and for F and later, 21. The preference for early types is less marked than was once supposed.

TABLE 25

MASSES OF SPECTROSCOPIC BINARIES

Mean Sp.	No.	Ecl. Bin.	$\overline{M_b^{1/3} \sin i}$	$\log \overline{M_b^{1/3}}$
O8.3	5	4	2.68	+0.498
B1.2	11	3	2.12	+ .396
B3	13	6	1.86	+ .341
B5.1	7	2	1.76	+ .316
B8.6	12	4	1.45	+ .232
A0	16	3	1.05	+ .091
A2.2	13	3	1.18	+ .142
A6.3	11	2	1.09	+ .108
F1.8	16	1	1.06	+ .096
F5.3	11	2	1.02	+ .079
dG5	4	2	1.01	+ .074
dK8	2	1	0.77	− .043
gG4	5	2	1.29	+0.180

Collecting the stars for which spectroscopic parallaxes are available for the brighter components, we find the results given in Table 26. The first four columns give the mean spectrum, the number of stars, the mean $\overline{M_b^{1/3} \sin i}$, and $\log \overline{M_b^{1/3}}$ (assuming a correction of $+0.070$ for $\overline{\sin i}$, as above). Then follows the mean spectroscopic absolute magnitude $\bar{S}$ of the brighter component. This must be reduced to bolometric magnitude.

We have first to reduce the values of *Mount Wilson Contribution* No. 511 to the scale here adopted, by the equation[5]

$$S'' = S + a(S - S_1) ,$$

where $a = 1.6$ for main-sequence stars and 0.6 for giants.

[5] *Ap. J.*, 87, 407, 418, 1938; *Mt. W. Contr.*, No. 589, pp. 19 and 30.

The bolometric magnitude M_b (including Eddington's temperature term) is then given by $M_b = S'' + C'$, where C' is taken from Table 40. The means of S_1 and S'' and of C' and M_b for the individual stars are tabulated. Then follow the values computed from the empirical relation (94), $\log \overline{M_b^{1/3}} = -0.040(\overline{M_b} - 5.20)$, and finally the residuals. The residuals are satisfactorily small—especially since these data have not been used in determining the constants of the equation. The inclusion of three dwarf stars of spectra G5–M1 in a single mean is justified by the linearity of the assumed relation.

TABLE 26

Sp.	No.	$\overline{M_b^{1/3} \sin i}$	$\log \overline{M_b^{1/3}}$ Obs.	$\overline{S}$	$\overline{S_1}$	$\overline{S''}$	$\overline{C'}$	$\overline{M_b}$	$\log \overline{M_b^{1/3}}$ Comp.	O−C
A5......	7	1.08	+0.103	1.59	1.84	1.19	+0.30	1.49	+0.148	−0.045
F2......	10	1.09	+ .108	2.97	2.96	2.99	+ .25	3.24	+ .078	+ .030
F5......	9	1.02	+ .078	3.32	3.35	3.27	+ .20	3.47	+ .069	+ .009
(dK4)...	3	0.80	− .026	6.70	6.79	6.56	− .82	5.74	− .022	− .004
gG2....	3	1.24	+0.164	1.00	0.56	1.26	−0.12	1.14	+0.162	+0.002

For the earlier spectral types the direct measures of parallax do not justify a similar comparison.

B. ECLIPSING BINARIES

50. *Data and Discussion.*—For eclipsing variables in which the spectroscopic orbits of both components have been determined, the masses are known, without statistical uncertainty, and also the radii. The absolute magnitudes may then be computed from the observed spectral types. The present stellar temperature scale—checked, as it is, by the direct determination of the surface brightness of the B3 star μ_1 Scorpii—appears to be good enough to lead to reliable values.

Table 27 gives the data adopted in the present work. The photometric orbits have been collated with the original references, and those elements adopted which were judged the best. The less reliable determinations are given half-weight, including some cases in which the photometric elements are by themselves satisfactory but make the secondary so faint that its spectrum should hardly be ob-

TABLE 27

MASSES OF ECLIPSING VARIABLES

Star	Sp.	P	L_1	r_1	r_2	b/a	i	Wt.	$\overline{R_b}$	$\overline{R_f}$	$M_b^{1/3}$	$M_f^{1/3}$	log $M_b^{1/3}$	M_b	Ref.
29 CMa	O8.5	4.393	0.65	0.471	0.376	0.73	64°0	....	18.6	14.8	3.54	3.21	+0.550	−7.8	1
Y Cyg	O9	2.996	.50	.206	.206	0.99	88.2	....	5.86	5.86	2.59	2.58	+ .412	−5.0	2
AH Cep	B0	1.775	.59	.353	.266	0.92	66.8	½	6.20	4.65	2.45	2.33	+ .390	−4.4	3
V Pup	B1	1.454	.59	.382	.374	0.83	76.9	....	6.03	5.90	2.74	2.47	+ .438	−4.0	4
AG Per	B3	2.029	.72	.263	.184	1.0	78.3	....	3.74	2.62	1.73	1.66	+ 238	−2.3	5
TT Aur	B3	1.333	.66	.36	.32	0.86	88.4	....	3.80	3.38	1.88	1.74	+ .275	−2.3	6
μ_1 Sco*	B3	1.446	.57	.360	.400	0.885	62.0	....	5.17	5.73	2.29	2.28	+ .360	−2.9	7
u Her	B3	2.051	.71	.318	.318	0.915	77.6	.	4.38	4.38	1.94	1.41	+ .288	−2.6	8
Z Vul	B3	2.455	.85	.323	.300	0.931	88.9	½	4.64	4.30	1.74	1.33	+ .241	−2.7	9
σ Aql	B3	1.950	.57	.242	.242	0.955	71.7	....	3.46	3.46	1.84	1.73	+ .265	−2.1	10
U CrB	B5	3.452	.74	.167	.273	1.0	81.5	½	3.36	5.49	1.86	1.34	+ .270	−1.4	11
U Oph	B5	1.677	.54	.252	.252	0.909	85.7	....	3.03	3.03	1.74	1.66	+ .241	−1.2	11
U Sge	B9	3.381	.92	.230	.295	1.0	90.0	....	4.48	5.77	1.88	1.27	+ .275	−1.0	12
AR Aur	B9	4.135	.50	.101	.110	1.0	88.8	....	1.84	2.01	1.35	1.32	+ .130	+0.9	13
TX UMa	B8	3.063	.86	.131	.241	1	83.5	½	1.84	3.39	1.45	0.96	+ .162	+0.7	14
GO Cyg	B9	0.718	.92	.478	.328	0.801	58.7	½	1.98	1.36	1.16	1.10	+ .065	+0.8	15
TV Cas	A0	1.813	.86	.285	.301	0.95	79.8	½	2.38	2.52	1.19	1.00	+ .076	+0.6	16
β Aur	A0	3.960	.50	.147	.147	0.99	77.4	....	2.58	2.58	1.33	1.33	+ .124	+0.5	11
RX Her	A0	1.779	.53	.204	.204	0.975	84.8	....	1.96	1.96	1.28	1.23	+ .108	+1.1	17
MR Cyg	A0	1.677	0.66	0.304	0.334	1.0	81.9	½	3.23	3.55	1.45	1.38	+0.162	0.0	18

* Mean of components.

TABLE 27—*Continued*

Star	Sp.	P	L_1	r_1	r_2	b/a	i	Wt.	$\overline{R_b}$	$\overline{R_f}$	$\mathfrak{M}_b^{1/3}$	$\mathfrak{M}_f^{1/3}$	$\log \mathfrak{M}_b^{1/3}$	M_b	Ref.
TX Her.......	A2	2.060	0.64	0.148	0.148	1.0	86°6	....	1.58	1.58	1.27	1.21	+0.105	+1.9	19
CM Lac.......	A2	1.605	.62	.150	.188	1.0	87 3	....	1.31	1.65	1.26	1.14	+ .100	+2.3	20
WW Aur......	A7	2.525	.56	.18	.17	1.0	87.7	....	2.26	2.13	1.30	1.24	+ .114	+1.7	21
S Ant.........	A8	0.648	.67	.50	.39	0.75	62.2	½	1.37	1.07	0.91	0.75	− .040	+2.9	22
Z Her.........	F2	3.993	.51	.097	.207	1.0	83.7	½	1.46	3.12	1.14	1.09	+ .056	+3.1	11
RS CVn......	F4	4.798	.71	.092	.296	0.95	80.0	....	1.63	5.26	1.23	1.20	+ .090	+3.1	23
W UMa......	F8	0.334	.60	.405	.304	0.806	74.6	....	0.78	0.58	0.90	0.81	− .046	+5.0	24
WZ Oph......	G0	4.183	.51	.081	.079	1.0	89.4	....	1.26	1.23	1.12	1.10	+ .050	+4.0	25
WW Dra......	gG5	4.630	.67	.228	.414	1.0	83.5	....	4.82	8.78	1.52	1.35	+ .182	+2.0	26
RT Lac.......	G9	5.074	.58	.294	.294	0.89	90.0	½	4.87	4.87	1 00	1.24	.000	+1.9	27
AR Lac.......	K0	1.983	.64	.159	.306	1.0	81.5	½	1.50	2.90	1.13	1.13	+ .053	+4.4	28
ζ Aur.........	cK4	973	.8:	.162	.0071	1.00	90	½	192	8.5	2.48	2.02	+ .393	−5.1	29
YY Gem*.....	dM1	0.814	0.50	0.160	0.160	1.0	86.5	....	0.62	0.62	0.86	0.83	−0.073	+7.7	30

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servable. AO Cassiopeiae, for which the photometric data are uncertain, was excluded. The successive columns give: (1) the designation, (2) spectral class (from Moore's catalogue[6] but taking Kuiper's value for 29 Canis Majoris), (3) the period in days, (4) the light of the bright component in terms of that of the system, (5) the longer radius of the brighter component, (6) the same for the fainter (with their mean distance as unit), (7) the ratio b/a of the axes in the orbit plane, (8) the inclination, and (9) the adopted weight. Then follow: (10) and (11) the mean radii $(a + 2b)/3$ of the two components, (12) and (13) the cube roots of the masses (all in "solar units"), (14) log $\mathfrak{M}_b^{1/3}$ for the bright component, and (15) the absolute magnitude M_b of this component.

The latter has been calculated by the equations

$$\left.\begin{aligned} M_b &= M + 2 \log\left(\frac{T_e}{5200}\right), \\ M &= M_\odot - 5 \log\left(\frac{R}{R_\odot}\right) - 10 \log\left(\frac{T_e}{T_\odot}\right). \end{aligned}\right\} \tag{95}$$

With Kuiper's values $M_\odot = +4.62$ and $T_\odot = 5713^\circ$, we have

$$M_b = 34.76 - 8 \log T_e - 5 \log\left(\frac{R}{R_\odot}\right) = C - 5 \log\left(\frac{R}{R_\odot}\right). \tag{96}$$

The values of C derived from Kuiper's temperature scale are:

Sp.	C	Sp.	C	Sp.	C
O8	−2.0	A0	+2.5	dK0	+5.2
B0	−0.4	A5	+3.3	dK6	+6.0
B1	−0.1	dF0	+3.8	dM2	+6.7
B3	+0.6	dF5	+4.3	gG0	+5.0
B5	+1.2	dG0	+4.5	gK0	+5.8
B8	+2.0	dG5	+4.9	gK5	+6.3
				gM2	+6.7

The absolute magnitudes thus derived are not purely "observational," since they involve an assumed temperature scale; but this scale is now well-enough known to make them trustworthy, except perhaps for class O.

Grouping the results by spectral type, we find the mean values given in Table 28. The subgiants RT Lacertae and WW Draconis

[6] *Lick Obs. Bull.*, 18, No. 483, p. 1, 1936.

(combined) and the supergiant ζ Aurigae are placed at the bottom of the table.

Means of the individual values of $M_b^{1/3}$ have been taken and the logarithms of these tabulated, as in § 45. The column headed "Comp." gives the values predicted by equation (94); and the last column, the residuals. These are remarkably small, considering that equation (94) was derived from entirely independent data and is extrapolated over a wide range.

TABLE 28

MASS-LUMINOSITY RELATION FOR ECLIPSING BINARIES

No.	Wt.	Sp.	$\overline{M_b}$	log $\overline{M_b^{1/3}}$ Obs.	log $\overline{M_b^{1/3}}$ Comp.	O−C
1	1	O8.5	−7.8	+0.550	+0.520	+0.030
3	2.5	B0	−4.5	+ .417	+ .388	+ .029
6	5.5	B3	−2.5	+ .284	+ .308	− .024
3	2.5	B7	−1.1	+ .260	+ .254	+ .006
7	5.0	A0	+0.7	+ .120	+ .180	− .060
6	5.0	A7	+2.4	+ .087	+ .112	− .025
3	2.5	dG1	+4.5	+ .013	+ .028	− .015
1	1	dM1	+7.7	− .073	− .100	+ .027
2	1.5	gG7	+2.0	+ .100	+ .128	− .028
1	0.5	cK4	−5.1	+0.393	+0.412	−0.019

The algebraic mean residual is −0.019 ± 0.006 (the probable error being found from the residuals). This could be reduced to zero by making the adopted values of $\overline{M_b}$ fainter by $0^m.48$ or by diminishing log T_e systematically by 0.06 or T_e by 13 per cent, but these changes are too great to be admissible.

It may be concluded, therefore, that the empirical equation

$$\log \overline{M_b^{1/3}} = -0.0400(\overline{M_b} - 5.20) \tag{94}$$

represents the course of the mass-luminosity relation over the whole range for which direct observational data are available, within little more than the casual error of the individual normals with which it is compared. The observed range covers 15^m in $\overline{M_b}$ or 17^m in the true absolute magnitude, and 0.72 in log $\overline{M_b^{1/3}}$, corresponding to a factor of 140 in the mass.

CHAPTER III

GENERAL DISCUSSIONS

In this chapter are collected various discussions which were omitted earlier in order not to interfere with the continuity of the presentation; and at the end some general conclusions are drawn.

A. STATISTICAL THEORY FOR SLOW-MOVING PAIRS

51. *Statistical Means of Different Kinds.*—It appears desirable to reproduce here an outline of the statistical discussion of slow-moving pairs.[1]

If we set

$$h_1 = l\left(\frac{sw^2}{4\pi^2}\right)^{1/3}, \tag{97}$$

where l is a constant to be determined, and

$$\sin i \sin^2 j\left(2 - \frac{r}{a}\right) = k^3, \tag{98}$$

where k varies from star to star, equation (6) becomes

$$h_1 = \mathrm{M}^{1/3} plk. \tag{99}$$

If we set

$$lk = 1 + z, \tag{100}$$

this goes over into equation (13). The condition that the mean value of z shall be zero (as assumed above) is

$$l\bar{k} = 1. \tag{101}$$

By defining l in this way, we may secure values of h_1 which will, in the mean, be the same as the values of $\mathrm{M}^{1/3}p$, provided that there is no correlation between $\mathrm{M}^{1/3}p$ and k. As the latter depends upon the orientation of the orbit and the position of the stars at the moment, this assumption is safe. If the values of p are known, this calibration will give a correct mean for $\mathrm{M}^{1/3}$. Writing it in the form

$$h_1\mathrm{M}^{-1/3} = p(1 + z),$$

[1] Russell, *A.J.*, 38, No. 897, p. 89, 1928.

we see that, if the masses are known or can be reliably estimated, it will give a correct mean parallax for stars selected *impartially with respect to the statistical error* z—that is, in practice, without regard to the apparent motion w. An example of observational selection in this respect and of its elimination by rejecting the biased data is discussed in §§ 34–35.

Other methods of calibration are possible; for example, we might require that the mean values of h_1 and of $M^{1/3}p$ should be the same for the stars for which h_1 is the same (and $M^{1/3}p$ therefore different). In this case we must write equation (99) in the form

$$lM^{1/3}p = \frac{h_1}{k}$$

and have in the mean

$$l' = \overline{k^{-1}} = l\overline{(1+z)^{-1}} = l(1 + \overline{z^2} + \overline{z^3} + \overline{z^4} + \dots) , \quad (102)$$

so that l' is always greater than l. In practice, however, stars will very rarely be grouped in accordance with their dynamical parallaxes.

Again we might demand that the mean distance, instead of the mean parallax, should be given correctly by the statistical formula. For an impartial selection we must then put equation (99) in the form

$$\frac{1}{h_1} = \frac{1}{M^{1/3}p}\,\frac{1}{lk}$$

and find

$$l' = \overline{k^{-1}} ,$$

as above. For a fixed value of h_1 we must write

$$\frac{k}{h_1} = \frac{1}{lM^{1/3}p}$$

and recover the relation (101).

The calibration which gives correct mean parallaxes for an unbiased grouping will give correct mean distances for a grouping according to h_1, and vice versa. Distances computed with the calibration factor l, which gives correct mean parallaxes, will average too great by the factor $\overline{k}\cdot\overline{k^{-1}}$. Correction should be made for this

in dealing with distances or space motions calculated from *individual* dynamical parallaxes.

If we seek the correct mean value of M, supposing the p's to be given, we have

$$h_1^3 = \text{M}p^3l^3k^3 ,$$

whence

$$l'^3\overline{k^3} = 1 = \frac{l'^3}{l^3}(1 + 3\overline{z^2} + \overline{z^3}) ,$$

which is the correction discussed in § 5.

Finally, we may determine a calibration factor l''' so that the mean absolute magnitude calculated from individual values of h_1 shall be correct, that is, so that

$$\overline{\log h_1} = \overline{\log \text{M}^{1/3}p} .$$

With impartial selection this gives

$$\log l''' = -\overline{\log k} = \log l - \overline{\log (1 + z)} = \log l + \tfrac{1}{2}\overline{z^2} - \tfrac{1}{3}\overline{z^3} + \tfrac{1}{4}\overline{z^4} .$$

52. *Mean Values of the Constants.*—The mean value of

$$k^3 = \sin i \sin^2 j \left(2 - \frac{r}{a}\right)$$

is easily found. Let φ be the angle between the orbit plane and the plane containing the radius vector and the line of sight, and J that between the radius vector and the direction of the orbital motion. Then all values of φ are equally probable, and

$$\cos j = \cos i \cos J + \sin i \sin J \cos \varphi .$$

Hence, if i and J are fixed and φ varies, the mean value of $\sin^2 j$ is $1 - \cos^2 i \cos^2 J - \frac{1}{2} \sin^2 i \sin^2 J$. Next let i vary, keeping J fixed. The mean value of $\sin i \overline{\sin^2 j}$ is then

$$\frac{\pi}{4} - \frac{\pi}{16} \cos^2 J - 3 \frac{\pi}{32} \sin^2 J ,$$

or

$$\frac{\pi}{32} (5 + \cos^2 J) .$$

In the elliptic orbit, if E is the eccentric anomaly, we have

$$\frac{r}{a} = 1 - e\cos E\,, \qquad \cos^2 J = \frac{e^2 \sin^2 E}{1 - e^2\cos^2 E}\,.$$

Integrating with respect to the time, we find

$$\left.\begin{aligned}\overline{k^3} = \frac{1}{2\pi}\int_0^{2\pi}\frac{\pi}{32}\left(5 + \frac{e^2\sin^2 E}{1 - e^2\cos^2 E}\right)(1 - e\cos E)\\ (1 + e\cos E)dE = \frac{\pi}{32}(5 - 2e^2)\,.\end{aligned}\right\} \qquad (103)$$

A direct attempt to find the mean values of k or $1/k$ leads, in this case, to integrals which can be evaluated only by quadratures, which, since three variables are involved, would be very laborious.

53. *Distribution of the Statistical Errors.*—If, however, we can find the statistical probability that k^3 is less than any assigned value, the mean of any function of k can then be found by a simple quadrature; and, what is more, the distribution of the statistical errors in individual cases, and the mean amount of these errors, can be determined.

Suppose that we have any function $F(x, y)$ of two variables, which themselves have such statistical distributions that the probability that $x < x_0$ is $P(x_0)$ and the probability that $y < y_0$, when $x = x_0$, is $Q(y_0, x_0)$. Required the probability that $F(x, y) < z$. Set $x = x_0$, and let the roots of $F(x_0, y) = 0$ be $y_1, y_2, y_3, \ldots\ldots$ Consider first the case where there is but one real root, and suppose that $F < z$ when $y < y_1$. Then the probability that $F < z$ when $x = x_0$ is $Q(y_1, x_0)$. Multiplying this by the probability that X lies between x_0 and $x_0 + dx_0$, and integrating, we find for the whole probability that $F < z$ the expression

$$\Phi(z) = \int Q(y_1, x_0)\frac{dP(x_0)}{dx_0}\,dx_0\,.$$

In the more general case, we will have

$$\Phi(z) = \int (Q_1 - Q_2 + Q_3 \ldots .)dP(x_0)\,, \qquad (104)$$

where $Q_1, Q_2, \ldots .$, correspond to the roots $y_1, y_2, \ldots .$, and the sign of Q is the same as that of $(\partial/\partial y)F(x_0, y)$, and the integration

extends over the whole permissible range of x_0. Equation (104) may be expressed geometrically as follows. The transformation $\xi = P(x)$, $\eta = Q(y, x)$, converts the part of the xy plane which we need to consider into a square in the $\xi\eta$ plane, bounded by the axes and the lines $\xi = 1$ and $\eta = 1$. The curve $F(x, y) = z$ goes over into a certain curve C in the $\xi\eta$ plane. The probability $\Phi(z)$ is equal to the fraction of the area of the square which lies on that side of C which corresponds to values of F which are less than z.

It then follows that the mean value of any function $f(z)$ is equal to the area included between the curve which has $\Phi(z)$ for abscissa and $f(z)$ for ordinate, the lines $\Phi = 0$ and $\Phi = 1$, and the axis $f = 0$.

Applying this first to the function

$$\begin{aligned} F &= \sin i \sin^2 j \\ &= \sin i\{1 - (\cos i \cos J + \sin i \sin J \cos \varphi)^2\} , \end{aligned}$$

we have, from geometrical considerations,

$$P(i) = 1 - \cos i , \qquad Q(\varphi) = \frac{\varphi}{\pi} ,$$

where i ranges from 0 to $\pi/2$ and φ from 0 to π, while J is supposed constant and positive. Setting $F = z$, we find that there are, in general, two values of φ given by the equations

$$\cos \varphi = -\cot i \cot J \pm \operatorname{cosec} i \operatorname{cosec} J \sqrt{1 - z \operatorname{cosec} i} .$$

If the integral is written in the form (104), considerable care about the limits is necessary; but if the curves C in the $\xi\eta$ plane are plotted, all difficulty disappears. The quadratures were made by drawing these curves on co-ordinate paper, and "counting squares." The resulting values of $\Phi(z)$ were smoothed graphically and interpolated to form a general table giving $\Phi(z)$ for all values of z and $\cos J$.

The next step was to introduce the orbital eccentricity. In this case the function for which the probability distribution is sought is

$$F' = \left(2 - \frac{r}{a}\right) \sin i \sin^2 J ,$$

and the appropriate variables are $x = E$ and $y = z = \sin i \sin^2 j$, so that $F' = z(1 + e \cos E)$. The probabilities are $P(x) = (E - e \sin E)/\pi$ and $Q(y, x) = \Phi(z, \cos J)$; also, $\cot J = e \sin E/\sqrt{1 - e^2}$. The probability $\psi(u)$ that $F < u$ was determined by another series of quadratures for various values of the orbital eccentricity e. As this function may be of some use to computers, it is tabulated here in summary form (sufficient for graphical interpretation) (Table 29).

TABLE 29

PROBABILITY ψ (u) THAT SIN i SIN^2 $j\{2 - (r/a)\} < u$

u	e						
	1.0	0.9	0.8	0.6	0.4	0.2	0.0
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.05	0.345	.185	0.109	0.050	0.034	0.028	0.027
0.10	0.453	.350	0.231	0.121	0.074	0.061	0.057
0.20	0.585	.526	0.455	0.294	0.197	0.145	0.134
0.30	0.671	.630	0.582	0.442	0.336	0.273	0.237
0.40	0.735	.704	0.671	0.581	0.473	0.415	0.409
0.50	0.784	.761	0.737	0.683	0.602	0.553	0.556
0.60	0.825	.807	0.788	0.755	0.724	0.678	0.667
0.80	0.885	.874	0.865	0.864	0.873	0.866	0.849
1.00	0.927	.923	0.921	0.935	0.947	0.964	1.000
1.20	0.957	.958	0.963	0.973	0.985	1.000	
1.40	0.978	.982	0.986	0.993	1.000		
1.60	0.991	.994	0.996	1.000			
1.80	0.998	0.999	1.000				
2.00	1.000						

Finally, it is necessary to take account of the probability of the occurrence of various values of e. This cannot be derived theoretically but must be found from observational data. The empirical function $P(e)$, given below, represents closely the distribution of eccentricities among the visual binaries for which reliable orbits are available. There is, of course, some doubt whether it is applicable to the physical pairs which have much longer average periods, but nothing better seems available at present. With these values of $P(e)$ and the values of $\psi(u, e)$ previously computed, the probability $X(k^3)$ that $\sin i \sin^2 j(2 - r/a) < k^3$ was determined by one more quadrature. It is given in Table 30. Anyone who prefers a different distribution of eccentricities may compute X for himself with the aid of Table 29.

With this distribution the mean value of e is 0.510, and that of e^2 is 0.312, giving a mean square value of 0.561 for e. The mean value of k^3 is found to be 0.4294, while the exact integral (eq. [103]) gives 0.4296, a satisfactory agreement. The mean value of k is 0.7022, that of $1/k$ is 1.620, and that of $\log_{10} k$ is -0.1761.

54. *Statistical Probable Errors.*—It would now be easy to find the average or the mean square deviations between the actual and statistical values of the parallax, mass, etc., and hence the "probable error" of the latter. Since, however, the distribution of these deviations is not Gaussian, the ordinary formulae for probable error are hardly applicable, and the following method has been preferred.

TABLE 30

STATISTICAL DISTRIBUTIONS

Observed		Computed Probability			
e	$P(e)$	k^3	X	k^3	X
0.0	0.000	0.00	0.000	1.0	0.942
0.1	0.021	.05	.063	1.1	0.963
0.2	0.085	.10	.132	1.2	0.979
0.3	0.191	.2	.275	1.3	0.989
0.4	0.336	.3	.411	1.4	0.995
0.5	0.511	.4	.537	1.5	0.997
0.6	0.669	.5	.647	1.6	0.999
0.7	0.783	.6	.739	1.7	1.000
0.8	0.869	.7	.811		
0.9	0.939	.8	.868		
1.0	1.000	0.9	0.911		

In a normal frequency distribution with mean value a and probable error r, half of the individual quantities will be less than r, one-fourth less than $a - r$, and so on—a definite probability X corresponds to any limit $a + br$. For a series of equidistant values of b, X can be found from tables of the probability integral. But we have already found (for each of our formulae) the relation between X and k, so that the corresponding values of k or any function $f(k)$ may be tabulated. If the distribution of the quantities f were Gaussian, the mean of the values of f corresponding to $b = \pm n/2$ would always equal a, and their difference would be nr. For other frequency distributions the deduced values of a will vary with n, at

a rate increasing with the skewness of the distribution, and those of r will vary at a rate increasing with the "excess."

Table 31 shows the result of this process, applied to k, $1/k$, k^3, and log k. The values of a corresponding to integral values of n from

TABLE 31

DISTRIBUTION OF STATISTICAL ERRORS

n	X	k	$1/k$	k^3	log k
−5	0.0459	0.332	3.02	0.037	−0.477
−4	.0887	0.407	2.46	0.069	− .387
−3	.1559	0.488	2.05	0.117	− .311
−2	.2500	0.567	1.76	0.183	− .246
−1	.3680	0.644	1.55	0.268	− .190
0	.5000	0.717	1.40	0.370	− .144
+1	.6320	0.785	1.27	0.485	− .105
+2	.7500	0.851	1.17	0.614	− .070
+3	.8441	0.910	1.10	0.756	− .040
+4	.9113	0.966	1.03	0.904	− .016
+5	0.9541	1.016	0.98	1.051	+0.007
a_0		0.717	1.40	0.370	−0.144
a_1		.715	1.41	.373	− .148
a_2		.709	1.46	.399	− .158
a_3		.699	1.58	.437	− .176
a_4		.687	1.75	.486	− .201
a_5		0.674	2.00	0.544	−0.235
Mean		0.700	1.60	0.435	−0.177
Mean by quadrature		0.702	1.62	0.429	−0.176
r_1		0.141	0.28	0.202	±0.085
r_2		.142	.29	.216	± .088
r_3		.141	.32	.213	± .090
r_4		.139	.36	.209	± .093
r_5		0.137	0.41	0.203	±0.097
Mean		0.140	0.331	0.209	±0.091
$\bar{r}/M$		0.200	0.204	0.487	

0 to 5 are given in the middle part of the table. They frequently show a wide range, indicating that the skewness is large. The mean of the six values is, however, close to the true mean M, found by quadratures, except when the skewness is extreme.

The lower part of the table gives the corresponding values of r. Here, again, the distribution is usually far from normal; but the

means, $\bar{r}$, have been taken as above. The ratio $\bar{r}/M$ of these to the true mean value may be taken as a measure of the statistical errors.

By a happy accident (if such a term may be applied to a purely mathematical relation) the distribution of the statistical errors in the determination of the parallax by this method is remarkably similar to the normal type, with a probable error of 20 per cent of the mean value.

We have, in fact, for the number of errors to be expected in 1,000 cases, the results shown in Table 32. Except for the excess of large negative over large positive errors, the distribution is as nearly

TABLE 32

STATISTICAL ERRORS OF THE FORMULA FOR PARALLAX

Limits of x	Positive	Negative	Both	Normal
0.0 –0 10	134	125	259	264
0.10–0.20	132	103	235	236
0.20–0.30	109	84	193	188
0.30–0.40	80	62	142	135
0.40–0.50	49	43	92	85
0.50–0.60	19	28	47	49
0.60–0.70	4	15	19	25
0.70–0.80	1	8	9	11
Over 0.80	0	4	4	7
All	528	472	1000	1000

normal as one might expect to find in a random sample, of moderate size, taken from a "population" of strictly normal distribution.

55. *Statistical Errors of the Adopted Dynamical Parallaxes.*—The calibration here has been made so that the mean dynamical parallax h_1 is correct for a group selected impartially with regard to the geometrical conditions in the orbit. The value of l is then $1/0.702 = 1.425$, and equation (97) becomes

$$h_1 = 0.418\sqrt[3]{sw^2},$$

as in equation (13).

The statistical probable error of h_1, arising from the averaging process, is ± 20 per cent of the mean value of h_1 for similar stars, that is, of the mean value of $M^{1/3}p$, so long as the selection is impartial. For stars selected by means of large or small values of h_1

it will be greater, and serious systematic effects of the selection may be expected.

It is shown in § 63 that the dynamical parallaxes, d, defined by equation (127), are so calibrated that, in the mean, they agree with the true parallaxes; but the effect of the accidental errors of h_1 (whether arising from errors of observation or from statistical causes) is increased, so that $(1 + z)$ is replaced by $(1 + z)^b$, where $b = 1.25$ (eq. [124]).

From Table 32 we find easily $\bar{z} = -0.0003$, $\overline{z^2} = 0.0844$, $\overline{z^3} = -0.0083$, $\overline{z^4} = 0.0195$, $\overline{z^5} = -0.0051$, $\overline{z^6} = 0.0071$, $\overline{z^7} = -0.0030$, and $\overline{z^8} = 0.0034$; whence $\overline{(1 + z)^{-1}} = 1.131$. The quadratures of Table 31 give $0.702 \times 1.62 = 1.138$. Similarly, we find (allowing, when necessary, for higher terms in the series)

$$\overline{(1 + z)^{1.25}} = 1.014\,, \qquad \overline{(1 + z)^{-1.25}} = 1.21\,, \qquad \overline{ln(1 + z)} = -0.068\,,$$

$$\overline{\log_{10}(1 + z)} = -0.0296\,.$$

The first of these quantities is allowed for by the correction B' (§ 63). It follows from the second that, if distances or transverse velocities are calculated from the individual dynamical parallaxes, d, their mean will be too great by the factor 1.21×1.014. To obtain correct means these distances must be multiplied by 0.815. Similarly, if absolute magnitudes are computed from the d's, they will, on the average, be too bright by $5 \times (0.0296 + \log_{10} 1.014) = 0^m.18$. No such correction is required when the dynamical parallax has been determined from an orbit, but an additional allowance should be made for the effects of errors of observation. This may be left for discussion when occasion arises—noting only that, since these errors cannot make a dynamical parallax negative, the integrals for the reciprocal or logarithm of $(1 + z)$ will still be convergent.

Barbier[2] has made the interesting remark that dynamical parallaxes determined by this statistical process cannot be regarded as the most probable parallaxes of the pairs. Even if no other information were available, this comment is strictly correct. By numerical

[2] D. Barbier, "Thèses présentées à la Faculté des Sciences de Paris" (No. d'ordre: 2326, Sér. A, No. 1462), p. 68, Paris: Gauthier-Villars, 1934.

differentiation of the probability given in Table 32, it is found that this is a maximum when $z = +0.07$, so that the most probable parallax is 1.07 times the value d defined by equation (127).

The most probable value, however, does not lead to correct mean values of any of the quantities depending on the parallax and is therefore not likely to be of practical use. Barbier's remark that the usefulness of dynamical parallaxes is comparable to that of small trigonometric parallaxes is somewhat too severe, for the percentage errors of the latter are much greater. Like all other determinations of small parallaxes, they are most valuable when combined into statistical averages—provided, of course, that any systematic effects of selection are allowed for.

The comparison (quoted by Barbier[3]) which Cecchini[4] has made between trigonometric and dynamical parallaxes is of interest. Grouping the stars in accordance with the values of d (from the older list by the writers), he finds:

$\bar{d}$........	0″000	0″010	0″030	0″050	0″070	0″090	0″100
$\bar{d}-\bar{i}$......	+ .004	+ .006	− .001	− .011	− .018	− .021	− .022
Predicted.	.000	− .002	− .006	− .009	− .013	− .017	− .019
O−C.....	+0.004	+0.008	+0.005	−0.002	−0.005	−0.004	−0.003

The discussion just given shows that, *for this method of selection*, agreement should be secured by multiplying the tabular values of d by 0.815. This removes by far the greater part of the discordance. The outstanding residuals are small. The application of an empirical correction to remove the whole discrepancy (as Cecchini has done) amounts to defining d so that the means will be correct when selected by the value of d. As we have seen, and as Barbier agrees, this will introduce systematic errors into means derived by other and more impartial methods of selection.

B. TRIPLE SYSTEMS

56. *Test of Statistical Formulae.*—A good test of the validity of the statistical process for slow-moving pairs may be made for triple

[3] *Ibid.*, p. 67.

[4] *Pub. d. r. osserv. astronom. di Merate*, 4, 22, 1931.

systems in which a close pair, for which an orbit has been calculated, is attended by a distant physical companion. There is no case in which an orbit may yet be derived for the wider pair, or is likely to be for a long time to come; but it is possible to calculate h_1 from the orbit of the close pair and the relative motion in the wide one.

If h_1 is the hypothetical parallax derived from the orbit and h_3 is that derived from the wide pair, if M_1 and M_2 are the masses of the close pair and M_3 is that of the companion, we have, by (11) and (13),

$$\left.\begin{aligned} h_1 = (M_1 + M_2)^{1/3}p\,, \qquad h_3 = (M_1 + M_2 + M_3)^{1/3}p(1+z)\,, \\ \frac{h_3}{h_1} = \left(\frac{M_1 + M_2 + M_3}{M_1 + M_2}\right)^{1/3}(1+z)\,. \end{aligned}\right\} \quad (105)$$

The values of M_1, M_2, and M_3 may be estimated from the mass-luminosity relation, and that of the statistical factor $1 + z$ thus determined in each individual case.

Adopting the empirical equation (94), we have

$$\log\left(\frac{M_2}{M_1}\right) = 0.12(M_{b1} - M_{b2})\,, \quad \log\left(\frac{M_3}{M_1}\right) = 0.12(M_{b1} - M_{b3})\,, \quad (106)$$

where M_{b1} and M_{b2} are the absolute bolometric magnitudes of the stars (with Eddington's correction).

But if m_1(bol) and m_2(bol) are the apparent bolometric magnitudes of the pair and m_1 and m_2 are the apparent visual magnitudes,

$$M_{b1} - M_{b2} = m_1(\text{bol}) - m_2(\text{bol}) = m_1 - m_2 + C_1' - C_2'\,,$$

where C' is the correction given in Table 40. When any of the components is a spectroscopic binary showing two spectra, we must make it brighter by $1^m.87$ (§ 64, end) to allow for the effect which would be produced by combining the two bodies into one mass. When only one spectrum is visible, the corresponding correction is $1^m.62$. (This is probably too great, but it will be used here for the sake of consistency.) The same correction should be applied to component C of ζ Cancri, which has an invisible optical companion.

Sixteen triple systems of this sort are available. Data for the orbits are given in Table 52. Those for the wide pairs are given in

Table 33. They have not been used for calculating dynamical parallaxes in the present work, since the orbits give better values; but most of them appear in the earlier lists. The values of h_3 from the slow motions, of h_1 from the orbits, and the ratios h_3/h_1 are given in the last three columns. The mean h_3/h_1 is 1.22, and the individual residuals give a standard deviation of ±0.45, corresponding to a probable error of ±24 per cent.

TABLE 33

TRIPLE SYSTEMS (WIDE PAIRS)

ADS	E	θ	$100\,\frac{d\theta}{dt}$	s	$100\,\frac{ds}{dt}$	$100w$	h_3	h_1	h_3/h_1
450...	15	207°.7	− 24°.6	2″.20	+0″.02	0″.95	0″.0244	0″.0298	0.82
1^h34...	00	15	+234	1.58	−0.12	6.48	.0788	.0630	1.25
1630...	75	63.0	0.0	10.24	−0.34	0.35	.0208	.0218	0.95
2200...	80	237.0	+ 0.3	13.99	−0.1	0.042	.0057	.0220	0.26
3^h10...	90	219	− 30	2.93	+0.71	1.72	.0399	.0445	0.90
3093...	70	106.7	− 3.5	83.15	−3.2	6.02	.281	.173	1.62
4929...	80	252.5	+ 10.0	2.85	+0.38	0.60	.0197	.0198	1.00
6175*..	75	165.6	+ 2.53	72.20	−0.97	2.60	.153	.120	1.27
6554...	00	193	+ 32	4.72	−0.14	2.61	.0617	.0305	2.02
6650...	75	133	− 53	5.48	0.00	5.14	.102	.0603	1.69
6993...	75	220	+ 54	3.33	−0.16	3.14	.0622	.0372	1.67
9247...	75	188.3	+ 5.0	6.29	+0.21	0.62	.0261	.0205	1.27
9909...	80	69.5	− 19	7.23	+0.68	2.44	.0682	.0569	1.20
10786...	70	244.9	+ 4.15	31.06	+3.02	3.78	.148	.105	1.41
12145...	80	225	− 21	4.11	+0.45	1.53	.0413	.0265	1.56
12973...	80	311.5	− 1.0	8.69	−0.16	0.176	0.0125	0.0182	0.69

* Motion of C relative to (A+B)/2.

A better test, however, is to compute the masses, as described above and exhibited in Table 34. The second column gives the spectra of the three components so far as known; the third, their visual magnitudes; and the fourth, the apparent bolometric magnitudes m(bol), obtained by adding the correction C' and the special corrections for spectroscopic binaries. For the white dwarf o^2 Eridani A (3093) this has been made brighter by 3^m to allow approximately for the known deviation of this star from the mass-luminosity law (cf. Fig. 1). Then follow the values of $\mathfrak{M}_2/\mathfrak{M}_1$ and $\mathfrak{M}_3/\mathfrak{M}_1$ (found by eq. [106]), of the correction factor in equation (105), and finally of $1 + z$. The mean value of z is +0.05, the average regardless of sign ±0.30 and the mean-square value ±0.38.

Both of the latter correspond to a probable error of ± 0.26. The value to be anticipated from the statistical process alone is ± 0.20; but the uncertainty of determination from 16 entries corresponds to

TABLE 34

MASSES IN TRIPLE SYSTEMS

ADS	Spectra		m			m(bol)			$\frac{M_2}{M_1}$	$\frac{M_3}{M_1}$	$\left(\frac{M_1+M_2+M_3}{M_1+M_2}\right)^{1/3}$	$1+z$
	1 2	3	1	2	3	1	2	3				
450..	G5	....	9.6	9.6	12.9	9.6	9.6	11.6	1.00	0.58	1.09	0.75
1^h34..	G5	....	7.8	8.2	12.1	7.8	8.2	10.6	0.90	0.46	1.08	1.16
1630..	B9	gK3	5.4	6 6	2.3	5.6	6.8	1.1	0.72	3.47	1.45	0.65
2200..	A6	....	5.7	6.7	9.3	6.0	7.0	9.2	0.76	0.41	1.07	0.24
3^h10..	F2	....	6.4^b	7.0	8.9	5.1	7.2	8.8	0.56	0.36	1.07	0.84
3093..	A2 M5	K0	9.7^c	11.5	4.5	7.0	8.4	4.3	0.68	2.11	1.31	1.24
4929..	A3	....	7.9	7 9	9.2	8.2	8.2	9.5	1.00	0.70	1.11	0.90
6175..	A2 A8	M1	2.0^b	2.8^b	8.9^a	0.7	1.5	5.1	0.80	0.30	1.05	1.21
6554..	K2	....	8.6	8.6	10.2	8.3	8 3	9.4	1.00	0.74	1.11	1.82
6650..	F7	G2	5.6	6.3	6.0^b	5.7	6.4	4.5	0.82	1.32	1.21	1.40
6993..	F8 ...	F7	3.8	5.3	6.8	3.9	5.2	6.9	0.70	0.44	1.08	1.55
9247..	F2	B9	6.8^b	8.5	5.1	5.4	8.6	5.3	0.41	1.03	1.20	1.06
9909..	F4	G7	4.8	5.1	7.2	5.0	5.3	7.1	0.92	0.56	1.09	1.10
10786..	M3	G4	10.2	10.7	3.5	7.7	8.2	3.5	0.87	3.19	1.39	1.02
12145..	K0	G4	8.9	8.9	8.2^b	8.7	8.7	6.6	1.00	1.79	1.24	1.26
12973..	A0	....	5.4	6.4	8.7	5.6	6.6	8.9	0.76	0.40	1.07	0.64

a. Spectroscopic binary, two spectra visible.
b. Spectroscopic binary, one spectrum visible.
c. White dwarf.

a probable error of 12 per cent, or ± 0.03, so that the outstanding discordance is not serious. The mean value of z, $+0.05 \pm 0.06$, is without significance.

C. MASS RATIOS

57. *Visual Binaries.*—The ratio of the masses in a visual binary is not easy to determine accurately. A critical study has recently been made by Kuiper.[5] We are interested here in the agreement of the results with the empirical mass-luminosity relation (94), which may be written, in this case, as in § 56:

$$\log\left(\frac{M_2}{M_1}\right) = 0.12(m_1 - m_2 + C_1' - C_2') . \tag{107}$$

[5] *Ap. J.*, 88, 479–482, 1938.

The data are given in Table 35. The second column gives the spectra, from Kuiper's Table 1;[6] the third, the difference of visual magnitude, from the same source; the fourth, that of the correction C', taken separately from Table 40 when both spectra are known and otherwise from Figure 4 (that is, guessing at the spectrum of the faint component on the assumption that it belongs to the main sequence, as the primary does in all these cases). The resulting values are given in parentheses. Kuiper, who purposely confines himself to the most reliable individual observed values of luminosity, reasonably rejects these faint components from his list; but, as estimates with the aid of Figure 4 have been made in large numbers in the present work, it is desirable to test their results.

The fifth column gives $\text{M}_2/(\text{M}_1 + \text{M}_2)$, from Kuiper's Table 5, except that an additional determination is given for α Geminorum (6175). The first value, from Boss's *General Catalogue*,[7] appears to be doubtful; so a rough solution was made from the measures of the distant companion C.[8] This gave $\text{M}_2/(\text{M}_1 + \text{M}_2) = 0.46 \pm 0.16$. A considerably better determination could be made by including the recent measures (which there was no time to collect). The last three columns give the value of log (M_2/M_1), corresponding to this, the value computed from equation (107) and the residual.

For 44 Bootis (9494) the fainter component is an eclipsing binary with nearly equal components. The computed M_2 is therefore twice the mean value calculated for a single component of this pair. The same assumption (following Kuiper) has been made for the faint component of 85 Pegasi (17175), with the results given in the last line of the table.

The two huge residuals, for o^2 Eridani (3093) and Sirius (5423), arise from the presence of white dwarfs. The large discordance for Procyon (6251) indicates that its companion is also a white dwarf (as Kuiper points out).

For 11 of the remaining 16 systems the data are free from uncertainty. The mean residual for these is $+0.042$ (corresponding to a ratio of 1.10), and the mean square is ± 0.067. The four pairs for

[6] These often disagree slightly with the values in Tables 52 and 53.

[7] 1, 155, 1937.

[8] See Russell, *Ap. J.*, 32, 369, 1910; measures to 1925 added from *Aitken Double Stars*, 1, 515, 1932.

which the data have been noted as doubtful have much larger residuals. Finally, for 85 Pegasi, Kuiper's assumption that the companion is double gives a good agreement. The mass ratio obtained from the distant companion of Castor (6175) agrees much better than that from the meridian observations.

TABLE 35

MASS RATIOS IN VISUAL BINARIES

ADS	Spectra		m_2-m_1	$C'_2-C'_1$	$\frac{M_2}{M_1+M_2}$	$\log\left(\frac{M_2}{M_1}\right)$ Obs.	$\log\left(\frac{M_2}{M_1}\right)$ Comp.	O−C
671	G0	K5+	$3^{m}74$	−0.95	0.392	−0.19	−0.33	+0.14
3093	B9	M5e	1.48	−3.3:	.31	− .35	+0.22	− .57
3841	G4	F4	0.15	+0.50	.443	− .10	−0.08	− .02
5423	A0	A5	10.06	+0.09	.295	− .38	−1.22	+ .84
6175	A0[b]	A1[b]	0.86	+0.03	.64:	+ .25	−0.11	+ .36:
					.46:	− .07		+ .04:
6251	F3	....	10.3	(−3.3)	.235	− .51	−0.84	+ .33
8630	F0	F0	0.03	0.00	.508	+ .01	0.00	+ .01
8891	A2	A2	0.0	0.00	.501	.00	0.00	.00
$14^{h}59$	G4	K1	1.37	−0.23	.446	− .09	−0.14	+ .05
9413	G8	K5	1.96	−0.46	.461	− .07	−0.18	+ .11
9494	G1	G5[a]	0.82	−0.09	.7:	+ .36:	+0.12	+ .24:
10157	G1	....	2.80	(−0.82)	.38	− .21	−0.24	+ .03
10660	G1	....	2.61	(−0.75)	.38	− .21	−0.22	+ .01
11046	K1	K4	1.68	−0.21	.450	− .09	−0.18	+ .09
11077	F5	....	3.40	(−0.84)	.26:	− .45:	−0.31	− .14:
11871	G0	....	1.78	(−0.43)	.45	− .09	−0.16	+ .07
14787	F0	....	2.82	(−0.47)	.328	− .31	−0.28	− .03
15972	M4+	M6	1.5	−0.5:	.359	− .25	−0.12:	− .13:
17175	G2	....	3.5:	(−1.1)	.51	+ .02	−0.29	+ .31:
			4.25	(−1.35)	0.51	+0.02	−0.03	+0.05:

a. Spectroscopic binary, showing two spectra.
b. Spectroscopic binary, showing one spectrum.

The rather large number of discordances with the simple mass-luminosity law, revealed in this short list, might be disquieting; but they arise mainly from the presence of white dwarfs which are observable only because they happen to be very near the sun. There is very little chance of finding white dwarfs in the general list. It is not at all improbable, however, that there are a good many pairs in the general list for which one component is an unresolved close pair. In this case the computed dynamical parallax will be too great.

58. *Eclipsing Binaries.*—Mass ratios are also known for the eclipsing spectroscopic binaries of Table 27. There are 33 of these; but for 2 of these—V Puppis and μ_1 Scorpii—the masses have been assumed either to be equal or to follow the mass-luminosity rela-

tion, which leaves 31 systems available. For these, M_2/M_1 is given directly by the spectroscopic observations. The difference of apparent magnitude is given by

$$m_1 - m_2 = 2.5 \log \left(\frac{1 - L_1}{L_1}\right) . \tag{108}$$

To find $C_1' - C_2'$, we may have recourse to the difference in surface brightness, which is given, in stellar magnitudes, by

$$J_1 - J_2 = m_1 - m_2 + 5 \log (r_1 - r_2) . \tag{109}$$

We have, also, by the definition of the bolometric correction C,

$$J = - 10 \log T_e - C - 0.02 .$$

The constant has been adjusted so that $J = 0$ for dG0 (Table 39). If now C' is plotted against J, we may take J_1 corresponding to the spectrum of the bright component and find J_2 from equation (109) and C_2' from the curve. When the observations of light are photographic, the value of J should be changed to the visual scale. This may be done, with sufficient accuracy for the present purpose, by multiplying by 0.8. For photoelectric observations J has been multiplied by 0.9 (as a compromise).

The results are given in Table 36. The "computed" value is derived from equation (107). The last column gives the logarithm of the ratio of the densities.

59. *Deviations from Mass-Luminosity Relation.*—It is apparent, on inspection, that there is a correlation between this and the deviations from the standard mass-luminosity relation. Grouping the stars according to ρ_2/ρ_1 and taking simple means, we find (excluding ζ Aurigae):

No.	Mean $\log\left(\frac{\rho_2}{\rho_1}\right)$	Mean O−C	$0.16 \log\left(\frac{\rho_2}{\rho_1}\right)$
6............	+0.30	+0.08	+0.05
9............	−0.03	− .02	.00
8............	−0.24	− .06	− .04
7............	−1.09	−0.17	−0.17

The relation between the means is smoother than might be expected from the scatter of the individual data. This relation is evidently

significant: it means that a star which has a lower density than normal for its spectral type will be brighter than normal for its mass. Let M_0 be the mass, and let r_0 and M_0 be the normal radius

TABLE 36

MASS RATIOS FOR ECLIPSING VARIABLES

Star	m_1-m_2	J_1-J_2	$C_1'-C_2'$	$\log\left(\frac{\mathrm{M}_2}{\mathrm{M}_1}\right)$ Obs.	$\log\left(\frac{\mathrm{M}_2}{\mathrm{M}_1}\right)$ Comp.	O−C	$\log\left(\frac{\rho_2}{\rho_1}\right)$
29 CMa	−0.65	−0.17	−0 28	−0.12	−0.11	−0.01	+0 18
Y Cyg	0.00	0.00	0.00	.00	.00	.00	0.00
AH Cep	−0.40	+0.11	+0.13	− .06	− .03	− .03	+0.31
AG Per	−0.99	−0.21	−0.18	− .05	− .14	+ .09	+0.41
TT Aur	−0.72	−0.47	−0.37	− .10	− .13	+ .03	+0.05
u Her	−1.00	−1.00	−0.62	− .42	− .19	− 23	−0.42
Z Vul	−1.55	−1.40	−0.70	− .35	− .27	− .08	−0.25
σ Aql	−0.27	−0.27	−0.23	− .08	− .06	− 02	−0.08
U CrB	−1.11	−2.17	−0.40	− .42	− .18	− .24	−1.06
U Oph	−0.18	−0.18	−0.10	− .06	− .03	− .03	−0.06
U Sge	−2.62	−3.16	+0.32	− .52	− .28	− .24	−0.85
AR Aur	+0.04	−0.14	−0.04	− .03	.00	− .03	−0.14
TX UMa	−1.95	−3.27	+0.25	− .52	− .20	− .32	−1.31
GO Cyg	−2.62	−1.80	−0.07	− .07	− .32	+ .25	+0.42
TV Cas	−1.95	−2.07	+0.08	− .26	− .23	− .03	−0.33
β Aur	0.00	0.00	0.00	− .01	.00	− .01	−0.01
RX Her	−0.12	−0.12	−0.03	− .05	− 02	− .05	−0.05
MR Cyg	−0.52	−0.72	−0.06	− .07	− .07	.00	−0 19
TX Her	−0.50	−0.50	−0.03	− .07	− .06	− .01	−0.07
CM Lac	−0.32	−0.80	0.00	− .12	− .04	− .08	−0 42
WW Aur	−0.26	−0.14	+0.01	− .06	− .03	− .03	+0.02
S Ant	−0.76	−0.22	+0.03	− .25	− .09	− .16	+0.08
Z Her	−0.03	−1.68	+0.48	− .09	+ .05	− .14	−1.07
RS CVn	−0.97	−3.51	+1.40	− .03	+ 04	− .07	−1.55
W UMa	−0.46	+0.16	+0.05	− .14	− .05	− .09	+0.23
WZ Oph	−0.03	+0.01	0.00	− .02	.00	− .02	+0.01
WW Dra	−0.27	−1.65	+0.77	− .15	− .06	− .09	−0.93
RT Lac	−0.35	−0.35	+0.14	+ .28	+ .03	+ .25	+0.28
AR Lac	−0.21	−1.63	+0.72	.00	+ .06	− .06	−0.85
ζ Aur	−1.8:	+5.0:	−1.8:	− .27	− .44	+ .17	+3.8:
YY Gem	0.00	0.00	0.00	−0.04	0.00	−0.04	−0.04

and absolute magnitude (with Eddington's correction) for the given spectral type. If the actual radius is r, we will have

$$M = M_0 - 5\log\left(\frac{r}{r_0}\right), \qquad \log\left(\frac{\rho}{\rho_0}\right) = -3\log\left(\frac{r}{r_0}\right).$$

If M is the mass derived for this star by the application of the standard mass-luminosity relation, we have

$$\mathrm{O} - \mathrm{C} = \log\left(\frac{\mathrm{M}_0}{\mathrm{M}}\right) = 0.12(M - M_0) = 0.20\log\left(\frac{\rho}{\rho_0}\right).$$

The observed coefficient, 0.16, indicates that the greater part, but not all, of the difference in density between the components arises from deviations of the kind here considered.

Eclipsing variables, as is well known, provide a powerful method for selecting stars which are, at the same time, larger and fainter than their main-sequence companions. Strömgren[9] noticed, some time ago, that some such stars were "too bright" for their masses, and had very small values of the calculated abundance of hydrogen; and Chandrasekhar has indicated that the principal component of ζ Herculis is a star of this sort.[10] These stars should provide an important test of theories of stellar energy.

Calculation of the masses and dimensions of eclipsing binaries, for which no radial velocities are available, may be systematically affected if the average mass-luminosity relation is applied to systems with large faint secondaries.

Even for visual binaries, observational selection operates strongly against the discovery of companions of small mass. Such practically dark bodies can produce observable effects only when their periods are short, so that their primaries are recorded as spectroscopic binaries of small velocity range and mass function. The studies mentioned in § 48 indicate that bodies of this sort are actually fairly numerous.

D. ASTROPHYSICAL CONCLUSIONS

60. *Relations between Mass, Luminosity, and Spectral Class.*—The principal result of the present investigation is that a simple linear mass-luminosity correlation has been shown to be of very general application. The results obtained by plotting log $\overline{\mathfrak{M}_b{}^{1/3}}$ against $\overline{M_b}$ for the various groups of stars which have been discussed are given in Figure 1. The heavy line represents the empirical linear formula (94); the dashed curve, Eddington's relation, with the zero-point derived in § 46. The general agreement of the points with the line is very striking. Almost all the large discordances correspond to small or poorly determined groups; and there is little evidence, if any, of systematic deviation from the line—except, of course, for

[9] *Zs. f. Ap.*, 7, 235, 1933.

[10] *Stellar Structure*, p. 279, Chicago: University of Chicago Press, 1938.

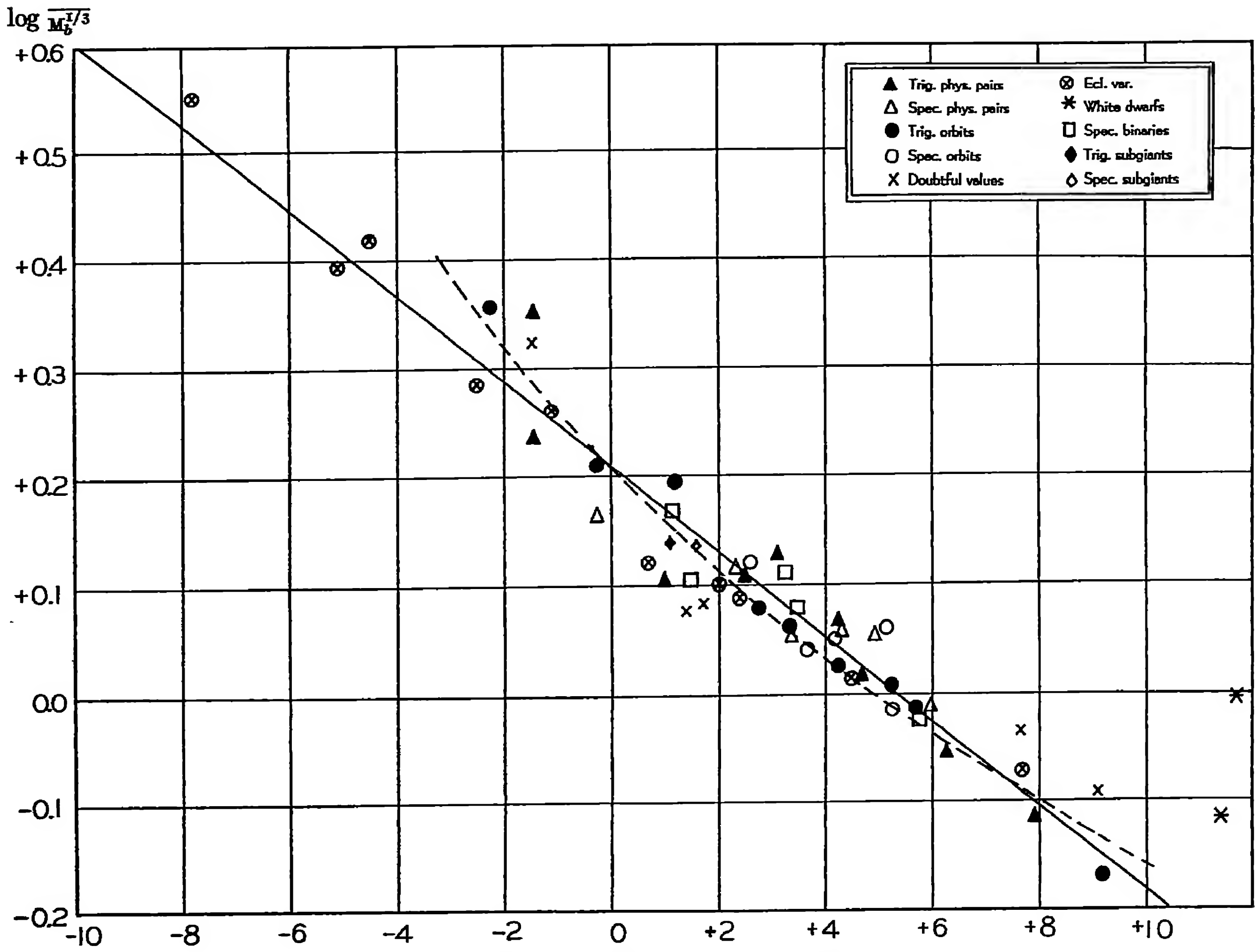

FIG. 1.—Relation between mass and luminosity

the white dwarfs. If $\log \overline{M_b^{1/3}}$ is plotted against the spectral type, Figure 2 is obtained. The mean values for spectroscopic binaries showing two spectra (Table 25) may be entered here but not in Figure 1 (since the absolute magnitudes are not reliably known).

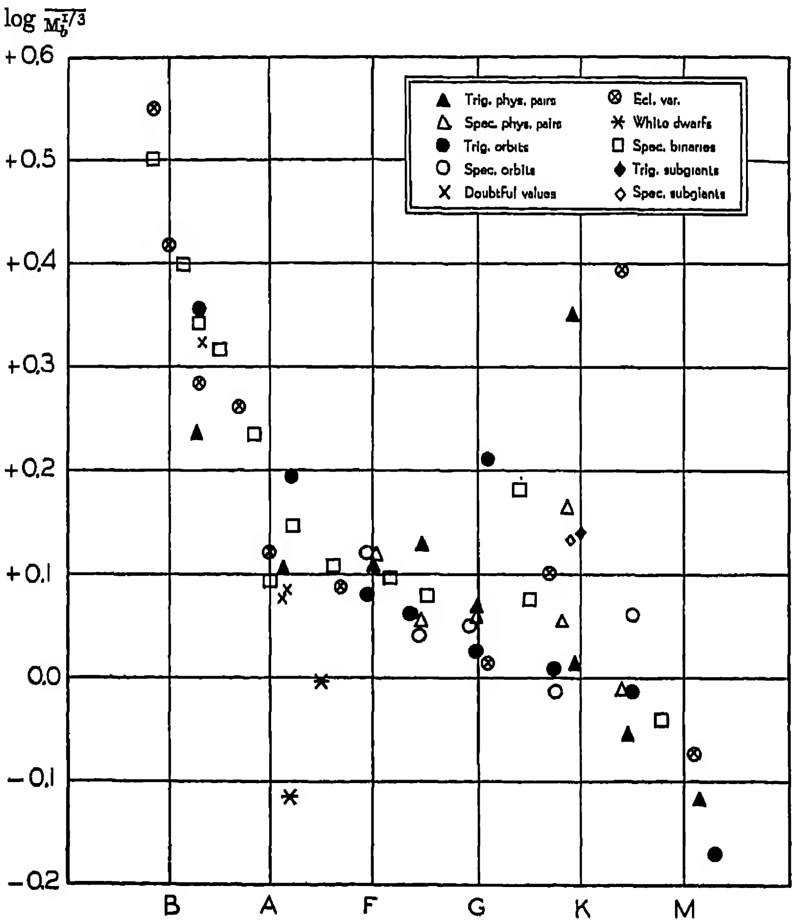

FIG. 2.—Relation between mass and spectral type

In general character this figure resembles the familiar spectrum-luminosity diagram—as it must, on account of the close correlation between mass and absolute magnitude. The main sequence is conspicuous; the giant series is well indicated; and the white dwarfs

stand alone.[11] It is noteworthy that Sirius B departs twice as far from the main sequence as o^2 Eridani B. The K stars, apparently intermediate between giants and dwarfs, represent the spectroscopic parallaxes for the giants (§ 40), the subgiants (§ 41), and the eclipsing variables RT Lacertae and WW Draconis, which are true subgiants, at least in density.

The quantity M_b which appears in equation (94) is not the bolometric absolute magnitude M, as commonly defined, but

$$M + 2 \log \left(\frac{T_e}{5200}\right) . \tag{110}$$

For the main-sequence stars, which form the great majority of our list, M is very nearly a linear function of M_b. We have, indeed, from Tables 23 and 28, the following (combining the orbits and physical pairs in the latter):

Eclipsing Binaries				Visual Binaries			
Sp.	$\overline{M_b}$	$2 \log \left(\frac{T_e}{5200}\right)$	O−C	Sp.	$\overline{M_b}$	$2 \log \left(\frac{T_e}{5200}\right)$	O−C
O8.5	−7.8	+1.67	−0.19	B2.9	−1.8	+1.11	+0.12
B0	−4.5	+1.37	− .01	A1.7	+1.1	+0.55	− .01
B3	−2.6	+1.11	+ .01	A9.8	+2.6	+0.32	− .02
B7	−1.1	+0.82	− .06	F4.2	+3.2	+0.21	− .05
A0	+0.7	+0.65	+ .03	F9.8	+4.2	+0.13	+ .02
A7	+2.4	+0.39	+ .01	G8.5	+4.9	−0.02	− .03
dG1	+4.5	+0.10	+ .03	K4.9	+5.9	−0.18	− .05
dM1	+7.7	−0.40	−0.01	M2.2	+8.5	−0.45	+0.06

From a plot of these data the empirical relation follows:

$$2 \log \left(\frac{T_e}{5200}\right) = -0.145(M_b - 5) . \tag{111}$$

The residuals from this are given in the table above. They are small and apparently of a random character.

[11] These are: Sirius B: $M_b = 11.7$, $\mathfrak{M} = 0.96$, $\log \mathfrak{M}^{1/3} = -0.005$, spectrum A5; o^2 Erid B: $M_b = 11.4$, $\mathfrak{M} = 0.44$, $\log \mathfrak{M}^{1/3} = -0.118$, spectrum A2.

From equations (111) and (110)

$$M = 1.145M_b - 0.725\,, \qquad M_b - 5.20 = 0.873(M - 5.23)\,, \qquad (112)$$

so that equation (94) becomes

$$\log \mathfrak{M}^{1/3} = -0.0349(M - 5.23)\,.$$

For the cube-root mean of the masses we have, therefore,

$$\log \mathfrak{M} = -0.1048(M - 5.23)\,. \qquad (113)$$

If L is the luminosity in terms of the sun,

$$\log L = -0.4(M - 4.62)\,;$$

whence

$$\log \mathfrak{M} = 0.262 \log L + 0.064\,, \qquad (114)$$

$$\log L = 3.816 \log \mathfrak{M} - 0.244\,. \qquad (115)$$

These equations give the final expression of the mass-luminosity relation, as derived in the present work. The additive constants indicate that the sun is not an "average" main-sequence star but is $0^m.6$ too bright for its mass—or has only 85 per cent of the mass normally corresponding to its light.

Measuring T_e and R also in solar units, we have from (111), (112), (113), and (114)

$$\log T_e = -0.0634(M - 5.23) - 0.055 = +0.604 \log \mathfrak{M} - 0.055\,,$$

$$\log R = 0.700 \log \mathfrak{M} - 0.022\,.$$

A standard main-sequence star of the sun's mass should therefore have 0.572 times the sun's light, 0.955 times its radius, 1.16 times its density, and 0.884 times its effective temperature, corresponding to G8 on Kuiper's scale (on which the sun is G2). A star of the sun's absolute magnitude should have 1.16 times its mass, 1.052 times its radius, 0.99 times its density, and 0.965 times its temperature (spectrum G3.5). Except for the spectrum, the latter would not be easy to distinguish from the sun by ordinary observations.

It is of interest to compare these results with the predictions of Bethe's theory.[12] For homologous stars in which radiation pressure is unimportant, he finds

$$L = \text{constant } M^{4.32}\mu^{6.04}y^{1.06}z^{-1.06} ,$$

$$R = \text{constant } M^{0.75}\mu^{0.57}(yz)^{0.05} ,$$

$$T_e = \text{constant } M^{0.70}\mu^{1.23}y^{-0.29}z^{-0.04} ,$$

where μ is the mean molecular weight, y the concentration of heavy elements (which produce opacity), and z the product of the concentrations of the active nuclei H and N^{14}.

If μ is taken as constant, the agreement is not close; but the assumption that $\mu \propto M^{-0.082}$ gives $L \propto M^{3.83}$, $R \propto M^{0.70}$, and $T_e \propto M^{0.60}$—an agreement too good to be wholly true, but doubtless significant.

Direct calculation of the hydrogen abundance by Strömgren's method[13] leads to the following results for "typical" stars (assuming helium to be practically absent):

log M	log R	log L	μ	X	log μ
+1.00	+0.678	+3.57	0.65	0.68	−0.185
+0.50	+ .328	+1.76	.79	.51	− .104
+0.00	− .022	−0.24	.88	.43	− .056
−0.50	−0.372	−2.15	0.97	0.35	−0.014

For the first entry, radiation pressure is important, though here neglected; and for the last, the limits of application of the theory are stretched, but the progression in the hydrogen abundance X is well illustrated. The deviation of the sun from standard corresponds to a difference in the computed hydrogen abundance ($X = 0.36$ instead of 0.43).

61. *Exceptions; The Trumpler Stars.*—Though these empirical relations hold good for all the groups of stars which have been investigated, it would be unjustified to assume that it was universally true. There are some stars, omitted from the preceding discussion because the data were judged to be of inadequate precision, which

[12] *Phys. Rev.*, 55, 434, 1939 (esp. p. 451).

[13] *Ap. J.*, 87, 520, 1938.

are very probably more massive in proportion to their luminosity. The remarkable eclipsing variable VV Cephei is one of these. The most noteworthy group is composed of the "Trumpler stars" in galactic clusters,[14] for which Trumpler has estimated the masses on the assumption that the observed discordances in radial velocity from the mean of the cluster arise from the Einstein shift predicted by general relativity.

The individual values for these stars are subject to serious uncertainty. Accepting their membership in the clusters, the absolute magnitudes derived for them with the aid of the cluster moduli, and the radial velocities for the individual stars and the cluster, we then have from the red shift the values of $\mathfrak{M}/R$ (which range from 8 to 24 for the individual stars), and also the photographic absolute magnitudes. Assuming the effective temperature, we find R from the photographic surface brightness J_p; and then M_{bol}.

Now

$$J_p = \text{constant } (e^{c_2/\lambda T} - 1)^{-1} ;$$

whence, if $c_2/\lambda T = z$,

$$\frac{d \log J_p}{d \log T} = \frac{z}{1 - e^{-z}} .$$

Setting $\lambda = 0.43$, $c_2 = 14{,}300$, and $T = 40{,}000°$ (Kuiper's value for class O8), we find

$$d \log J_p = 1.48 d \log T .$$

But $L_p = J_p R^2$; hence,

$$d \log R = 0.74 d \log T ;$$

also, $L_{bol} = R^2 T^4$, so that

$$d \log L_{bol} = +2.52 d \log T .$$

Finally,

$$M_b = -2.5 \log L_{bol} + 2 \log T + 4.62 , \qquad (116)$$

if L is in solar units. Hence, if L_p and M/R are given,

$$dM_b = -4.3 d \log T , \qquad d \log \mathfrak{M} = -0.74 d \log T .$$

[14] *Pub. A.S.P.*, 47, 254, 1935.

An increase in log T by 0.1 will therefore decrease the value of log $M^{1/3}$ derived from the observed data by -0.025 and will increase that deduced from the mass-luminosity relation by 0.017, changing the residual by -0.042. This corresponds on Kuiper's scale to a change of one-tenth of a spectral class and is probably of the order of magnitude of the uncertainty of the temperature estimates.

The numerical data for these stars (Table 37) have been taken from Chandrasekhar[15] (representing values revised by Kuiper since the publication of his list[16]). The sixth column gives M_b calculated from equation (116); and the eighth, $M_b^{1/3}$ from equation (94). The

TABLE 37

THE TRUMPLER STARS

No	Sp.	log M	log L	log T_e	M_b	log $M^{1/3}$ Obs.	log $M^{1/3}$ Comp.	O−C
1	O6	1.74	5.88	4.8:	−7.9	0.58	0.52	+0.06
2	O9	1.99	4.72	4.50	−5.6	.66	.43	+ .23
3	O7+	2.14	5.60	4.68	−7.5	.72	.51	+ .21
4	O8.5	2.45	5.76	4.55	−8.1	.82	.53	+ .29
5	O9w	2.35	5.36	4.50	−7.2	.78	.50	+ .28
6	Bo	2.60	5.04	4.40	−6.6	.87	.47	+ .40
7	O9	1.89	4.92	4.50	−6.1	0.63	0.45	+0.18
Mean					−7.0	0.75	0.49	+0.25

last line contains the means (giving half-weight to the data for the three preceding stars, which are described by Trumpler as less reliable). The logarithms of the means of the individual values of $M^{1/3}$ are given, as usual.

The great discordance with the mass-luminosity relation for other stars has been pointed out and discussed by Kuiper and Chandrasekhar. An increase in the assumed temperatures would diminish this; but no permissible change will remove it.

The weakest link in the chain of reasoning which leads to these results appears to be the assumption that the observed red shifts arise entirely from the relativity effect. It is well known that almost all studies of the B stars show an unexplained positive K-term much greater than can be accounted for by relativity. The assumption

[15] *Stellar Structure*, p. 313. [16] *Ap. J.*, 88, 506, 1938.

that the whole K-term comes from this cause would lead to a comparable overestimate of the masses of these stars.

Despite this, the data afford prima facie evidence for the existence of "overmassive" or "underluminous" stars (compared with the previous empirical standard). The best hope of obtaining more specific evidence of the existence and properties of such bodies is in the photometric study of spectroscopic binaries of great mass, on the chance of finding eclipses.

While there is thus strong evidence of the existence of stars which deviate greatly from the empirical mass-luminosity relation described above, the proportion of those which agree with it is so great that the relation may obviously safely be used as a basis for the calculation of dynamical parallaxes.

For the discordant stars, the estimated masses would be too small and the parallaxes too great; but these bodies are so few and so remote that the chance of their appearing in lists of visual double stars showing any trace of motion is, for the present, negligible.

CHAPTER IV

THE CALCULATION OF DYNAMICAL PARALLAXES

A. GENERAL TREATMENT

62. *General Equations.*—We have now to devise a system of dynamical parallaxes, taking into account what we may know of the masses of the stars, which will give results that, in the mean for impartially selected groups, will be equal to the trigonometric parallaxes which we have taken as our standard system of reference.

We have, for each star, by equations (21) and (35),

$$h_1 = \mathrm{M}^{1/3} pH(1 + z) ,$$

where M is the combined mass and p the true parallax, while H represents the systematic, and z the accidental, part of the observational error, so that $\bar{z} = 0$. Similarly, for the trigonometric parallaxes,

$$t = pT(1 + x) .$$

We have assumed $T = 1$, taking the trigonometric parallaxes as standard, and can be sure that H is very nearly 1 for the orbits. For physical pairs it may be slightly greater, owing to the observational selection discussed and very largely eliminated in § 36.

The values of $\overline{\mathrm{M}_b^{1/3}}$ given in Table 23 have been derived from equations (88) and (90), setting $H = T = 1$. If H is sensibly different for orbits and physical pairs, the means of the residuals O − C_2 of log $\overline{\mathrm{M}_b^{1/3}}$ from the linear formula should differ. The weighted means are:

	Orbits		Phys. Pairs	
	Mean Res.	Wt.	Mean Res.	Wt.
Trigonometric parallaxes........	+0.001	7.0	−0.005	2.6
Spectroscopic parallaxes........	+0.003	2.3	+0.009	2.7
Both together........... ..	+0.001	9.3	+0.002	5.3

The mean difference of 0.001 in the logarithms corresponds to $H = 1.003$. We may therefore safely set $H = 1$, so that

$$h_1 = \mathfrak{M}^{1/3} p(1 + z) .$$

63. *Solution Neglecting Mass of Companion.*—In applying this equation, we will first discuss the simplified case of a star attended by a companion of negligible mass. If M is its absolute magnitude and $\mathfrak{M}_a$ the mass corresponding to this, then, according to the accepted linear mass-luminosity relation,

$$\log \mathfrak{M}_a^{1/3} = - a(M - A) , \tag{117}$$

where $a = (2/15)n$ and A is a constant. We have, as in equation (24),

$$\mathfrak{M}^{1/3} = \mathfrak{M}_a^{1/3}(1 + u) , \tag{118}$$

$$h_1 = \mathfrak{M}_a^{1/3} p(1 + u)(1 + z) . \tag{119}$$

If M_1 is the absolute magnitude corresponding to h_1,

$$\left.\begin{aligned} M_1 = m + 5 + 5 \log h_1 &= M + 5 \log h_1 - 5 \log p \\ &= M - 5a(M - A) + 5 \log (1 + u)(1 + z) , \end{aligned}\right\} \tag{120}$$

or

$$(M_1 - A) = (1 - 5a)(M - A) + 5 \log (1 + u)(1 + z) . \tag{121}$$

From equations (117), (119), and (121), setting $1/(1 - 5a) = b$, we find

$$\log p = \log h_1 + ab(M_1 - A) + b \log (1 + u)(1 + z) . \tag{122}$$

If

$$ab(M_1 - A) = \log n_a , \tag{123}$$

we have

$$h_1 n_a = p(1 + u)^b (1 + z)^b . \tag{124}$$

Then, since $\overline{u} = \overline{z} = \overline{uz} = 0$,

$$\overline{h_1 n_a} = \overline{p}\{1 + \tfrac{1}{2}b(1 - b)(\overline{u^2} + \overline{z^2}) + \ldots .\} .$$

Now, according to §§ 24, 27, and 38,

$$\overline{u^2} + \overline{z^2} = \frac{Z_1^2}{p_h}; \qquad Z_1^2 = 0.141 .$$

Also, by equation (94), $a = 0.040$, $n = 0.300$, $b = 1.250$; whence

$$\overline{h_1 n_a} = \bar{p}\left(1 + \frac{0.022}{p_h}\right) .$$

The values of p_h assigned in § 12, the relative correctness of which has been confirmed in the course of the discussion, are 10, 2.5, and $\frac{2}{3}$ for good, fair, and poor orbits, and 1, $\frac{2}{3}$, and $\frac{1}{3}$ for physical pairs of the same quality. We then have

$$\bar{p} = B\overline{h_1 n_a} , \tag{125}$$

where $\log_{10} B$ has the values -0.0009, -0.0038, and -0.0141 for the three grades of orbits, and -0.0095, -0.0141, and -0.0278 for the physical pairs. The mean value of $Bh_1 n_a$ will then be statistically equal to that of the true parallax p, for any impartially chosen group, in which errors of one sign or the other in h_1 are not selectively favored.

From equations (94) and (123)

$$\log n_a = 0.0500(M_1 - 5.20) .$$

We may set

$$\log n_0 = \log (Bn_a) = 0.0500(M_1 + B' - 5.39) , \tag{126}$$

where B' is $+0.17$, $+0.11$, and -0.09 for good, fair, and poor orbits, and 0.00, -0.09, and -0.36 for physical pairs of the same grade.

If now we set

$$d = h_1 n_0 , \tag{127}$$

the mean of the dynamical parallaxes d will be equal to that of the true parallaxes p, so long as the stars are chosen in an impartial manner.

The calibration by means of M_1—an unavoidable necessity in practice—therefore introduces no systematic error into the dynamical parallaxes. The accidental errors of the double-star data, and also those arising from deviations from the mass-luminosity law, are, however, increased by 25 per cent (eq. [124]). The statistical approximations for physical pairs therefore produce in the final dynamical parallax a probable error of ± 25 per cent. The value found in I (p. 97) is almost the same but varies with the absolute magnitude on account of the changes in n resulting from Eddington's formula.

64. *Solution for Components of Comparable Mass.* The treatment of a pair with both components visible is simple if it is assumed that the linear relation between log M and M is satisfied. The difference in bolometric magnitude ΔM gives the mass ratio, and it is easy to compute the change D in the combined magnitude which would result if the components were combined into one mass.

Assuming $\mathrm{M} = cL^n$, let the bolometric luminosities be L_1 and xL_1. The masses will be M_1 and $\mathrm{M}_1 x^n$; the combined light, $L = L_1(1 + x)$; the combined mass, $\mathrm{M}_1(1 + x^n)$; and the light corresponding to a single star of this mass, $L' = L_1(1 + x^n)^{1/n}$. Hence, since $n = 2/7$ (very nearly),

$$D = -2.5 \log \left(\frac{L'}{L}\right) = -2.5\{\tfrac{7}{2} \log (1 + x^{2/7}) - \log (1 + x)\} , \quad (128)$$

where $\Delta M = -2.5 \log x$.

When one or both of the components are spectroscopic binaries, let ΔM denote the difference in the *combined* light of these pairs. If the brighter component is a spectroscopic binary, with two spectra visible, we have assumed that the mass of the companion is $0.9\mathrm{M}_1$. The combined light will be $(1 + 0.9^n)L_1$ or, when $n = 2/7$, $1.692L_1$, corresponding to a single star of mass $1.162\mathrm{M}_1$. For the other visual component, if single, the light will be $1.692xL_1$ and the mass $1.162x^n\mathrm{M}_1$. We then have $L = 1.692(1 + x)L_1$. The combined mass is $(1.900 + 1.162x^n)\mathrm{M}_1$ or $1.162\mathrm{M}_1(1.635 + x^n)$. Hence,

$$D = -2.5\{\tfrac{7}{2} \log (1.635 + x^{2/7}) - \log (1 + x)\} . \quad (129)$$

When the bright component is a spectroscopic binary showing only one spectrum, we have assumed a mass ratio of 0.6; whence it follows, similarly, that

$$D = -2.5\{\tfrac{7}{2}\log(1.531 + x^{2/7}) - \log(1+x)\}\,. \qquad (130)$$

If the fainter component is a spectroscopic binary, we find, in the same way, when two spectra appear,

$$D = -2.5\{\tfrac{7}{2}\log(1 + 1.635\, x^{2/7}) - \log(1+x)\}\,; \qquad (131)$$

and if only one spectrum is observed,

$$D = -2.5\{\tfrac{7}{2}\log(1 + 1.531\, x^{2/7}) - \log(1+x)\}\,. \qquad (132)$$

These equations can be obtained by substituting $1/x$ for x in equations (129) and (130).

Allowance for the division of the mass of the system into two or more parts may thus be made simply by adding the correction D to M_1 in equation (126), obtaining

$$\log n_0 = 0.0500(M_1 + B' + D - 5.39)\,.$$

The correction D is always negative. If we set $D = -1^m\!.88 + D'$, D' will be positive for all simple pairs, and almost always negative when one or both components are spectroscopic binaries. We then have

$$\log n_0 = 0.0500(M_1 + B' + D' - 7.27)\,. \qquad (133)$$

The values of D' are given in Table 38.

When both components are spectroscopic binaries, showing two spectra, the mass of each pair is 1.9 times that of the brighter component, or 1.635 times that of a single star of the same brightness. To combine each pair into one star would change the magnitude of each by $-1^m\!.87$, which must be added algebraically to the correction D' for a simple pair (Table 38). When both components are binary and show one spectrum, the correction is $-1^m\!.62$. The rare case where one component shows one spectrum and the other two, can be treated with sufficient accuracy by adopting the mean correction $-1^m\!.74$.

TABLE 38

CORRECTIONS D′ TO M_1, DEPENDING ON BOLOMETRIC MAGNITUDE DIFFERENCE ΔM

ΔM	Simple Pair	Bright Component Spectroscopic Binary 2 Spectra	Faint Component Spectroscopic Binary 2 Spectra	Bright Component Spectroscopic Binary 1 Spectrum	Faint Component Spectroscopic Binary 1 Spectrum
0ᵐ0.........	0ᵐ00	−1.05	−1.05	−0.89	−0.89
2.0.........	+0.28	−1.00	−0.53	− .82	−0.41
4.0.........	+0.77	−0.70	+0.19	− .49	+0.28
6.0.........	+1.17	−0.44	+0.78	− .21	+0.84
8.0.........	+1.44	−0.25	+1.19	− .03	+1.24
10.0.........	+1.62	−0.15	+1.46	+ .09	+1.48
12.0.........	+1.72	−0.09	+1.62	+ .15	+1.64
14.0.........	+1.79	−0.05	+1.73	+0.20	+1.74

65. *The Bolometric Corrections.*—The corrections from visual to bolometric magnitude, which should be made in calculating dynamical parallaxes, are not the ordinary values but incorporate a correction for the effect of the radius upon the luminosity.

The theoretical mass-luminosity relation for homologous stars may be written

$$L = \text{constant } M^3\mu^4\beta^4k^{-1} ,$$

where μ is the molecular weight,

$$\frac{1-\beta}{\beta^4} = \text{constant } M^2\mu^4 ,$$

and k is the mass coefficient of opacity. According to the accepted extension of Kramers' theory,

$$k = \text{constant } \rho T^{-3.5}\tau^{-1} ,$$

where τ is the "guillotine factor." If R is the radius, we find easily that

$$L = \text{constant } M^{5.5}\mu^{7.5}\beta^4R^{-0.5}\tau .$$

If T_e is the effective temperature,

$$L = \text{constant } R^2 T_e^4 .$$

Eliminating R, we have

$$L = \text{constant } \mathrm{M}^{4.4} \mu^6 \beta^{3.2} T_e^{0.8} \tau^{0.8} .$$

(The dependence of the constant upon the composition does not concern us at present.)

For stars of a standard mass and composition the absolute magnitude will be

$$M = M_m - 2 \log \left(\frac{T_e}{T_0} \right) - 2 \log \tau ,$$

where M_m is the absolute magnitude corresponding to M for the standard model and composition, and a standard effective temperature T_0. Following Eddington, we will take $T_0 = 5200°$.

If C is the ordinary bolometric correction,

$$M = M_{\text{vis}} + C . \tag{134}$$

If, also,

$$M_b = M + 2 \log \left(\frac{T_e}{T_0} \right) \tag{135}$$

and

$$C' = C + 2 \log \left(\frac{T_e}{T_0} \right) , \qquad T_0 = 5200° , \tag{136}$$

then

$$M_b = M_{\text{vis}} + C' , \qquad M_m = M_b + 2 \log \tau . \tag{137}$$

Note that M_b is the quantity given in Table 23 and appearing in equation (92).

The values of C' used in the present work were computed with Eddington's bolometric correction[1] and the temperature scale given below. New values of these quantities, based on much more extensive material, have recently been given by Kuiper.[2] The results

[1] *The Internal Constitution of the Stars*, p. 139, Table 16.

[2] *Ap. J.*, 88, 429, 1938.

from the two sets of data are given in Table 39. Doubtful or interpolated values are in parentheses. If the new values C'_k are sub-

TABLE 39

BOLOMETRIC CORRECTIONS

Sp.	Older Values			Kuiper				
	T_e	C	C'	T_e	C	C'_k	$C'_k+0.20$	$\Delta \log \overline{\mathrm{M}^{1/3}}$
B0.......	23000	−2.17:	−0.88	25000	−2.70	−1.34	−1.14	+0.010
B5.......	15000	−1.04	−0.12	15500	−1.58	−0.64	−0.44	+ .012
A0.......	11000	−0.43	+0.23	10700	−0.72	−0.10	+0.10	+ .005
A5.......	8600	−0.11	+0.32	8500	−0.31	+0.11	+0.31	.000
F0.......	7400	−0.02	+0.29	7500	−0.16	+0.15	+0.35	− .002
F5.......	6500	0.00	+0.19	6500	−0.04	+0.14	+0.34	− .006
dG0.......	6000	−0.02	+0.10	6000	−0.06	+0.06	+0.26	− .006
dG5.......	5600	−0.07	−0.01	5400	−0.10	−0.08	+0.12	− .005
dK0.......	5100	−0.16	−0.18	4900	−0.11	−0.17	+0.03	− .008
dK5.......	4400	−0.40	−0.55	4150*	−0.85	−1.03	−0.83	+ .011
dM0.......	3400	−1.14	−1.57	3600*	−1.43	−1.74	−1.54	+ .001
dM5.......	....	....	....	2850†	−3.1:	−3.6:	−3.4:	....
gG0.......	5600	−0.07	0.00	5200	−0.25	−0.25	−0.05	+ .002
gG5.......	4700	−0.27	−0.36	4600	−0.39	−0.50	−0.30	− .002
gK0.......	4200	−0.50	−0.69	4200	−0.59	−0.73	−0.53	− .006
gK5.......	3400	−1.14	−1.57	3600	−1.35	−1.68	−1.48	− .001
gM2.......	3100	−1.53	−1.98	3200	−1.95	−2.37	−2.17	+ .007
gM5.......	(2850)	−2.03	−2.55	2840†	−3.4:	−3.9:	−3.7:	(+0.045)
gM7.......	2700	−2.25	−2.83	....	....	....	....	....

* Kuiper gives T_e = 3000° for dK5+, 3200° for dM2. Plotting the former as K6, and M0 as K8, we interpolate these values.

† Extrapolated.

stituted for the old in Table 23, giving new absolute magnitudes M_k, and if the linear least-squares solution is repeated, we find

$$\log \overline{\mathrm{M}_k^{1/3}} = (-0.0387 \pm 0.0015)\overline{M_k} + 0.1954 . \qquad (138)$$

The old solution gave

$$\log \overline{\mathrm{M}_b^{1/3}} = (-0.0391 \pm 0.0014)\overline{M_b} + 0.2032 . \qquad (92)$$

The coefficients of M_b and M_k are substantially identical, and we may write

$$\log \overline{\mathfrak{M}_k^{1/3}} - \log \overline{\mathfrak{M}_b^{1/3}} = -0.039(\overline{M_k} - \overline{M_b}) - 0.0078$$
$$= -0.039(C'_k - C' + 0.20) .$$

The change from our former constants to Kuiper's may therefore be made simply by substituting $C'_k + 0.20$ for C' and using exactly the old form of the mass-luminosity relation. The effect of this substitution on $\log \overline{\mathfrak{M}^{1/3}}$ is shown in the last column. The average discordance (excluding the doubtful value for gM5) is ± 0.005, corresponding to 1.2 per cent, which is practically negligible.

For the late M stars the bolometric corrections are large and uncertain, depending largely on band absorption in the spectra. The older tabular values are too small. The new values would diminish the dynamical parallaxes of giants beyond M2, thereby somewhat improving the agreement with observation (§ 39). For dwarfs of this class better parallaxes can be obtained by assuming that the absolute magnitude depends only on the spectral class.[3]

66. *Allowance for Guillotine Factor.*—The function τ, which is defined by very complicated equations, has been tabulated by Strömgren.[4] For giant stars, $\log \tau$ is very nearly zero; for those of the main sequence, it is considerable. From Strömgren's calculations[5] we have for typical and well-determined stars:

	Sun	Sirius A	U Oph br	u Her br
Spectrum	G0	A0	B8	B3
T_e	5750	11000	13000	18000
M	+4.6	+0.5	−2.2	−3.7
M_b*	+4.7	+1.0	−1.3	−2.6
$2 \log \tau$	1.56	1.12	0.94	0.96
Computed	1.49	1.19	0.99	0.88
O−C	+0.07	−0.07	−0.05	+0.08

* $M_b = M + 2 \log (T_e/T_0)$.

The empirical formula

$$2 \log \tau = 1.10 + 0.085 M_b \tag{139}$$

[3] W. W. Morgan, *Ap. J.*, 87, 589, 1938.

[4] *Zs. f. Ap.*, 4, 118, 1932; 7, 222, 1933.

[5] *Ap. J.*, 87, 520, 1938.

represents these satisfactorily. How good it would be for the fainter dwarfs is unknown, as the theory for these dense stars is imperfect; but it is undoubtedly much better to use it than to ignore τ, as has previously been done.

Bethe's approximation,[6]

$$\tau = \text{constant } \rho^{1/2} T^{-3/4} ,$$

is a good representation of the prevailing average conditions for main-sequence stars later than A0 or thereabouts. It gives

$$\begin{aligned} L &= \text{constant } \mathfrak{M}^{5.25} \mu^{6.75} \beta^4 R^{-1.25} \\ &= \text{constant } \mathfrak{M}^{3.23} \mu^{4.15} \beta^{2.46} T_e^{1.54} . \end{aligned}$$

The present approximation, which permits a more direct connection between giants and main-sequence stars, appears to be preferable for the purpose in hand.

We have, then, by equation (137),

$$M_m = 1.085 M_b + 1.10 , \qquad 2 \log \tau = 0.0 . \tag{140}$$

Now we have adopted

$$\log \overline{\mathfrak{M}_b^{1/3}} = -0.0400\, \overline{M_b} + 0.2080 = -0.0400(\overline{M_b} - 5.20) ; \tag{94}$$

whence

$$\log \overline{\mathfrak{M}_b^{1/3}} = -0.0369\, \overline{M_m} + 0.2486 = -0.0369(\overline{M_m} - 6.74) . \tag{141}$$

If we should correct the individual values of M_b for the main sequence, adding $2 \log \tau$ and obtaining M_m, and then solve by least squares, we would obtain equation (141) exactly. Proceeding as in § 63, we would have new values of a, M, A, and b in equation (122); but, as may readily be verified, we would arrive at exactly the same numerical values. This equation, and its consequences, equations (126) and (133), may then be applied to all main-sequence stars, using the bolometric corrections C', and will give correct dynamical parallaxes.

[6] *Phys. Rev.*, 55, 434, 1939.

This gives for the main sequence

$$\log n_0 = 0.0500(M_1 + B' + D' - 7.27) . \tag{133}$$

For the giants, however, $a = 0.0369$, $A = 6.74$, $n = 0.276$, $b = 1.227$, and $ab = 0.0452$. Introducing B' and D' as above, we have

$$\log n_0 = 0.0452(M_1 + B' + D' - 8.81) . \tag{142}$$

NOTE.—In strict accuracy, the values of B' and D' should have been recalculated with the new value of n. This would change the constant in equation (126) by $-0^{m}.02$ and the individual values of B' by $+0.02$, $+0.01$, -0.01, 0.00, -0.01, and -0.04, which would alter n_0 in the most extreme case by 0.6 per cent, which is quite negligible. The values of D' were calculated with $n = 0.286$, almost midway between the two adopted values, and likewise need no change.

The values of n_0, given by equations (133) and (142), are as shown in the table below. For subgiants the values for the giants may be used.

$M_1+B'+D'$	n_0		$M_1+B'+D'$	n_0	
	Main Sequence	Giants		Main Sequence	Giants
−6.0	0.217	0.214	+4.0	0.686	0.606
−5.0	.244	.238	+5.0	0.770	.673
−4.0	.273	.264	+6.0	0.864	0.746
−3.0	.307	.293	+7.0	0.969	
−2.0	.344	.325	+8.0	1.088	
−1.0	.386	.360	+9.0	1.220	
0.0	.433	.400	+10.0	1.369	
+1.0	.486	.444	+11.0	1.536	
+2.0	.545	.492	+12.0	1.724	
+3.0	0.612	0.546			

It should be emphasized that this table assumes that the conditions in the star's interior—density model and atomic composition—are of the average type for main-sequence stars of the same absolute magnitude.

The table of n_0 given in our original discussion[7] takes no account of the correction B'. To obtain comparable results from the present

[7] *A.J.*, 38, No. 897, p. 98, 1928, Table B.

table, we should set $B' = +0.19$ (the value for a perfectly accurate orbit). We then find for the ratio $n_0(\text{new})/n_0(\text{old})$ the following values:

M_1	-3.0	-1.0	1.0	3.0	5.0	7.0	9.0	11.0
Main sequence	1.660	1 170	1.002	0.959	0.996	1.030	1.096	1.167
Giants	1.581	1.089	0.914	0.855	0.853	....	....	....

The differences arise from the curvature of Eddington's relation, on which the old values were based. Between $M_1 = 0$ and $M_1 = 7$—that is, for a large majority of the main-sequence stars—the average difference, regardless of sign, is ± 3.2 per cent. For the bulk of the giants the new values of n_0 are smaller than the old, since allowance for the function τ makes the luminosity less for a given mass.

If $M_1 < -1$, the new values of n_0 are much larger than the old. This is an advantage, for almost always, in practice, when M_1 comes out very bright, this arises not from extreme luminosity of the stars but from unusual diminution of the apparent separation or motion of a physical pair by foreshortening. In such cases the assumption of the large value of the exponent n, which is theoretically to be anticipated for great luminosity and mass, does harm.

67. *Estimation of Spectra of Unobserved Components.*—When the visual magnitudes and spectra of both components have been observed, their separate bolometric magnitudes, the combined bolometric magnitude, and the bolometric ΔM can be calculated. But the spectra of the companions of the great majority of double stars are unknown. In this case we may plot the values of C' against the absolute visual magnitudes M_0 (Table 1) and obtain a mean curve giving the relation of the two. Given the spectrum of the brighter component, we may find from this curve what difference $\Delta C'$ may be expected to correspond to the observed visual Δm (when both stars are on the main sequence or are giants). Then

$$\Delta M_{\text{bol}} = \Delta m_{\text{vis}} + \Delta C' .$$

In many cases only the spectrum of the combined light is known. No significant error will be committed by taking this as the spectrum of the brighter component.

Much time may be saved, for a large number of stars, by constructing a diagram see (Fig. 4, p. 137), in which the co-ordinates are spectral type and Δm_{vis}, and the contours are drawn for specified values of $\Delta C'$.

For the giants, whenever Δm_{vis} is considerable, the fainter component is always of early type—for which C' is small; while for the bright star, C'_b is often negative and large. It has therefore been assumed here that $C' = 0$ for the companion whenever $\Delta m_{\text{vis}} > 1^{\text{m}}$, so that $\Delta C' = -C'_b$. When $\Delta m < 1^{\text{m}}$, it has been assumed as a rough approximation that $\Delta C' = -C'_b \Delta m_{\text{vis}}$.

The determination of ΔM_{bol} and that of the correction D' may be combined in a single step, by constructing new contour diagrams, in which spectral type and visual Δm are co-ordinates, and contours are drawn for D'. The preparation of these diagrams (Figs. 3*a* and 3*b*, pp. 134 and 135) presents no difficulty. The discontinuity in the upper part of that for the giants corresponds to the change in the formula for $\Delta C'$ when $\Delta m_{\text{vis}} = 1^{\text{m}}$.

These diagrams were prepared with provisional values of Δm and C', differing slightly from those finally adopted; but the corrections which they give will be substantially the same, except for late M spectra—for which the corrections themselves are uncertain.

Similar diagrams could be prepared for the cases when a component is a spectroscopic binary, but did not appear to be worth the trouble.

In this case Δm_{vis} is to be taken as the observed visual-magnitude difference between the telescopically observed components (regardless of whether they are spectroscopic binaries or not). The corresponding bolometric difference ΔM_{bol} may be obtained with sufficient accuracy by adding C', determined as above.

68. *Faint Stars with Unknown Spectra.*—For some faint pairs the spectral type is unknown. Rough values of the dynamical parallax may be found for these by estimating the spectrum from the observed value of $M_1(\text{vis})$.

From (17) and (120) we find

$$M_1(\text{vis}) = m + 5 + 5 \log h_1 = M'_1(\text{vis}) + \Delta \,,$$

where

$$M'_1(\text{vis}) = M_0 + 5 + 5 \log h'_1 \,, \qquad \Delta = m - m_b \,.$$

From the data of Table 18 (using the groups with trigonometric parallaxes), we find:

Group	Mean Sp.	M_1' (vis)	Group	Mean Sp.	M_1' (vis)	Group	Mean Sp.	M_1' (vis)
P........	B2.8	0.8	O.......	F3.7	3.9	P.......	G9.5	5.4
P.	A1.3	1.8	P.....	F4.7	4.0	P.......	K4.7	7.2
O.	A2.1	2.4	O.......	F9.8	4.8	O.......	K5.1	6.8
O........	A9.5	3.4	P......	F9.9	4.9	P.	M1.5	9.5
P..... ..	F0.0	3.2	O.......	G7.4	5.9	O......	M2.9	10.6

These data give a smooth curve from which the spectra corresponding to given values of M_1'(vis) may be read. They are given in Table 44 (p. 139).

For main-sequence stars, with M_1' fainter than $+3$, this should give a fairly good estimate of the spectrum, and so of d.

For the 55 giants (with trigonometric parallaxes) in Table 18, the mean spectral class is G9.1; the mean $M_0 = +0.5$; and $M_1' = +1.4$—corresponding to a main-sequence star of class B9. For the latter the correction C' is $+0.19$, while for gG9 it is -0.62. Taking the typical values $B' = 0$, $D' = +0.5$, we have for the giant $M_1 + B' + D' = +1.3$; while, if treated as a dwarf, it is $+2.1$. The corresponding values of n_0 are 0.458 (giant) and 0.551 (main sequence). This rough method will therefore estimate the values of d for giants about 20 per cent too great—more for the redder stars. Very few true giants, however, for which the relative motion can be reliably detected by the available observations, will be so faint that their spectra have not been observed.

For these faint stars the tendency of observational selection to favor the "interesting" pairs (§ 14) is a much more serious source of error.

B. PRECEPTS FOR CALCULATING DYNAMICAL PARALLAXES

69. *Instructions for the Computer:*

1. For each system the following photometric and spectroscopic data are required:

   Visual magnitude, m, corresponding to the combined light;

   Difference in visual magnitude Δm;

Spectral class, of the brighter component, or of the combined light;

Information whether either component is a *spectroscopic binary*, and, if so, whether one or two spectra are visible.

It is desirable, but not necessary, to know the spectral class of the fainter component.

When the spectrum is unknown, an approximate value may be found, as described in Precept 10.

2. The necessary double-star data are either—

A. The period P (in years) and mean distance a'' (in seconds of arc) for pairs for which orbits are available; or

B. The apparent distance s in seconds of arc and relative motion w (in seconds of arc per year), for slow-moving physical pairs, for which reliable orbits cannot be computed.

The hypothetical parallax h_1 is then given by the equations

$$\text{A)}\quad h_1 = \frac{a''}{P^{2/3}},$$

$$\text{B)}\quad h_1 = 0.418\sqrt[3]{sw^2}\,.$$

Grading the orbits as "good," "fair," or "poor" is a matter of judgment. The number and accuracy of the observations, the length of the observed arc, and the possible uncertainties of quadrant must all be considered. Orbits for which only the elements have been published, without a detailed comparison with the observations, can therefore not be graded and should usually be rejected, and the system treated as a physical pair.

3. To find s and w for a physical pair:

Plot the observed position angles θ and distances s separately against the time. Draw straight lines to represent these data—or curves, if straight lines are inadequate. From these lines—or the tangents to the curves—determine the yearly variations, $d\theta/dt$ and ds/dt, at a suitable epoch

(the same for both). In the absence of curvature, this should be the mean epoch of the observations—otherwise, some date at which the motions are best defined by the observations.

Read the values of s and θ at this epoch from the plots. Then

$$w^2 = \left(\frac{ds}{dt}\right)^2 + \left(\frac{s}{57}\frac{d\theta'}{dt}\right)^2,$$

where

$$\frac{d\theta'}{dt} = \frac{d\theta}{dt} - 0.0056 \sin \alpha \sec \delta$$

(the last term being the correction for precession).[8]

Knowledge of the relative weight of the observations is essential for drawing the "best" lines or curves.

The grading of these values of h_1 is again a matter of judgment. A determination may be called "good" if the adopted lines or curves for both θ and s are determined by the plots with little or no freedom of choice; moderate latitude is "fair"; and considerable uncertainty, "poor."

4. When h_1 has been found, calculate M_1(vis) by the equation

$$M_1(\text{vis}) = m + 5 + 5 \log h_1 .$$

Giants and main-sequence stars are now to be separated. "Giants" (from the standpoint of the present tables) include stars of classes G0–K0 with M_1(vis) brighter than $3^m.0$, and of classes K1–M1 with M_1(vis) brighter than $4^m.0$. All others are reduced by the "main-sequence" tables.

5. Find the bolometric value of M_1 by the equation

$$M_1 = M_1(\text{vis}) + C' .$$

Take C' from Table 40 with the spectral type of the brighter component (or of the combined light) as argument.

[8] For convenience of tabulation the quantities ds/dt, $d\theta/dt$, and w given in our earlier publications are *centennial* motions.

TABLE 40

Sp.	C′	Sp.	C′	Sp.	C′	Sp.	C′	Sp.	C′	Sp.	C′	Sp.	C′
B0	−0.88	A0	+0.23	F0	+0.28	gG0	0.00	gK0	−0.72	dG0	+0.10	dK0	−0.18
B1	− .69	A1	+ .26	F1	+ .27	gG1	− .07	gK1	−0.84	dG1	+ .08	dK1	−0.22
B2	− .52	A2	+ .29	F2	+ .25	gG2	− .14	gK2	−0.99	dG2	+ .06	dK2	−0.29
B3	− .36	A3	+ .30	F3	+ .23	gG3	− .21	gK3	−1.15	dG3	+ .03	dK3	−0.35
B4	− .22	A4	+ .31	F4	+ .22	gG4	− .28	gK4	−1.32	dG4	+ .01	dK4	−0.43
B5	− .12	A5	+ .32	F5	+ .20	gG5	− .35	gK5	−1.49	dG5	− .01	dK5	−0.55
B6	− .02	A6	+ .32	F6	+ .18	gG6	− .42	gM0	−1.66	dG6	− .04	dK6	−0.85
B7	+ .06	A7	+ .32	F7	+ .16	gG7	− .48	gM1	−1.82	dG7	− .06	dK8	−1.18
B8	+ .13	A8	+ .31	F8	+ .14	gG8	− .55	gM2	−1.98	dG8	− .09	dM0	−1.57
B9	+ .19	A9	+ .30	F9	+ .12	gG9	− .62	gM3	−2.2	dG9	− .13	dM1	−1.9
A0	+0.23	F0	+0.28	dG0	+0.10	gK0	−0.72	gM4	−2.4	dK0	−0.18	dM2	−2 2
								gM5	−2.6			dM3	−2.5
								gM6	−2.8			dM4	−2.8
								gM7	−3.0			dM5	−3.1

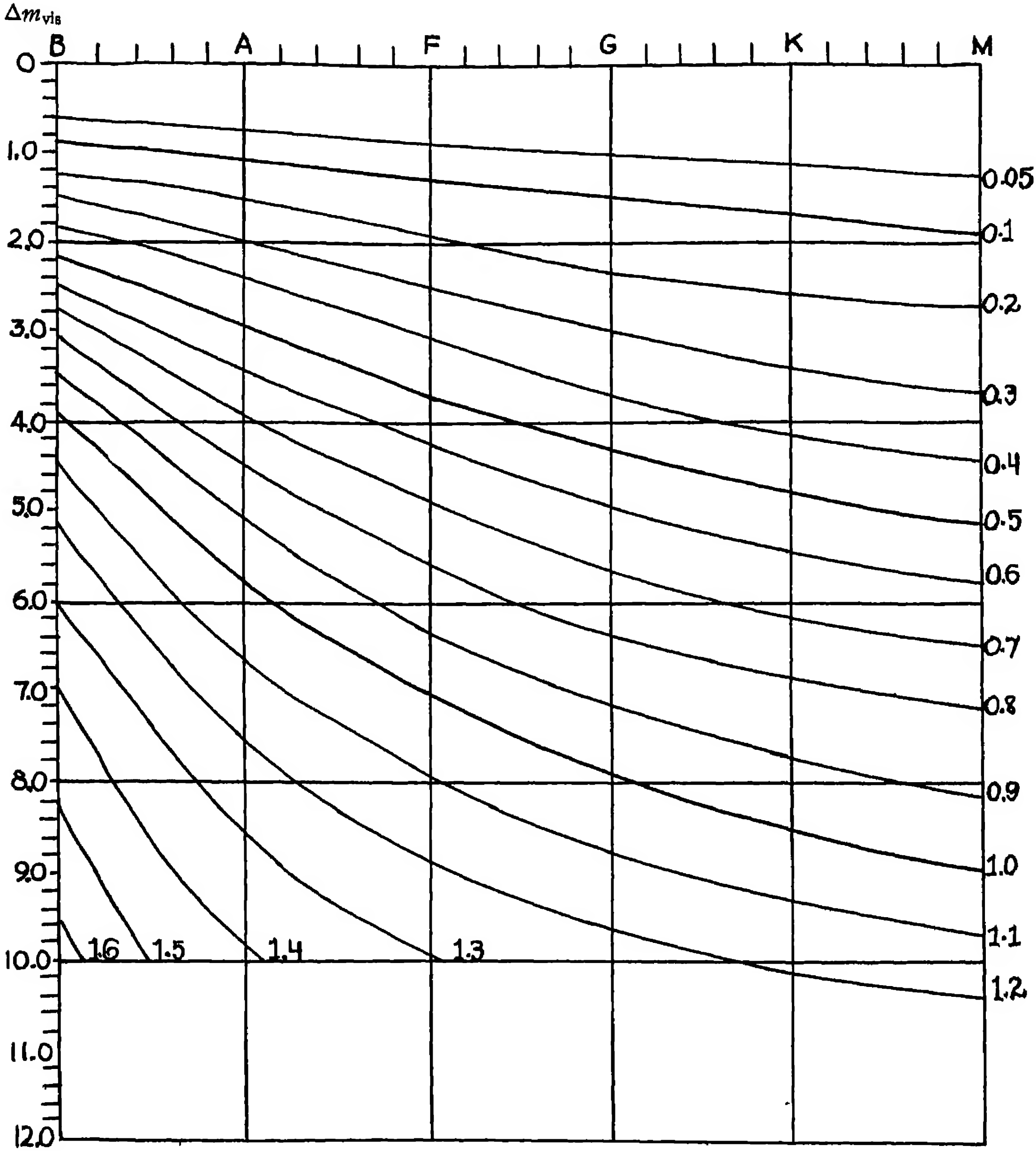

FIG. 3a.—D' for main-sequence stars

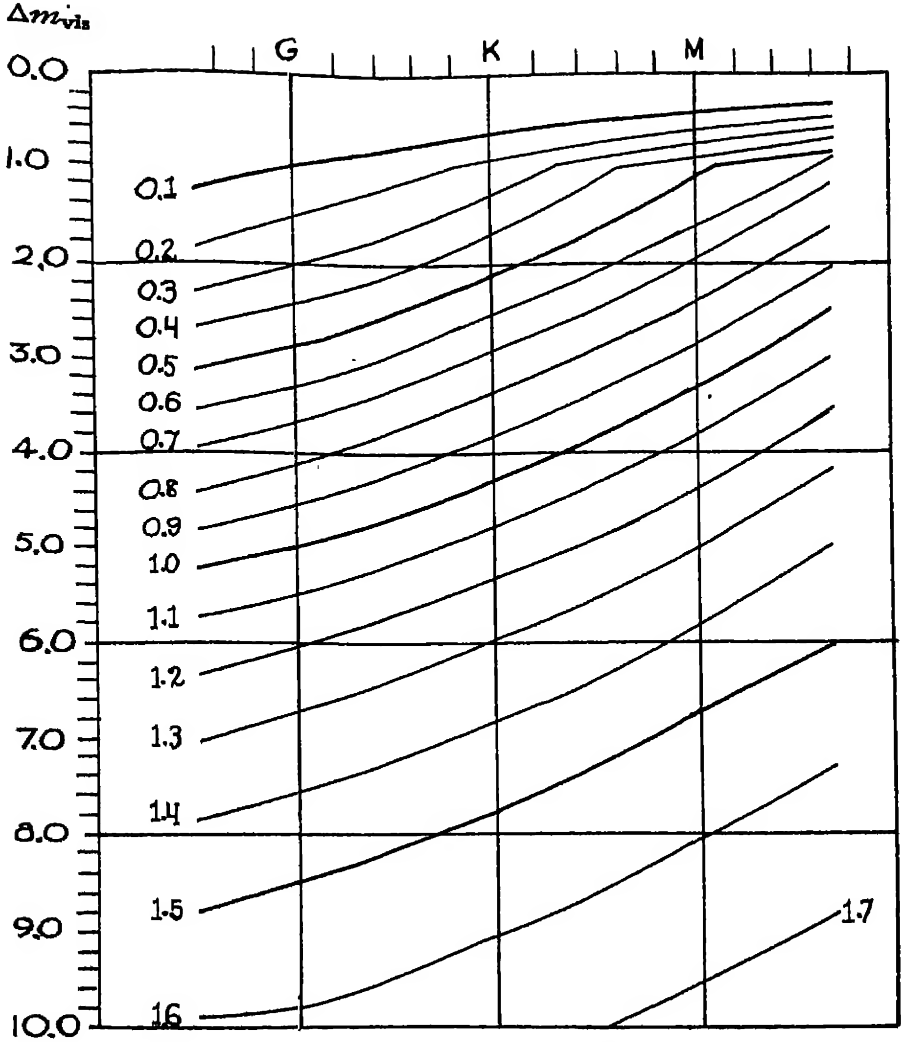

FIG. 3b.—D' for giant stars

6. Take the correction B', depending upon the grade assigned to the pair, from Table 41.

TABLE 41

	Good B'	Fair B'	Poor B'
Orbits..............	+0.17	+0.11	−0.09
Physical pairs........	0.00	−0.09	−0.36

7. Find the quantity D', as follows:
   A. When only one spectrum is known, and neither component is a spectroscopic binary, read D' from Figure 3*a* or Figure 3*b*, finding the point with the spectral class as horizontal and Δm(vis) as vertical co-ordinate, and reading the value of D' from the contours. Separate diagrams are given for giants and main-sequence stars.
   B. If both spectra are known, find ΔM(bol) by the equation

$$\Delta M(\text{bol}) = \Delta m(\text{vis}) + C'_f - C'_b ,$$

where C' and C'_b are the values taken from Table 40 for the spectral classes of the fainter and brighter components. Then take D' from Table 42, with argument ΔM(bol), using the appropriate column according to the presence or absence of spectroscopic binaries. If both components are spectroscopic binaries, take the value of D' from the column for a simple pair, and add, algebraically, the correction $-1^{m}.87$, -1.74, or -1.62, according as both, one, or neither of the components show two spectra.
   C. If the spectrum of the fainter component of the visual pair is not known but the brighter is a spectroscopic binary, use the equation

$$\Delta M(\text{bol}) = \Delta m(\text{vis}) + \Delta C' .$$

When the brighter component belongs to the main sequence, read $\Delta C'$ from the contour diagram, Figure 4.

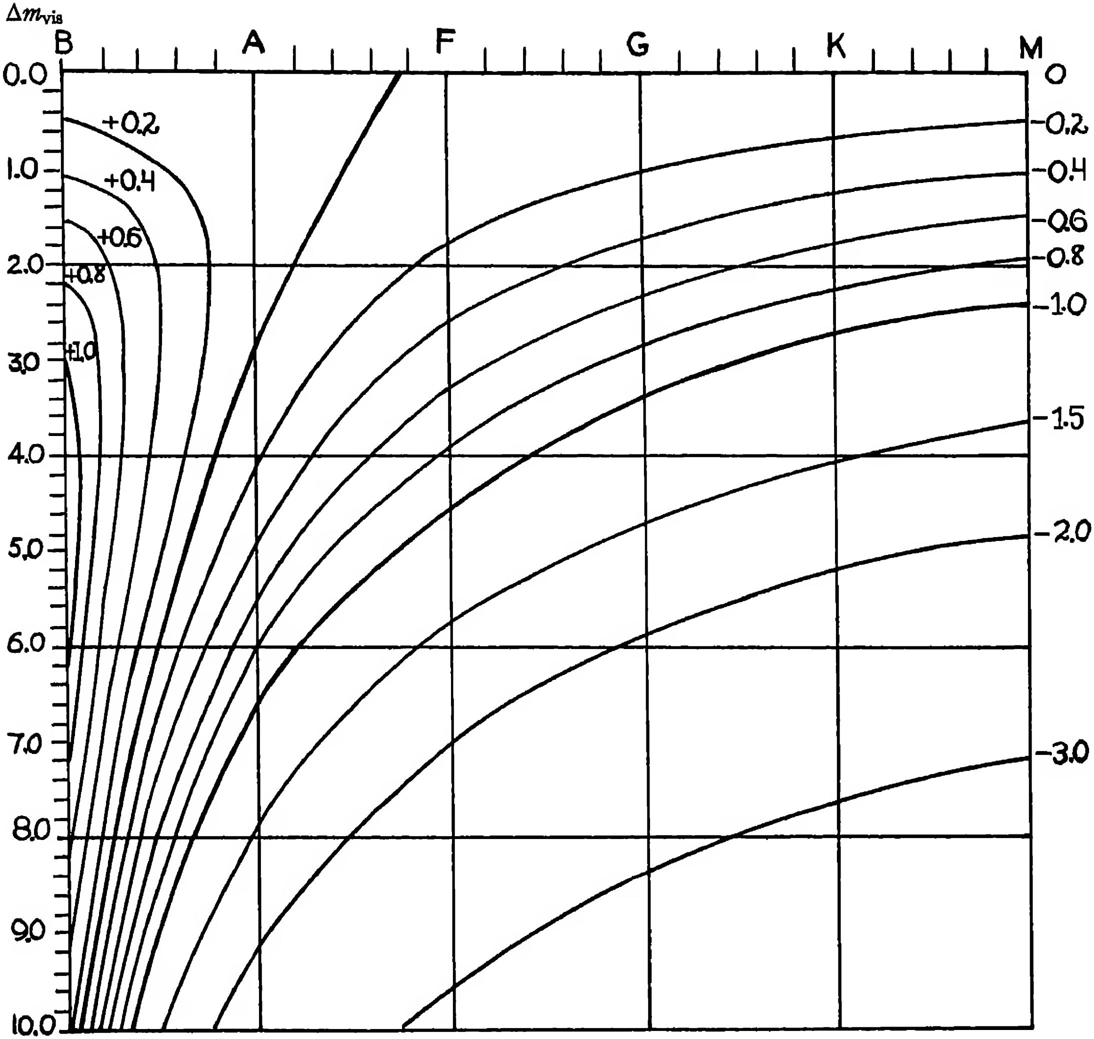

FIG. 4.—$\Delta C'$ for main-sequence stars

When the brighter component is a giant and Δm(vis) > 1^m, take $\Delta C' = -C'_b$; if Δm(vis) < 1^m, take $\Delta C' = -C'_b$ Δm(vis). With this value of ΔM(bol) proceed as in (B), above, to find D'.

TABLE 42

CORRECTIONS D' TO M_1, DEPENDING ON BOLOMETRIC MAGNITUDE DIFFERENCE ΔM

ΔM(bol)	Simple Pair	Bright Component Spectroscopic Binary 2 Spectra	Faint Component Spectroscopic Binary 2 Spectra	Bright Component Spectroscopic Binary 1 Spectrum	Faint Component Spectroscopic Binary 1 Spectrum
$0^m.0$...	$0^m.00$	−1.05	−1.05	−0.89	−0.89
0.5...	+0.02	−1.10	−0.98	− .93	−0.82
1.0...	+0.08	−1.09	−0.86	− .92	−0.71
1.5...	+0.17	−1.06	−0.69	− .87	−0.57
2.0...	+0.28	−1.00	−0.53	− .82	−0.41
2.5...	+0.40	−0.93	−0.34	− .74	−0.23
3.0...	+0.52	−0.85	−0.16	− .66	−0.05
3.5...	+0.64	−0.77	+0.02	− .57	+0.11
4.0...	+0.77	−0.70	+0.19	− .49	+0.28
4.5...	+0.88	−0.63	+0.35	− .42	+0.43
5.0...	+0.99	−0.57	+0.50	− .35	+0.57
5.5...	+1.07	−0.50	+0.64	− .27	+0.72
6.0...	+1.17	−0.44	+0.78	− .21	+0.84
6.5...	+1.24	−0.39	+0.90	− .16	+0.96
7.0...	+1.31	−0.33	+1.00	− .11	+1.06
8.0...	+1.44	−0.25	+1.19	− .03	+1.24
9.0...	+1.55	−0.18	+1.35	+ .04	+1.40
10.0...	+1.62	−0.15	+1.46	+ .09	+1.48
11.0...	+1.67	−0.11	+1.55	+ .13	+1.59
12.0...	+1.72	−0.09	+1.62	+0.15	+1.64

8. Having found M_1, B', and D', enter Table 43 with argument ($M_1 + B' + D'$) (using the appropriate column for main-sequence or giant stars), and find n_0.

TABLE 43

$M_1+B'+D'$	n_0 M.S.	Diff	n_0 Giants	Diff	$M_1+B'+D'$	n_0 M.S.	Diff	n_0 Giants	Diff	$M_1+B'+D'$	n_0 M.S.	Diff
−6.0	0.217	13	0.214	12	+0.5	0.459	27	0.421	23	+ 6.0	0.864	51
−5.5	.230	14	.226	12	+1.0	.486	29	.444	23	+ 6.5	0.915	54
−5.0	.244	14	.238	12	+1.5	.515	30	.467	25	+ 7.0	0.969	58
−4.5	.258	15	.250	14	+2.0	.545	32	.492	27	+ 7.5	1.027	61
−4.0	.273	16	.264	14	+2.5	.577	35	.519	27	+ 8.0	1.088	64
−3.5	.289	18	.278	15	+3.0	.612	36	.546	29	+ 8.5	1.152	68
−3.0	.307	18	.293	15	+3.5	.648	38	.575	31	+ 9.0	1.220	73
−2.5	.325	19	.308	17	+4.0	.686	41	.606	33	+ 9.5	1.293	76
−2.0	.344	20	.325	17	+4.5	.727	43	.639	34	+10.0	1.369	
−1.5	.364	22	.342	18	+5.0	.770	46	.673	36			
−1.0	.386	23	.360	20	+5.5	.816	48	.709	37			
−0.5	.409	24	.380	20	+6.0	0.864		0.746				
0.0	0.433	26	0.400	21								

TABLE 44

M_1' (vis)	Sp.	M_1' (vis)	Sp.
0.0	B2	+ 6.0	K0
+1.0	B7	7.0	K5
2.0	A2	8.0	M0
3.0	A8	9.0	M2
4.0	F5	+10.0	M3
+5.0	G3	....	....

9. The dynamical parallax d is then given by

$$d = n_0 h_1 .$$

10. When the spectral type of the star is unknown, a rough dynamical parallax may be found by assuming that it be-

TABLE 45

CALCULATION OF DYNAMICAL PARALLAXES

ADS	433	1798	996	6263
m	10.3	8.9	5.18	5.81
Δm	3.0	0.8	0.92	0.3
Spectrum	*M3*	G5	A5, *F6*	A0
Spectroscopic binary notes	....	....	b, d	b
s	2.65	2.28	23.60	1.30
w	0.118	0.00113	0.00302	0.00462
h_1	0.139	0.0060	0.025	0.0127
Grade	f	p	f	g
ΔM(bol)	....	....	0.78[2]	0.4[4]
M_1(vis)	11.02	2.80*	2.17	1.33
C' (Table 40)	−2.5	−0.35	+0.32	+0.23
M_1	+8.52	+2.45	+2.49	+1.56
B' (Table 41)	−0.09	−0.36	−0.09	0.00
D'	+0.23[1]	+0.10[1]	−1.57[3]	−0.91[5]
$M_1+B'+D'$	+8 66	+2.19	+0.83	+0.65
n_0	1.174	0.502	0.476	0.466
d	0.164	0.0030	0.012	0.0059

NOTES TO TABLE 45

b. Brighter component spectroscopic binary, 1 spectrum visible.
d. Fainter component spectroscopic binary, 1 spectrum visible.
* Reduced as giant.
[1] From Fig. 3.
[2] From Table 40.
[3] +0.05 (Table 42); −1.62 (p. 136).
[4] From Fig. 4.
[5] From Table 42.

longs to the main sequence and by using the correlation between the mean values of M_1'(vis) and the spectral type (§ 68). We have

$$M_1'(\text{vis}) = M_1(\text{vis}) - m + m_b ,$$

where m_b is the apparent magnitude of the brighter component. With M'_1(vis) as argument, the estimated spectrum may be taken from Table 44; and the calculation proceeds as described under (5).

As an illustration of the method, the essentials of the calculation for a few cases of different types for which the data may be found in Tables 46 and 53 are given in Table 45. The notes appended should make the process clear.

CHAPTER V

GENERAL CATALOGUE OF DYNAMICAL PARALLAXES

The development of the improved method for determining dynamical parallaxes (chap. iv) clears the way for the preparation of a general catalogue based upon a uniform treatment of all available material.

The four existing catalogues[1] are very nearly homogeneous—since Finsen deliberately adopted the methods used in the other three; but the new calibration introduces certain refinements and should give better results for the stars of high luminosity.

A. SUMMARIES OF OBSERVATIONAL DATA

70. *Double-Star Data.*—In any general discussion or catalogue based on the collation of the observational data available at the time of its preparation, it is the obvious duty of the authors to place on record, as far as possible, what data have been used and how they have been corrected and weighted.

It is obviously impracticable to reprint the individual double-star measures—most of which are to be found in the catalogues of Burnham, Aitken, and Innes. When orbits have been published, the observations are practically always given by the computers; these may be found in the references listed at the end of Table 52. These orbits have been adopted as published, leaving the matter of rejection of discordant observations, etc., to their authors. The judgment of the present writers has been exercised in the selection of the best available orbit and the grading of its quality, or sometimes in the rejection of a poor orbit, leaving the star in the list of "physical pairs."

For the great number of slow-moving pairs we have been obliged to confine ourselves to tabulating the adopted values of the relative positions and motions at a given epoch which have been used in calculating the dynamical parallaxes in our *General Catalogue.* For the majority of these the data already published[2] were found to require no change, and reference may be made to these papers. All cases in which the previously published values have been altered are listed in Table 46. The columns contain, respectively: the ADS

[1] *A.J.*, 39, No. 930, pp. 165–207, 1929; *Lick Obs. Bull.*, Nos. 451 and 485, 1933 and 1937; *Union Obs. Circ.*, No. 93, p. 139, 1935.

[2] *Ibid.*

TABLE 46

STARS FOR WHICH PUBLISHED MOTION HAS BEEN CHANGED OR NO MOTION PREVIOUSLY PUBLISHED

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
48.......	95	125°	+ 80°	4".60	+1".40	6".60
144......	75	292.0	− 0.9	7.55	+0.37	0.39
191.......	70	149.4	− 1.8	11 56	+0 29	0.47
220.......	80	133	+ 3.5	2.30	+0.34	0.37
283.....	70	88.1	+ 0.1	1.37	−0.89	0.89
433*......	30	107	+200	2.65	+7.2	11.78
566.......	70	320 8	0.0	5.7	+2.3	2.3
618.......	75	324.7	+ 3.5	2.05	−0.11	0.16
659.......	75	146 9	+ 0.4	2.21	−0.04	0.04
662.......	09	166.5	− 5.0	1.22	+0.50	0.51
673......	00	221	− 44	0.65	+0.44	0.67
683......	80	117.4	− 1.6	4.48	−0.07	0.15
716.......	00	268	− 28	2.28	−0.36	1.18
805.......	00	104	+ 50	0.67	−0.64	0.87
806.......	80	214.4	+ 2.3	6.40	+0.22	0.32
899.......	75	160.0	− 0.5	29.96	−0.08	0.32
923......	80	322.5	− 7.3	3.92	+0.66	0.83
996.......	75	63.6	− 0.2	23.60	+0.26	0.30
1030.......	75	317	− 29	0.69	+0.26	0.45
1023.......	75	308.4	− 1.1	4.78	+0.06	0.12
1040.......	75	305.0	− 11.5	0.57	0.00	0.12
1477†......	90	205.2	+ 0.4	18.35	−0.13	0.18
1187.......	70	282.9	− 0.6	8.63	+0.38	0.40
1287.......	75	173.8	+ 3.8	3.37	+0.08	0.22
1339.......	75	88.0	+ 1.0	3.70	−1.25	1.25
1411.......	75	126	+ 49	0.50	−0.29	0.51
1406......	75	253	+ 15	2.04	0.00	0.53
1457......	70	169.9	− 4.3	2.74	+0.24	0.32
1567.......	00	13.5	+ 8.0	2.87	+0.26	0.47
1582......	75	52.7	− 5.9	2.37	−0.22	0.34
1703.......	80	230.6	+ 3.2	15.75	+0.63	1.02
1733.......	00	350	+120	2.13	−0.54	4.53
1798.......	70	264.6	+ 1.7	2.28	+0.10	0.11
1801.......	00	205.0	+ 15.6	1.10	+0.26	0.39
1860.......	80	262.5	− 27	2.12	+0.45	1.13
1851.......	05	337.5	+ 40	1.05	+0.50	0.88
1868.......	70	342.0	+ 2.0	1.84	0.00	0.05
1933.......	70	40.0	− 3.8	1.84	+0.36	0.39
1924.......	70	218.7	+ 1.8	13.52	0.00	0.35
1953.......	80	347.5	− 3.5	3.64	−0.14	0.28

* Also ADS 440

† Polaris. Mean values of θ have been corrected for precession.

TABLE 46—*Continued*

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
1971.......	75	83°0	− 0°7	7″72	+0″05	0″15
2004.......	75	173.3	− 9.5	1.85	0.00	0.32
2008.......	75	73.4	− 1.2	6.53	0.00	0.18
2042.......	80	118.9	− 0.8	3.35	+0.16	0.17
2046.......	75	326	− 18	4.62	−0.60	1.60
2218.......	75	216.9	+ 1.7	8.40	−1.80	1.81
2261.......	75	319.5	− 9.0	1.52	−0.23	0.34
2279.......	80	64	− 11	1.80	+0.24	0.43
2336.......	75	89.5	+ 9.8	0.57	−0.41	0.42
2397.......	80	304	+ 14	1.35	−0.11	0.34
2443.......	75	28.0	− 0.8	3.38	+0.14	0.16
2475.......	70	251.5	+ 0.1	6.88	+0.13	0.14
2504.......	75	94.5	+ 10.0	0.91	+0.12	0.19
2546.......	70	162.4	− 1.2	6.81	+0.10	0.22
2554.......	80	102.9	− 7.0	1.80	−0.20	0.31
2577.......	75	43.5	+ 10.0	2.60	+0.59	0.73
2584.......	75	178.9	− 4.0	2.88	−0.16	0.28
2618.......	80	185.0	− 0.6	7.30	+0.36	0.38
2628.......	00	51	− 40	0.61	+1.00	1.09
2801.......	80	102.4	− 4.5	1.16	−0.15	0.18
2850.......	75	347.2	+ 0.4	6.83	+0.26	0.26
2894.......	00	28.2	− 14.0	11.10	0.00	2.82
2959.......	16	194	−220	0.70	+2.00	3.37
2993.......	75	94.6	− 7.3	1.70	−0.23	0.33
2999.......	75	219.0	+ 3.8	3.78	+0.17	0.28
3015.......	80	177	− 22	2.25	+0.55	1.05
3095.......	75	12.8	− 1.8	3.47	−0.43	0.45
3141.......	75	255.5	+ 5.0	2.06	+0.13	0.20
3158.......	00	170.4	− 2.4	2.01	−0.20	0.22
3187.......	05	147.5	− 10.8	0.76	0.00	0.15
3207.......	80	299.0	+ 18.0	0.99	+0.20	0.36
3197.......	75	353.8	− 5.2	2.49	+0.70	0.74
3258.......	20	344	+155	0.27	−0.10	0.73
3224.......	75	186.7	− 4.2	6.88	+0.30	0.64
3330.......	85	317	+ 26	1.75	+0.60	0.98
3338.......	75	176.3	− 8.6	1.55	−0.17	0.30
3358.......	12	148.2	+ 70	0.23	+0.45	0.53
3497.......	00	178.9	+ 3.2	1.11	0.00	0.05
3568.......	80	174.6	− 11.8	2.54	−0.10	0.56
3589.......	80	245	+ 44	2.90	+0.56	2.27
3623.......	75	49.3	+ 1.1	14.12	+0.27	0.31
3712.......	80	298.3	+ 6.0	2.75	+0.16	0.31
3744.......	75	71.4	− 7.0	4.70	+0.18	0.66
3764.......	75	183.2	− 2.6	1.72	+0.03	0.10
3824.......	70	225.3	− 0.2	14.66	−0.04	0.21

TABLE 46—*Continued*

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
3823	80	201°3	+1°9	9″54	0″00	0″23
3870	75	280.0	+18.0	1.27	−0.30	0.48
3955	70	275.4	−10.0	0.97	+0.32	0.37
4032	05	237.8	+38.0	0.64	+0.74	0.85
4204	85	349.4	+10.8	2.23	+0.40	0.56
4193	75	141.8	−0.4	11.41	+0.06	0.20
4222	00	358	+26	0.77	+0.54	0.64
4390	70	203.7	+8.3	1.62	−0.24	0.32
4421	75	62.0	+0.3	3.71	0.00	0.02
4499	75	288.8	−1.0	4.30	+0.05	0.13
4762	75	193.0	+12.7	1.38	+0.26	0.31
4730	75	243.0	−2.4	0.93	+0.06	0.08
4773	70	354.0	+2.0	7.94	−0.26	0.31
4841	00	288	−60	1.02	+0.44	1.17
4840	75	262.8	−0.4	5.48	+0.12	0.15
5088	75	121.8	+11.4	1.94	+0.08	0.37
5276	80	263	+16	1.50	−0.15	0.42
5436	70	258.0	+0.2	5.06	−0.09	0.11
5447	90	310	−80	0.56	+0.11	0.80
5570	80	146.0	−8.0	3.20	−0.95	1.07
5608	75	169.5	−5.1	3.81	+0.01	0.38
5645	00	265.6	+10.0	0.64	−0.10	0.14
5746	80	155.9	−8.0	3.40	+0.64	0.83
5816	80	356.0	−0.4	7.00	+0.35	0.37
5813	75	25.0	+2.4	1.98	−0.21	0.22
5996	75	135	+41	0.57	+0.26	0.48
6038	80	225.0	+8.5	1.50	+0.33	0.39
6117	75	125	+64	0.70	+0.10	0.78
6140	75	243.9	−1.2	4.41	+0.23	0.26
6155	75	16.0	+2.0	3.22	−0.30	0.31
6185	80	332.0	−4.8	0.62	+0.30	0.31
6180	75	109.1	−5.8	1.77	+0.03	0.20
6263	80	142	+17	1.30	−0.27	0.46
6321	75	233.7	+7.2	6.47	+0.65	1.00
6454	85	249	−30	1.30	−0.42	0.81
6499	75	98.0	+4.4	2.16	0.00	0.15
6538	80	73.8	−1.5	0.77	−0.05	0.06
6569	75	352.6	−3.4	3.54	−0.12	0.27
6623	80	52.5	−36	2.07	+0.90	1.61
6659	75	181.4	+2.8	6.53	+0.30	0.40
6727	70	180	−15	0.99	+0.45	0.53
6815	75	214.6	+4.1	4.85	+0.29	0.42
6988	65	307.4	−0.7	30.52	0.00	0.64
7016	65	62.4	+11.5	1.35	−0.20	0.32
7034	80	277.8	+0.4	3.53	+0.27	0.27

TABLE 46—*Continued*

ADS	E	θ	$100 \frac{d\theta}{dt}$	s	$100 \frac{ds}{dt}$	$100w$
7046.	05	349°.2	− 2°.6	0″.79	−0″.10	0″.11
7049.	75	99	− 19	1.68	+0.50	0.76
7050.	00	148	− 60	0.66	+1.14	1.34
7118.	75	73.2	+ 2.5	2.56	−0.57	0.58
7152.	00	261.2	+ 16.0	1.08	−0.10	0.31
7187.	75	200 0	− 0.9	7.39	+0.48	0.51
7198.	75	141.2	− 9.5	6.79	+0.70	1.37
7215.	80	60.8	+ 7.0	1.43	0.00	0.17
7251.	75	59	+ 25	19.3	−1.6	8.4
7270.	00	222	− 26	1.45	+0.24	0.71
7281.	75	19.4	+ 6.3	5.73	+0.24	0.64
7276.	00	129	+ 34	2.37	−0.42	1.46
7292.	75	237.5	− 7.3	2.85	+0.21	0.44
7303.	85	120	+ 15	5.60	0.00	1.41
7307.	75	149	+ 66	1.65	−0.31	1.92
7354.	70	311.6	+ 1.2	5.67	0.00	0.06
7441.	20	33.5	− 13.3	4.25	−10.0	10.02
7500.	70	140.8	− 2.2	3.58	+0.67	0.69
7503.	75	313.5	− 4.4	5.13	+0.16	0.46
7551.	80	316	− 29	2.02	+0.84	1.34
7566.	75	294.4	− 4.1	1 96	+0.10	0.20
7613.	85	267	− 9	0.97	+0.20	0.26
7632.	75	227.6	− 1.4	1.08	−0.13	0.13
7685.	80	110	− 32.5	0.92	−0.24	0.58
7695.	75	260.4	− 1.4	2.39	−0.07	0.10
7715.	70	330.7	+ 0.6	4.42	+0.12	0.12
7721.	80	72.5	− 55	1.31	+0.37	1.32
7724.	75	111.5	+ 16.8	3.25	+1.6	1.86
7738.	90	178	− 34	1.81	+0.24	1.11
7802.	80	120	− 27	1.84	−0.35	0.95
7827.	75	158.5	+ 3.8	4.91	+0.37	0.48
7833.	75	124.6	− 1.2	4.36	+0.09	0.14
7837.	75	158.4	− 3.5	2.45	−0.24	0.29
7860.	80	314.5	+ 13.5	3.10	−0.86	1.12
7878.	75	166.7	− 2.9	3.36	+0.12	0.22
7926.	80	195	− 25	0.44	+0.29	0.33
7929.	85	328	− 51	0.79	+0.19	0.74
7939.	75	197.0	+ 1.0	6.64	+0.35	0.36
7936.	75	359	+ 12	2.10	+0.41	0.60
7979.	80	105.8	+ 5.8	6.33	+0.22	0.66
8043.	80	285	+ 15	1.14	+0.14	0.33
8047.	05	226	+ 20	0.47	+0.50	0.53
8083.	85	264.5	− 0.8	35.84	−1.70	1.80
8100‡.	85	299.0	+ 18.0	7.55	−1.40	2.71
8102.	75	237.8	− 1.0	0.61	−0.14	0.14

‡ Pair AC.

TABLE 46—*Continued*

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
8131......	75	251°8	+ 2°8	9″50	+0″11	0″46
8140.....	75	330	− 14	5.03	+0.30	1.28
8148....	14	43	−100	2.03	−4.00	5.36
8196.......	75	323.0	+ 5.2	15.44	+0.22	1.32
8236....	80	169.6	− 3.9	5.64	+0.65	0.76
8355.....	80	125	+ 18	1.51	+0.48	0.68
8414....	80	93.2	− 0.2	7.64	+0.14	0.14
8440...	70	91.9	− 4.3	11.40	−1.40	1.64
8446..	80	338	− 33	1.16	−0.49	0.83
8450...	80	223.0	− 1.9	11.50	+1.96	2.00
8460....	75	15.5	− 6.6	1.48	−0.30	0.34
8477...	75	283.0	− 11.0	7.51	−0.57	1.56
8531....	70	336.4	0.0	19.82	+0.96	0.96
8535...	75	308	− 36	0.50	−0.12	0.34
8553.....	80	47.5	− 44	2.00	+0.15	1.55
8606.....	80	235	+ 17	2.52	−0.06	0.75
8625...	75	195.5	− 4.3	1.66	−0.12	0.17
8690.....	75	201.2	+ 1.0	15.97	+0.25	0.40
8710.....	80	285.9	− 6.0	3.48	+0.44	0.56
8749....	80	249.9	+ 0.5	2.39	+0.48	0.48
8751......	85	350.0	− 10.2	1.01	−0.66	0.68
8755......	05	312.5	− 1.6	2.02	+0.66	0.66
8796.....	75	340.0	− 6.0	3.34	−0.30	0.46
8864.....	75	195	− 14	0.99	+0.31	0.39
8883.......	75	75.8	− 0.7	27.05	−0.55	0.60
8890......	80	351.6	+ 1.0	1.23	−0.16	0.16
8919.....	75	144.5	− 11.0	1.55	0.00	0.29
8934......	75	132.8	− 2.0	4.32	+0.03	0.14
8949.....	00	77	+ 40	2.43	+1.24	2.11
8981.....	80	27.5	+ 6.0	2.50	−0.64	0.70
9004.......	80	70	− 11	2.02	+0.14	0.41
9025......	75	352	+ 16	9.3	−3.8	4.64
13^h79....	80	110.2	− 3.6	8.12	−0.55	0.72
9048......	75	325.8	− 1.4	6.28	+0.46	0.48
9053......	80	70	+ 31	2.72	+0.60	1.61
9069......	75	14.2	− 4.8	7.24	+0.13	0.45
9121.......	85	196.5	+ 1.0	0.56	+0.08	0.08
9165.......	00	62	− 32	0.69	−0.66	0.76
9168.......	80	251.5	+ 4.5	2.12	+0.39	0.43
9167.....	75	67	+ 42	2.33	−0.18	1.74
9265.......	75	14.0	+ 3.6	2.78	−0.19	0.27
9269.......	08	78	− 70	0.55	+0.50	0.84
9306.......	80	131.0	− 10.5	5.20	+0.90	1.28
9312.......	75	35.5	+ 3.4	2.50	+0.70	0.72
9329.......	80	95	− 30	0.61	−0.04	0.32

TABLE 46—*Continued*

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
9338	75	102°.0	+ 5°.7	5″.89	0″.00	0″.63
9387	00	247	+ 50	1.48	+1.00	1.64
9396	00	342.5	+ 30	1.66	+0.88	1.25
9453	00	316	+ 40	0.92	+0.24	0.70
9493	75	346.9	+ 0.7	9.84	+0.44	0.48
9497	00	304	− 34	1.65	+0.54	1.11
9507	75	210.7	+ 1.3	4.20	+0.48	0.50
9517	95	37.6	− 2.6	3.81	−1.24	1.25
9535	75	9.8	− 0.5	24.60	−0.84	0.84
9553	85	257.0	− 7.1	1.22	−0.76	0.77
9554	80	128.3	+ 9.0	0.89	+0.14	0.20
9586	00	322	− 36	1.04	−0.54	0.84
9580	75	171.4	− 3.4	13.27	+0.23	0.73
9584	75	39.0	− 4.0	10.55	+0.98	1.18
9630	80	228.0	− 9.2	1.50	−0.23	0.32
9639	80	313	− 38	1.64	+0.20	1.09
9728	75	188.0	+ 0.6	11.90	+0.17	0.28
9727	80	293.0	+ 3.7	4.55	+0.58	0.67
9696	75	82.0	− 2.5	30.58	+0.90	0.91
9737	80	302.3	+ 3.0	6.20	+0.26	0.47
9751	00	99.9	+ 4.6	2.40	−0.34	0.40
9758	05	362	+ 19	0.52	+0.04	0.18
9842	80	333	+ 16	5.77	+0.34	1.70
9850	80	262.5	− 7.5	2.72	−0.44	0.55
9880	90	137	+ 50	0.80	+0.38	0.80
9853	80	115.7	− 4.2	1.35	−0.14	0.14
9930	75	67.4	+ 4.0	2.52	+0.22	0.30
9951	00	362	+ 2	0.85	+0.60	0.60
9951§	00	48.0	+ 7.0	2.10	+0.24	0.37
9958	85	262.5	+ 2.5	5.43	+0.18	0.34
9966	80	135	+ 10	2.68	−0.26	0.56
9974	80	230	− 10	1.70	+0.26	0.38
10052	80	361	− 10.5	1.10	+0.13	0.22
10058	80	142.5	− 2.5	5.0	+0.9	0.91
10070	75	211	− 15	1.13	+0.20	0.35
10105	70	195.0	+ 0.8	16.35	+0.30	0.53
10111	85	152	− 20	0.88	+0.04	0.30
10129	75	114	− 7.5	3.65	−0.23	0.48
10140	00	220	−540	0.20	0.00	1.89
10171	80	290.3	− 2.0	5.78	+0.05	0.15
10169	75	305	+ 7	1.11	−0.13	0.20
10193	80	85.5	− 5.7	2.11	−0.04	0.19
10225	75	38.4	+ 3.2	5.48	−0.30	0.47
10260	75	67.5	+ 3.0	2.32	−0.05	0.15
10294	85	5.5	+ 7.0	0.57	+0.14	0.16

§ Pair CD.

TABLE 46—*Continued*

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
10312.......	80	153°	+34°	1″.29	−0″.08	0″.79
10345.......	80	164	−92	2.69	−1.04	4.43
10374.......	05	250	−120	0.48	+0.90	1.35
10425.......	10	323	+74	0.37	+0.44	0.66
10526.......	75	311.0	+6.0	3.82	+0.38	0.59
10597.......	75	264.3	−4.0	3.18	+0.02	0.18
10614.......	85	336	+17	0.94	+0.06	0.30
10722.......	80	324	−22	0.72	−0.06	0.28
10728.......	85	347	−22	2.25	−0.49	0.96
10769.......	85	304	+25	2.17	−0.64	1.17
10912.......	75	276	+8	1.02	−0.21	0.26
10993.......	80	260.2	−2.7	6.21	+0.18	0.29
11002.......	00	116.4	+9.0	1.44	−0.60	0.65
10988.......	75	264	+6	2.30	+1.13	1.17
11090.......	80	150	−13	2.15	+0.10	0.47
11110.......	85	85	−14	1.00	−0.28	0.37
11127.......	00	224	−56	0.90	+0.54	1.03
11123.......	80	234	−15	1.14	+0.07	0.30
11160.......	80	65	−0.5	1.22	+0.65	0.65
11240.......	06	149	−35	0.33	+3.50	3.52
11273.......	70	236.0	+3.0	5.05	+0.25	0.40
11318.......	80	198.4	−1.1	6.09	−0.10	0.12
11344.......	85	22.5	−10	0.59	+0.25	0.27
11372.......	80	190.8	−1.8	5.54	−0.17	0.21
11524.......	18	176	−110	0.24	+0.10	0.47
11529.......	80	55	+10	1.01	0.00	0.19
11534.......	80	181.4	+4.0	4.10	+0.30	0.45
11642.......	75	338.0	−2.5	4.23	+0.12	0.19
11635.......	75	138	−40	2.48	−0.40	1.76
11667.......	80	121.1	−0.7	13.10	−0.22	0.22
11685.......	70	267.0	−0.6	3.82	+0.21	0.21
11715.......	80	37.9	−1.4	4.20	+0.25	0.26
11722.......	80	204	+8	0.95	+0.48	0.50
11697.......	75	95.2	−5.0	1.50	+0.06	0.12
11750.......	80	183.0	−2.4	3.60	+0.20	0.23
11816.......	80	296.0	−5.2	2.00	0.00	0.16
11811.......	00	132	+30	1.28	+0.54	0.88
11916.......	75	259.7	0.0	16.84	0.00	0.16
11914.......	70	360.0	+2.1	1.89	−0.13	0.16
11956.......	75	67	−30	0.90	−0.34	0.58
12010.......	80	263.2	−0.6	12.30	+0.34	0.34
11997.......	70	64	+14	2.38	−0.56	0.84
12029.......	80	154.0	−0.8	9.78	−0.70	0.70
12037.......	70	291.7	−1.3	8.07	0.00	0.11
12101.......	80	260.0	+2.5	17.00	−0.64	1.13

TABLE 46—*Continued*

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
12129.......	80	123°6	+ 1°3	8"00	+0"90	0"94
12228.......	75	327	+ 17.5	1.44	+0.27	0.53
12239.......	85	154.8	+ 4.0	0.84	+0.10	0.12
12246.......	80	213	+ 17	1.11	+0.10	0.36
12289.......	80	10.0	− 3.5	3.25	−0.12	0.21
12287.......	00	126.2	+ 4.0	1.80	−0.42	0.44
12288.......	08	36.2	− 7.5	0.69	+0.50	0.51
12322.......	75	66.0	− 1.2	12.12	−0.05	0.15
12296.......	80	340	− 17	0.94	+0.77	0.81
12414.......	80	316.9	+ 8.0	2.21	−0.04	0.33
12449.......	75	234.1	− 1.5	1.90	−0.25	0.25
12540.......	80	55.2	− 0.9	34.48	+0.23	0.29
12561§......	80	52.5	+ 0.6	6.00	−0.15	0.20
12594.......	75	148.0	− 2.2	5.11	0.00	0.15
12580......	80	254.4	0.0	11.46	+0.28	0.32
12623.......	85	144	+ 30	0.60	+0.14	0.35
12638.......	85	231.9	+ 6.0	2.69	+0.13	0.34
12679.......	75	208	− 20	1.10	−0.20	0.42
12667.......	85	41	− 14	0.87	+0.20	0.28
12697.......	80	42.7	+ 0.6	6.68	−0.26	0.29
12687.......	75	287.8	+ 2.9	1.30	+0.07	0.11
12715.......	80	319	+ 7	3.65	+0.20	0.52
12608.......	80	21.5	− 2.6	11.35	+0.09	0.09
12789......	75	28.1	− 2.7	18.15	+0.20	0.58
12815.......	85	135.2	− 2.0	38.05	+1.40	1.61
12850.......	95	294.2	− 8.0	0.51	−0.44	0.44
12851.......	80	195.0	− 4.0	0.95	+0.23	0.24
12913.......	75	72.0	− 2.8	25.80	+0.18	1.01
12966.......	05	311	− 8	0.79	+0.06	0.12
13028.......	00	318	− 56	0.77	−0.18	0.77
13149.......	00	208.2	− 6.6	7.46	+1.78	1.94
13186.......	85	305	− 42.5	0.38	−0.06	0.29
13196.......	75	134.2	+ 5.0	1.19	0.00	0.12
13236.......	00	155.8	− 18.8	1.47	−0.28	0.55
13256.......	70	349.9	0.0	4.60	−0.40	0.40
13251.......	00	243	− 21	3.08	+1.00	1.49
13307.......	80	115.1	− 0.8	5.45	+0.35	0.35
13334.......	00	166.4	+ 14.0	1.62	0.00	0.41
13393.......	75	339.5	− 3.2	4.56	+0.21	0.30
13371.......	80	22.5	− 9.5	5.19	+0.56	0.92
13434.......	80	13.1	− 0.9	5.9	−1.0	1.00
13442.......	75	327.0	+ 1.1	11.50	+0.25	0.41
13473.......	80	278.4	− 2.1	4.51	+0.02	0.12
13449.......	80	265	− 35	0.32	0.00	0.19
13560.......	80	120	− 14	5.40	−0.10	1.26

TABLE 46—*Continued*

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
13649	05	211°	+ 38°	0".77	+0".16	0".54
13648	00	65.9	− 3.6	5.88	+0.14	0.35
13692	80	339.5	− 3.8	3.14	+0.53	0.56
13750	80	291	+ 27	0.90	−0.22	0.48
13767	80	332.0	− 5.0	2.57	−0.10	0.23
13830	00	196.0	− 2.0	1.35	+0.26	0.26
13920	00	345.2	+ 5.6	0.88	+0.22	0.24
13986	00	45	− 52	0.59	−0.20	0.57
14004	80	303.7	− 2 0	4.24	+0.35	0.37
14054	80	194.0	+ 2.2	9.58	0.00	0.44
14126	80	20	− 19	0.67	+0.18	0.28
14206	80	182.5	− 5.0	3.92	+0.19	0.37
14233	80	96	+ 23	1.30	−0.20	0.57
14280	00	278	− 30	1.62	+0.46	0.96
14259	80	61.5	+ 8.5	6.40	−0.40	1.09
14286	05	224.7	+ 19	1.24	+0.30	0.52
14293	00	187.9	+ 8.4	1.55	−0.20	0.31
14295	85	95.0	− 3.0	9.99	+0.26	0.50
14377	00	61.2	− 4.2	2.87	+0.28	0.34
14370	00	28.0	+ 12.0	0.70	+0.64	0.66
14397	85	32.5	− 8.0	0.67	+0.10	0.13
14420	80	35.8	− 12.8	1.70	−0.15	0.40
14504	75	31.7	− 5.0	1.93	−0.09	0.17
14556	80	222.5	− 4.8	2.70	+0.10	0.23
14558	75	288	+ 27	0.99	+0.16	0.50
14569	80	260.0	+ 4.7	4.60	+0.15	0.44
14590	00	109.8	+ 3.6	4.78	0.00	0.34
14600	00	251.6	− 6.0	1.06	0.00	0.10
14602‖	05	157	+ 44	0.90	−1.00	1.22
14702	95	274.2	− 5.4	2.28	+0.67	0.70
14804	75	171.1	− 1.4	3.10	−0.24	0.25
14847	00	237.5	+ 36	2.9	+2.5	3.11
14856	75	184.7	+ 1.0	2.55	+0.34	0.35
14894	85	208	+ 24	0.61	+0.10	0.28
14864	80	45.2	− 2.9	4.58	+0.09	0.20
14889	80	50	− 37	1.53	+0.95	1.36
14878	80	116.3	− 0.8	6.12	+0.46	0.46
14977	80	215.5	+ 3.0	3.35	+0.38	0.43
15026	00	128	− 18	1.12	0.00	0.35
15060	75	10.0	− 1.0	3.86	−0.07	0.08
15208	85	323.0	+ 5.0	2.76	−0.41	0.49
15215	80	236	− 25	0.69	+0.10	0.32
15313	80	110	+ 17.5	0.99	−0.10	0.33
15229	80	302.5	− 33	2.49	+0.64	1.48
15392	75	338.6	− 5.6	9.0	+0.30	0.89

‖ Pair BC.

TABLE 46—*Continued*

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
15401	00	175°0	−24°0	0″75	+0″40	0″51
15452	85	179.2	− 3.4	1.08	−0.18	0.19
15494	80	302.0	+ 3.9	1.16	−0.10	0.13
15491	80	265	−13	1.35	+0.51	0.59
15562	85	243.2	+ 2.2	3.98	+0.10	0.20
15599	00	355.0	+ 4.0	0.51	−0.12	0.13
15614	80	200.2	− 2.2	1.20	+0.15	0.16
15707	85	59	+21	0.62	−0.50	0.55
15769	80	102.5	−15	1.60	−0.05	0.42
15828	75	193.2	+ 0.5	15.45	+0.22	0.31
15870	80	355.9	− 5.0	3.85	+0.60	0.67
15881	75	95.2	+ 0.5	4.23	0.00	0.08
15902	00	7	−60	0.63	+0.14	0.68
15905	75	146.6	0.0	2.98	+0.58	0.58
15935	95	219.0	− 5.0	2.84	+0.88	0.91
15962	00	268	+52	1.12	−0.62	1.20
15966	80	342.4	− 8.9	5.37	+0.12	0.83
15971	80	329	−57	3.25	−0.90	3.36
16069	75	143.8	− 0.2	13.62	+0.04	0.04
16111	17	34.0	+30.0	0.28	+0.95	0.96
16185	80	155	−67	1.01	−0.40	1.25
16199	00	235.0	+16.0	0.43	−0.06	0.14
16228	80	279.5	− 4.2	2.88	+0.18	0.27
16270	75	255	+19	3.7	−1.25	1.76
16291	75	70	−12.5	3.5	+0.9	1.16
16298	80	4.6	− 0.8	2.71	+0.03	0.04
16345	00	227.5	+95.0	0.89	−1.54	2.14
16407	80	4.6	− 1.5	5.42	+0.20	0.22
16519	75	145.8	− 2.2	8.48	−0.04	0.31
16557	75	253.0	+ 2.0	15.18	+0.50	0.77
16561	00	125	−34	0.46	0.00	0.27
16642	80	29.2	+ 2.7	4.80	+0.46	0.52
16638	00	152	−110	0.31	−0.52	0.79
16672	75	346.2	+ 3.3	13.54	−0.34	0.88
16666	75	191	+28	2.61	+0.60	1.42
16730	80	229.8	+ 0.4	7.00	+0.30	0.31
16733	05	89.2	+ 4.0	2.34	+0.44	0.47
16775	80	32.0	− 6.4	2.15	−0.44	0.50
16818	75	310.6	− 2.8	4.89	+0.05	0.24
16877	85	321	+32.5	0.50	+0.19	0.34
16958	75	7.7	− 4.3	3.26	−0.20	0.32
16979	85	138.8	− 5.0	5.90	+1.08	1.20
17020	85	254	+57	0.55	+0.05	0.55
17022	80	196.2	− 0.8	1.68	+0.05	0.06
17079	80	282.0	0.0	18.85	+0.56	0.56

TABLE 46—*Continued*

ADS	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$
17092.......	75	263°.0	+ 3°.5	1".68	+0".06	0".12
17107.......	80	244	+ 26	3.0	+0.8	1.58
17126......	80	68.8	+ 4.8	1.10	−0.08	0.12
17140.......	75	325.4	+ 2.6	3.10	+0.31	0.34
1.......	80	70.0	+ 0.6	15.24	+0.20	0.26
9.......	00	202	− 58	1.30	+0.64	1.47
32.......	80	150	− 15	0.60	+0.13	0.20

number (*New General Catalogue of Double Stars*, Carnegie Institution of Washington Publication No. 417, [Washington, 1932]); the epoch E (95 = 1895; 30 = 1930, etc.); the position angle, θ, at the epoch E; the centennial motion in angle, $100(d\theta/dt)$ (not corrected for precession); the distance s at the epoch; the centennial motion in distance, $100(ds/dt)$; and the relative centennial motion, $100w$.[3]

The values of h_1 given in Table 53 are derived by equation (13) from these data, or from the preceding references when no change was required. Cases in which other data, given in Table 53, differ from those adopted in the earlier lists are discussed on pages 177 ff.

71. *Parallax Data.*—For the stars used in the discussion of the masses, additional data are required. The trigonometric parallaxes are taken, without exception, from Schlesinger's catalogue.[4] The cases in which these values have been combined with moving-cluster parallaxes (see § 7), or in which the latter alone were available, are listed in Table 47, which explains itself.

The spectroscopic parallaxes have been taken, by preference, from *Mount Wilson Contribution* No. 511.[5] The systematic corrections to these values derived by the present writers[6] have not been applied to the values listed in this chapter but were taken into account in

[3] When referring to the earlier lists, it should be borne in mind that the columns there headed "$d\theta/dt$," "ds/dt," and "w" give *centennial* motions and should have been headed "$100(d\theta/dt)$," "$100(ds/dt)$," and "$100w$."

[4] *General Catalogue of Stellar Parallaxes*, 1935.

[5] *Ap. J.*, 81, 187, 1935.

[6] *Ap. J.*, 87, 389, 1938; *Mt. W. Contr.*, No. 589.

TABLE 47

STARS FOR WHICH CLUSTER PARALLAXES, OR A MEAN OF TRIGONOMETRIC AND CLUSTER PARALLAXES, HAVE BEEN USED

(Unit, 0".001)

ADS	Sp.	Trig. Par.	Cluster Par.	Adopted
2999	F5		26 ±5.5	
3135	F6	46±6	26 ±5	34 ±4
3169	F8	34±7	23 ±5	27 ±4
3210	G5		24 ±3.9	
3264	A6		23 ±2.6	
3475	F7	36± 6	30 ±6	33 ±4
3483	F5	18± 8	32 ±6	27 ±5
4002	B1	5± 9	5.7±1.6	5.7±1.6
4115	B3	2± 9	15.6±4.4	15.1±4.2
4123	B2		5.7±1.6	
4131	B9		6.1±2.4	
4187	B2		5.1±1.4	
4193	O9		5.1±1.4	
4241	B0		5.1±1.4	
4263	B0	14± 9	5.7±1.6	6.5±1.5
4265	B3	− 7±13	11.5±2.1	11.4±2.1
6^h33	B3		9.6±2.1	
6^h58	B3		5.2±2.1	
5654	B1		5.6±2.1	
6255	B8		9.8±2.1	
8^h96	B9		10.5±4.9	
9^h47	B3		5.6±4.3	
10^h31	B5		7.0±2.5	
*	B5		13 ±3	
11^h55	B9		14.4±2.3	
11^h66	B9		14 ±3	
12^h41	B1	30±14	14.2±2.6	14.7±2.5
12^h68	B3		11.5±2.4	
13^h14	B8		17.5±3.3	
13^h63	B9		18 ±3	
13^h79	B5		15 ±2	
14^h116	B5		10 ±2	
15^h27	B8		14.8±2.4	
15^h35	B5		10 ±2	
15^h38	B3		6.8±1.4	
15^h55	B3		9.4±1.4	
15^h56	B3		8.6±2.3	
9823	B3		8.5±1.4	
15^h101	B3		10.8±1.7	
9913	B0	24±14	8.5±1.2	8.5±1.2
9951	B2	22± 7	8.8±1.4	8.9±1.4
10049	B4		5.9±1.1	

* π Cent: α $11^h16^m.4$, δ $-53°56'$ (1900).

the discussion (§§ 17, 30, etc.). In a few cases (Table 48) these spectroscopic parallaxes were altered to correspond to improved values of the visual magnitudes (see § 9). The reasons for the change

TABLE 48

Changes Made in Mount Wilson Spectroscopic Parallaxes

(Unit, 0".001)

ADS	Sp.	Mount Wilson	Adopted	Note	ADS	Sp.	Mount Wilson	Adopted	Note
237	*G4*	21	18	1	8450A	*K2*	60	44	1
246A	*M3*	263	312	2	8450B	......	44	32	1
246B	......	174	189	2	8477A	*G5*	24	18	1
497A	*G4*	20	16	1	8477B	......	32	24	1
497B	......	16	13	1	8506A	*G1*	7	5	1
582A	*A1*	6	5	1	8506B	......	9	7	1
616A	*F2*	7	12	3	8804	*F4*	33	47	8
1081A	*G3*	10	8	1	9346A	*K0*	36	31	1
2204	*G1*	20	17	1	9346B	......	19	16	1
2257A	*A4*	11	15	4	9617A	*F9*	44	51	12
2257B	......	10	14	4	9696A	*G3*	40	33	1
2409B	*F3*	10	7	1	9842A	*F7*	27	23	1
2499A	*A5*	5	4	1	9909AB	*F4*	29	44	12
3409B	*G6*	12	14	5	9910A	*G6*	30	24	1
3514A	*K2*	36	27	1	9910B	......	46	36	1
3514B	......	27	21	1	9979A	*F6*	24	28	13
3572B	*A1*	10	17	6	10094A	*F1*	10	9	1
3588A	*F2*	32	26	7	(7929)A*	*K5*	116	166	14
5816A	*K5*	4	3	1	10781A	*G1*	17	14	1
5816B	......	4	3	1	11468AB	*G5*	10	7	1
6650A	*F7*	32	40	8	12101A	*G1*	35	30	1
6650C	......	48	44	8	14054A	*F1*	14	12	1
7139A	*K6*	60	44	1	14073	*F3*	42	52	10
7187A	*F3*	21	14	9	14592	*G4*	8	9	7
7187B	......	21	14	9	14878A	*F6*	22	16	1
7873A	*K5*	3	2	1	14878B	......	29	19	1
7873B	......	3	2	1	15881A	*G6*	6	5	15
7902A	*K4*	7	6	1	15881B	......	4	3	15
8022A	*F4*	7	6	1	15966A	*G8*	26	21	1
8035	*G7*	35	41	10	15966B	......	30	24	1
8119A	*G0*	69	83	11	16173AB	*G3*	48	34	1
8119B	......	63	76	11	16665A	*K1*	27	23	1
8131A	*F6*	27	23	1	16836AB	*K4*	10	7	1
8131B	......	23	20	1	17149A	*G0*	25	35	7
8162A	*K0*	63	54	1	17149B	......	26	37	7

* Burnham's *General Catalogue* number: $\alpha 17^h 12^m.1$, $\delta -34°53'$ (1900).

NOTES TO TABLE 48

1. Magnitude adopted at Mount Wilson interpreted by us as referring to combined light.
2. Harvard photometric magnitude 7.73 used for m_b.
3. Misprint in Mount Wilson tabular m_b.
4. Not spectroscopic binary.
5. Harvard photometric magnitude for star B preferable.
6. Magnitudes in *Harvard Annals*, Volume 64, adopted.

[Notes to table continued on following page]

NOTES TO TABLE 48—*Continued*

7. Error in calculation of m_b.

8. Henry Draper magnitude represents light of brighter star.

9. Potsdam magnitudes reduced to Harvard scale, adopted; i.e., 7.11 and 7.59. Harvard magnitudes inconsistent with combined light in *Henry Draper Catalogue*.

10. Spectroscopic binary observations apply to visual pairs; no correction applied by us.

11. Star B spectroscopic binary of very small range; no correction applied.

12. Spectroscopic binary orbit very uncertain; no correction applied.

13. Spectroscopic binary correction to m_b too large (Joy).

14. Misprint in published spectroscopic parallax.

15. Error in *Henry Draper Catalogue;* 7.1 and 8.4 adopted magnitudes.

TABLE 49

SPECTROSCOPIC PARALLAXES

(Unit, 0.″001)

ADS	Sp.	*Mt. Wilson Contr.* Nos.		Other Sources*	Adopted	ADS	Sp.	*Mt. Wilson Contr.* Nos.		Other Sources*	Adopted
		244	262					244	262		
122...	*B9*	....	10	10	10	8347..	*A1*	8	.....	....	8
659...	*A2*	8	......	6	7	8706..	*A1*	34	.....	36	35
940...	*B8*	10	......	12	11	8801..	*A2*	20	.....	17	18
1630...	*B9*	12	......	32†	20	8824..	*A0*	8, 8	8	....	8
1683...	A0	8	......	....	8	9737..	*B6*	.....	9	8	8
2436...	*A0*	7	......	8	7	9757..	*A0*	26	.....	21	23
2832...	*B7*	....	8	....	8	9778..	*A1*	30	.....	32	31
3182...	*A3*	17	......	20	18	9823..	*B3*	.....	8	14	11
4002...	*B1*	....	7	8	7	9913..	*B0*	.....	7	7	7
4068...	*B9*	....	10	14	12	9951..	*B2*	.....	7	8	7
4123...	*B2*	....	3	4	3	10049..	*B4*	.....	6	....	6
4187...	*B2*	....	6	5	5	10087..	*A1*	29	.....	34	31
4263...	*B0*	....	11, 5	8	8	10230..	*A2*	12	.....	18	15
4950...	*A1*	10	......	13	11	10355..	*A0*	12	.....	....	12
5487...	*B6*	....	8	8	8	10526..	*B9*	14	.....	22	18
5961...	*A2*	38	......	37	37	10990..	*B9*	18	.....	29	23
6012...	*B8*	8	......	7	7	11504..	*B8*	.....	7	8	7
6175...	*A2*	66	......	72‡	69	12026..	*B9*	36	.....	41	38
7158...	*B9*	26	......	26	26	12287..	*B4*	.....	6	3	4
7292...	*B9*	21	......	41	29	12880..	*A1*	42	.....	38	40
7555...	*A0*	10	......	....	10	13148..	*A0*	16	.....	23	19
7979...	*B9*	18	......	20	19	13672..	*B2*	.....	3	3	3
8175...	*B9*	12	......	14	13	13728..	*A0*	9	.....	....	9
8220...	*B3*	....	5	4	4	14504..	*B8*	.....	8	7	7
8231...	*A1*	11	......	....	11	15032..	*B1*	.....	8	6	7

* Schlesinger's catalogue unless otherwise noted.

† Parallax of A: *Mt. Wilson Contr.*, No. 511; Pair BC used in study of masses.

‡ Parallax of C: *Mt. Wilson Contr.*, No. 511.

are described in the notes. A few spectroscopic parallaxes of late-type stars were added from Schlesinger's catalogue (§ 8).

For early-type stars spectroscopic parallaxes were taken from this catalogue and from *Mount Wilson Contributions* No. 244 and No. 262. The stars for which data from these sources were combined and the adopted mean spectroscopic parallaxes are listed in Table 49.

72. *Data Used in Discussion of Masses.*—The fundamental data which were used in the discussion of the masses of visual double stars are summarized in Table 50. Stars with known orbits, main-sequence stars without orbits, giants, subgiants, and supergiants are successively listed; in each part the stars are grouped according to the spectral class, as in the final means in Table 23. The limits of our subdivisions are marked by rules. The successive columns give the ADS number; the spectral class (Mount Wilson values being in italics); the visual magnitude m_b of the brighter component (including the light of both components of a spectroscopic binary); the modulus $m_b - M_0$ (Table 1); and then the "reduced" values (§ 10) h'_1, t', and s' of the hypothetical, trigonometric, and spectroscopic parallaxes. The last two columns contain the weight, p (§ 12), which was employed in combining the data with those for other stars. This is different for the comparison with trigonometric and with spectroscopic parallaxes.

TABLE 50

DATA FOR DETERMINATION OF STELLAR MASSES FOR PHYSICAL PAIRS

(Unit, 0".001)

ADS	Sp.	m_b	Mod.	h'_t	t'	s'	Tr. Wt. p	Sp. Wt. p
I. Binaries with Orbits								
1630[1]	B9	5.39	4.99	239	50	199	0.47	2.0
3093[2,3]	B9	9.91	....	....	....	....	....	....
746	A0	6.10	5.40	228	108	168	0.036	0.4
7555	A0	5.77	5.07	206	....	103	....	0.9
9757[1a]	A0	4.6	3.9	172	157	139	0.57	0.9
12973[1]	A0	5.31	4.91	355	96	153	0.50	0.9
10087[b]	A1	4.3	3.4	177	38	149	1.16	0.7
12880	A1	2.98	2.08	117	50	104	1.89	0.7
0^h27	A2	4.84	3.74	223	179	84	0.34	0.7
2616	A2	6.62	5.52	127	63	....	0.13	....
4617[b]	A2	4.61	3.51	202	136	146	1.10	0.7
5423	A2	−1.58	−2.68	164	109	106	9.95	2.0
6175[bd]	A2	2.29	1.19	208	126	120	7.44	2.0
8891	A2	2.92	1.82	180	92	74	1.72	1.25
9343	A2	4.43	3.33	111	37	162	0.23	1.25
11897	A2	6.80	5.70	193	−207	....	0.024	....
102	A3	6.83	5.53	217	−13	....	0.073	....
4929	A3	7.91	6.61	420	84	....	0.027	....
7545	A3	5.03	3.73	84	101	101	0.14	2.0
8974	A3	5.3	4.0	208	170	152	0.52	2.0
9301	A3	6.56	5.26	248	124	147	0.075	1.25
11950[1]	A4	3.07	1.57	153	45	99	3.57	2.0
10360	A5	6.13	4.43	200	123	146	0.26	0.9
1598	A6	4.71	2.81	151	95	84	1.96	2.00
2200[1]	A6	5.66	3.76	85	73	73	0.47	2.00
3264[1,4a]	A6	6.3	4.4	280	175	114	1.11	1.25
8539[1]	A6	6.62	4.72	145	....	97	....	1.25
8987	A6	6.29	4.39	211	91	83	0.52	2.00
9^h40	A7	3.80	1.80	195	160	126	1.52	2.00
11111[1b]	A7	5.91	3.91	139	79	79	0.12	1.25
8630[1]	F0	3.65	1.15	197	151	117	3.57	2.0
9769	F0	7.29	4.79	136	45	100	0.14	1.25
14499	F0	5.82	3.32	139	60	106	0.91	2.0
14787	F0	3.92	1.42	135	96	100	4.35	2.0
16497	F0	6.31	3.81	151	104	....	0.60	....
8189[a]	F1	8.1	5.4	216	445	120	0.028	2.0

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h_1'	t'	s'	Tr. Wt. p	Sp. Wt. p
1709	*F2*	6.42	3.62	175	249	127	0.23	2 0
3159	*F2*	6.63	3.83	158	....	123	....	1.25
3588	*F2*	6.15	3.35	178	108	121	0.31	1.25
3711[a]	*F2*	6.7	3.9	163	84	90	0.094	1.25
9247[2b]	*F2*	7.6	4.8	182	64	110	0.56	2.0
12972	*F2*	6.88	4 08	144	85	131	0.069	1.25
15281[1b]	*F2*	5.1	2.3	124	69	110	2.86	2.0
15988	*F2*	5.73	2.93	110	93	81	1 16	1.25
6251	*F3*	0.48	−2.47	116	93	106	9 80	2.0
11005[b]	*F3*	6.1	3.15	150	239	81	0.54	1.25
14073[1]	*F3*	4.01	1.06	87	44	85	4.54	2.0
14360	*F3*	6.28	3.33	114	121	106	0.91	2.0
14773[1]	*F3*	5.14	2.19	231	165	151	4.00	2.0
7203	*F4*	4.91	1.81	120	122	115	1.54	1.25
8148[1]	*F4*	4.12	1.02	96	....	50	....	0.5
8197	*F4*	6.08	2.98	178	....	138	....	2.0
8804	*F4*	5.22	2.12	202	152	125	1.78	2.0
9909	*F4*	4.77	1.67	123	78	89	5.15	2.0
3483[1,5]	*F5*	6.77	3.57	150	140	104	1.04	2.0
8419	*F5*	7.89	4.69	126	....	96	....	1.25
9094	*F5*	8.50	5.30	212	172	92	0.16	2.0
9378	*F5*	7.77	4.57	135	....	107	....	2.0
10235	*F5*	6.76	3.56	129	108	118	0.61	1.25
11077	*F5*	5.22	2.02	198	125	152	3.12	2.0
16800[a]	*F5*	8.4	5.2	165	− 88	110	0.25	2.0
588	*F6*	7.76	4.41	252	99	107	0.20	0.5
3135[1,5]	*F6*	7.05	3.70	146	187	121	1.39	2.0
5871	*F6*	7.19	3.84	208	123	123	0.17	1.25
9979[1a]	*F6*	6.5	3.15	274	205	120	0.80	2.0
221	*F7*	8.16	4.66	154	....	94	....	2.0
490[b]	*F7*	5.9	2.4	203	157	139	1.20	2.0
3475[1,5]	*F7*	7.71	4.21	202	229	111	0.91	2.0
6650	*F7*	5.56	2.06	155	101	108	4.35	2.0
8337	*F7*	7.07	3.57	109	197	98	0.34	2.0
18^h113	*F7*	5.01	1.51	172	....	102	....	2.0
15176	*F7*	7.50	4.00	202	215	126	0.16	2.0
3169[3b]	*F8*	7.6	3.95	173	167	95	0.83	1.25
6993	*F8*	3.53	−0.12	35	11	28	7.07	2.0
7390	*F8*	5.94	2.29	101	81	109	0.86	2.0
9578[1]	*F8*	7.08	3.43	165	....	102	....	0.5
11520	*F8*	7.13	3.48	164	149	109	0.80	2.0
671[1]	*F9*	3.68	−0.12	184	172	150	7.81	2.0
1538[1b]	*F9*	7.3	3.5	215	205	130	0.20	1.25
9617	*F9*	5.58	1.78	173	155	116	1.96	2.0
12447	*F9*	8.46	4.66	188	94	94	0.048	2.0

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h'	t'	s'	Tr. Wt. p	Sp. Wt. p
3082	*G0*	8.15	4.20	118	62	97	0 15	1.25
8119[b]	*G0*	4.7	0.75	235	196	112	5.20	2.0
9626[2]	*G0*	7.16	3.21	154	158	127	1.04	2.0
10157	*G0*	3.04	−0.91	83	72	91	8.78	2.0
11871	*G0*	5 25	1.30	151	106	138	4.88	2.0
9380	*G1*	7.59	3.49	125	....	120	....	2.0
9494[1]	*G1*	5.28	1.18	177	136	119	5.16	2.0
10660	*G1*	5 33	1.23	152	116	106	2.00	1.25
11060[a]	*G1*	7.9	3.8	236	....	80	. .....	2.0
17175	*G1*	5.86	1.76	207	194	128	3.03	2.0
5234	*G2*	6.99	2.79	127	105	98	0.83	1.25
6420[1]	*G2*	5 83	1.63	178	127	134	5.20	2.0
61	*G3*	6.44	2.04	164	36	100	1.57	2.0
16173	*G3*	6.56	2.16	108	86	92	2.33	2.0
14^h59	*G4*	0.33	−4.17	139	111	117	9.87	2.0
13461	*G4*	7.50	3.00	88	163	104	0.33	2.0
1^h34	G5	7.76	3.11	264	151	....	0.45	....
1^h37	G5	6.01	1.36	391	302	....	1.63	....
3210[1, 4]	*G5*	8.21	3.56	139	124	108	1.67	2.0
9413[1]	*G5*	4.80	0.15	183	158	123	6.58	2.0
10598	*G6*	6.00	1.20	143	87	104	6.15	2.0
520[1]	*G7*	6.37	1.42	148	138	122	3.70	2.0
17^h31	K0	5.66	0.16	137	142	....	2.20	....
12145[2]	*K0*	8.95	3.45	130	54	110	1.64	2.0
13125	K0	8.11	2.61	97	− 6.7	....	0.41	....
11046[1]	*K1*	4.28	−1.42	119	101	101	9.46	2.0
16665[1]	*K1*	8.96	3.26	182	58	103	0.73	2.0
17178	*K1*	9.17	3.47	114	....	109	....	1.25
6554	*K2*	8.65	2.75	108	146	85	1.52	2.0
10075	*K2*	7.79	1.89	146	141	96	2.86	2.0
7284[1]	*K4*	7.87	0.87	87	90	72	3.57	2.0
9716	*K4*	7.39	0.39	66	55	65	1.82	1.25
(7929)[6]	*K5*	6.05	−0.95	97	95	107	8.40	2.0
10585[7]	K5	9.67	2.67	123	188	....	0.75	....
9031	*K6*	7.87	0.87	130	90	99	3.17	2.0
9982[1]	*K6*	9.13	2.13	115	5.3	96	0.58	1.25
10188	*K6*	8.83	1.83	86	32	98	0.91	2.0
11^h22[8]	K8	7.89	0.89	161	101	....	1.10	....
$-58°7893$	Ma	9.5	0.0	60	51	....	0.64	....
$-\ 8°4352$	*M3*	9.86	.36	156	173	163	2.3	1.25
10786	*M3*	10.21	.71	146	151	164	5.5	2.0
15972	*M3*	9.64	0.14	202	278	170	8.3	2.0

ADS	Sp.	m_b	Mod.	h'_1	t'	s'	Tr. Wt. p	Sp. Wt. p
II. Physical Pairs—Main Sequence								
4193[4b]	O9	3.19	6.39	285	97	....	0.4	....
4241[4b]	O9	4.32	7.12	241	135	....	0.5	....
4263[5]	*B0*	2.05	4.85	122	61	75	0.53	0.5
9913[5b]	*B0*	3 2	6.00	206	135	111	0.58	.4
2726[a]	*B1*	4 56	6.86	306	424	141	0.037	.4
2843	*B1*	2.91	5.21	309	110	77	0.078	.25
3274[b]	B1	6.16	8.46	740	....	148	....	....
4002[5a]	*B1*	4.4	6.7	318	125	153	0.51	.5
5654[4]	B1	1.63	3.93	32	34	61	0.31	.4
12^h41[5bkl]	B1	1.88	4.18	207	101	103	0.72	.5
15032	*B1*	3.33	5.63	147	94	94	0.15	.25
3734	B2	6.78	8.58	210	....	156	....	....
4123[4]	*B2*	5.81	7.61	123	189	100	0.25	.25
4187[4b]	*B2*	4.98	6.78	465	116	114	0.4	.4
5107[2]	B2	5.21	7.01	427	578	151	0.025	.5
9951[5b]	*B2*	4.69	6.49	258	178	139	0.78	.5
13672[b]	*B2*	6.26	8.06	390	123	123	....	....
14749	B2	6.01	7.81	199	....	73	....	.4
17140	B2	5.08	6.88	334	....	95	....	.4
2772[a]	B3	6.76	8.16	428	....	128	....	....
4115[5]	B3	4.56	5.96	311	234	172	0.51	.5
4150	B3	6.68	8.08	219	....	166	....	....
4265[5b]	B3	5.83	7.23	230	320	140	0.71	.5
6^h33[4]	B3	6.02	7.42	336	292	61	0.48	.4
6^h58[4]	B3	6.01	7.41	297	158	61	0.59	.4
9^h47[4]	B3	5.84	7.24	729	157	280	0.12	.5
8220[d]	*B3*	6.02	7.42	210	....	122	....	.4
12^h68[4b]	B3	4.17	5.57	469	150	143	0.62	.5
15^h38[4a]	B3	4.55	5.95	365	105	171	0.28	.25
15^h55[4, 9]	B3	3.61	5.01	353	94	131	1.43	.7
15^h56[4]	B3	4.92	6.32	478	157	129	0.26	.4
9823[4]	*B3*	4.72	6.12	218	142	185	0.56	.4
15^h101[4]	B3	3.64	5.04	193	110	92	0.30	.25
10449[a]	B3	5.21	6.61	463	− 21	126	0.046	.4
14296	*B3*	4.83	5.53	217	38	128	0.070	.5
15417	B3	8.40	9.80	1280	....	92	....	....
17006[b]	B3	6.24	7.64	203	....	101	....	.4
10049[4]	*B4*	5.22	6.22	455	104	105	0.7	.5
12287	*B4*	5.43	6.43	270	....	77	....	.4
10^h31[4]	B5	4.70	5.40	180	84	84	0.3	.4
———[4, 10]	B5	4.87	5.57	391	169	91	0.5	.4
13^h79[1, 4]	B5	4.73	5.43	403	183	85	0.3	.25
14^h116[4]	B5	4.72	5.42	242	122	121	0.8	.5
15^h35[4]	B5	5.24	5.94	308	154	200	0.6	.5
13198	B5	6.62	7.32	269	....	145	....	.4
14099	B5	5.59	6.29	181	....	127	....	0.5
14585[b]	*B5*	6.82	6.62	122	− 21	84	....	....

TABLE 50—*Continued*

ADS	Sp	m_b	Mod.	h'_i	t'	s'	Tr. Wt. p	Sp. Wt. p
5487^b	*B6*	5 8	6.15	163	....	136	....	0.4
9737^a	*B6*	5 67	6.02	344	272	128	0.11	.5
2832	*B7*	5.50	5.60	370	....	105	....	0.4
434	B8	5.49	5.29	106	....	125	....	0.5
940	*B8*	4.50	4.30	87	22	80	0.16	.5
1^h8^a	B8	4.77	4.57	230	....	57	....	.25
2336^1	B8	5.37	5.17	119	....	86	....	.25
2440	*B8*	6.68	6.48	152	730	....	.019	....
3^h44	B8	4.81	4.61	435	176	....	.14	....
3823	*B8*	0.34	0 14	17	6.4	6.4	.93	.5
4615^b	B8	5.29	5.09	77	....	63	....	.25
5289	B8	6.03	5.83	60	....	103	....	.5
6012^a	*B8*	6.21	5.51	177	127	89	.086	.4
6255^4	B8	4.51	4.31	157	71	58	.5	.4
13^h14^4	B8	4.82	4.62	210	147	143	.5	.4
15^h27^4	B8	4.46	4.26	178	105	142	.8	.5
11504	*B8*	5.47	5.27	107	....	79	....	.4
14126	B8	6.54	6.34	136	....	112	....	.5
14504	*B8*	5.77	5.57	98	....	91	....	.5
122	*B9*	5.53	5.13	287	....	106	....	.4
197	B9	7.30	6.90	115	144	....	....	....
836	B9	7.42	7.02	220	−203	....	....	....
2185	B9	7.08	6.68	238	−217	....	....	....
4068	*B9*	5.86	5.46	297	....	148	....	.5
4131^{4bd}	B9	6.36	5.96	311	95	125	.3	.4
4768	*B9*	7.29	7.09	146	78	....	....	....
6569	B9	6.55	6.15	204	....	119	....	.5
7158	*B9*	4.34	3.94	141	49	160	.36	.5
8^h96^4	B9	5.64	5.24	157	117	168	.3	.5
7292^b	*B9*	4.30	3.90	97	199	175	.31	.5
7979	*B9*	4 51	4.11	180	....	127	....	.5
8175^b	*B9*	5.63	5.23	401	....	145	....	.5
11^h55^4	B9	4.97	4.57	165	118	107	.8	.5
11^h66^4	B9	5.47	5 07	227	145	176	.3	.25
13^h63^4	B9	5.66	5.26	82	202	124	.5	.4
10526	*B9*	4.50	4.10	139	− 20	119	.15	.5
10990^b	*B9*	4.75	4.35	134	163	171	.14	.4
12026^b	*B9*	3.32	2.92	160	138	146	.41	.4
12160	*B9*	6.73	6.33	59	− 18	....	.021	....
12654^b	B9	5.03	4.63	44	....	152	....	.25
13524	B9	4.43	4.03	96	0	83	.23	.5
14575	B9	6.10	5.70	121	....	97	....	.5
16877	B9	6.25	5.85	112	....	104	....	.5
220	A0	7.05	6.35	244	....	187	....	.5
1183	A0	7.41	6.71	264	395	....	....	....
1507	A0	4.75	4.05	227	....	123	....	.5
1588^b	A0	7.18	6.48	227	138	....	0.074	....
1683	A0	6.05	5.35	212	....	94	....	0.4

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h_t'	t'	s'	Tr. Wt. p	Sp. Wt. p
2436	Ao	6.84	6.14	202	....	118	....	0.5
3355	Ao	7.32	6.62	337	....	549	....	....
6263^{b}	Ao	6.72	6.02	208	160	160	0.036	.5
6862	Ao	5.87	5.17	195	−32	....	.074	....
10^{h}79	Ao	6.46	5.76	255	....	184	....	.4
8824^{b}	Ao	7.12	6.42	396	....	154	....	.4
9258^{a}	Ao	7.69	6.99	217	....	450	....	....
9338^{b}	Ao	5.24	4.54	210	81	105	.33	.5
15^{h}97	Ao	5.35	4.65	272	179	....	.16	....
10105	Ao	5.65	4.95	314	....	127	....	.7
10225	Ao	5.96	5.26	237	....	135	....	.5
10355^{b}	Ao	6.48	5.78	222	....	172	....	.5
10795^{b}	Ao	6.04	5.34	112	....	105	....	.5
11353^{a}	Ao	6.08	5.38	179	167	....	.13	....
11311^{a}	Ao	5.03	4.33	66	66	103	.23	.5
11640^{b}	Ao	6.77	6.07	221	....	164	....	.5
11870^{bd}	Ao	6.82	6.12	132	−117	217	.042	.5
13148	Ao	4.90	4.50	119	8	151	.20	.5
13728^{1}	Ao	6.21	5.51	228	....	114	....	.4
13920	Ao	6.08	5.38	86	−24	....	.043	....
14293	Ao	5.63	4.93	97	−9.7	116	.17	.4
14682^{b}	Ao	6.03	5.33	187	−117	....	.10	....
15670^{a}	Ao	8.15	7.45	241	....	431	....	....
22^{h}55^{b}	Ao	4.55	3.85	165	237	100	.16	.5
17020	Ao	6.84	6.14	186	152	....	.033	....
582	A1	7.93	7.03	143	....	127	....	....
824^{ad}	A1	6.5	5.6	158	....	132	....	.5
988	A1	6.93	6.23	502	....	142	....	.5
3572^{b}	A1	5.38	4.68	320	....	147	....	.7
4566	A1	2.73	1.83	130	49	93	.80	.7
4950	A1	6.26	5.36	114	....	130	....	.5
8231	A1	6.39	5.49	251	....	138	....	.5
8347^{1}	A1	6.56	5.66	163	....	109	....	.4
8706^{d}	A1	2.90	2.00	83	63	88	.82	.5
8954	A1	6.34	5.44	108	....	147	....	.3
9778	A1	3.74	2.84	103	126	115	.42	.4
11558	A1	7.52	6.62	167	....	231	....	....
659	A2	7.15	6.05	47	....	114	....	.4
899	A2	5.55	4.45	218	....	78	....	.4
1359	A2	6.40	5.30	276	−57	....	.10	....
1615^{bd}	A2	4.8	3.7	236	....	105	....	.7
1938	A2	7.04	5.94	94	....	138	....	.7
2080	A2	3.68	2.58	62	131	112	.82	.7
2^{h}50^{a}	A2	4.02	2.92	142	123	....	.35	....
5400	A2	5.28	4.18	199	....	130	....	.7
5961	A2	3.65	2.55	65	130	120	.46	.4
11^{h}31	A2	6.25	5.15	161	257	....	.052	....
8801^{b}	A2	4.75	3.65	59	75	97	.30	.4
8966	A2	5.98	4.88	166	152	114	.077	0.4
15^{h}106^{1}	A2	5.57	4.47	485	133	....	.13	....

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	μ'_l	t'	s'	Tr. Wt. p	Sp. Wt. p
10129	A2	5.56	4.46	141	....	94	....	0.5
10230	A2	5.64	4.54	81	81	121	0.16	.5
10374	A2	3.20	2.10	50	129	116	.71	.7
11336[b]	A2	5.24	4.14	175	208	135	.17	.5
11635	A2	5.4	4.3	196	109	130	.48	.7
19^h61	A2	5.75	4.65	119	77	....	.093	....
16467[b]	A2	5.46	4.36	119	388	142	.18	.5
862[a]	A3	7.2	5.9	166	75	121	.020	.5
1055[b]	A3	6.72	5.42	194	....	73	....	.4
1860[b]	A3	4.9	3.6	142	116	79	.26	.7
2755	A3	8.56	7.26	252	57	....	....	....
2799[I]	A3	5.99	4.69	243	....	191	....	.5
3182	A3	5.81	4.51	67	....	144	....	.5
6815	A3	6.31	5.01	186	....	141	....	.7
8939	A3	7.33	6.03	90	....	145	....	.7
8991	A3	5.73	4.43	209	....	54	....	.7
11089	A3	5.92	4.62	177	....	168	....	.4
11635[II]	A3	5.16	3.86	225	89	107	.58	.7
12104	A3	7.70	6.40	178	210	....	.026	....
12708	A3	7.53	6.23	194	....	159	....	.5
13371	A3	6.24	4.94	311	....	97	....	.5
14196	A3	7.29	5.99	174	....	142	....	.4
15600	A3	4.61	3.31	267	152	120	.50	.7
2257	A4	5.25	3.75	130	62	79	.14	.7
7114	A4	3.12	1.62	238	142	91	.56	.7
7286	A4	6.37	4.87	104	....	123	....	.7
7402	A4	3.76	2.26	164	105	119	.73	.7
9396	A4	5.77	4.27	193	29	50	.19	.7
10227	A4	4.88	3.38	171	19	57	.35	.7
996[bd]	A5	5.87	4.17	171	150	150	.16	.5
1392[I]	A5	8.84	7.14	220	375	....	....	....
2499	A5	8.55	6.85	936	....	94	....	....
3358[I]	A5	5.56	3.86	136	112	118	.18	.7
4390	A5	6.02	4.32	81	190	95	.13	.7
6004[bd]	A5	7.6	5.9	121	30	106	.046	.3
11^h2	A5	5.38	3.68	71	−11	82	.19	.5
8153	A5	4.14	2.44	92	86	101	.21	.25
9701[b]	A5	4.5	2.8	156	51	95	.71	.7
11853	A5	4.49	2.79	123	94	76	.80	.5
12594	A5	7.44	5.74	132	42	70	.083	.3
21^h15	A5	4.70	3.00	267	152	....	.38	....
15182	A5	9.17	7.47	344	−125	....	....	....
16979	A5	5.67	3.97	249	37	118	0.24	0.5
218[b]	A6	8.1	6.2	348	296	87	0.049	0.5
8311	A6	5.93	4.03	198	301	122	0.10	.7
11667	A6	5.94	4.04	109	....	109	....	0.5

ADS	Sp.	m_b	Mod.	h_1'	t'	s'	Tr. Wt. p	Sp. Wt. p
3824[12lxl]	*A7*	5.4	3.4	81	....	62	....	0.5
5559	*A7*	4.82	2.82	235	143	114	0.62	.7
6028	*A7*	7.11	5.11	116	....	105	....	.7
6871	*A7*	6.90	4.90	325	....	95	....	.7
8759	*A7*	7.26	5.26	107	....	101	....	.7
9173[lxl]	*A7*	4.9	2.9	130	38	76	.52	.7
9747	*A7*	8.14	6.14	155	....	84	....	.5
11977[a]	*A7*	9.1	7.1	1210	....	105	....	....
12061	*A7*	5.07	3.07	169	29	103	.40	.7
13692[d]	*A7*	6.0	4.0	120	0	95	.30	.5
14233	*A7*	7.16	5.16	162	....	119	....	.7
5983[b]	*A8*	3.8	1.6	121	126	113	.88	.7
8406	*A8*	6.01	3.81	54	....	81	....	.7
9959	*A8*	8.56	6.36	487	....	150	....	.5
15407	*A8*	7.07	4.87	151	....	75	....	.7
16599	*A8*	7.61	5.41	375	....	109	....	.5
16644	*A8*	8.96	6.76	405	675	360	....	....
191	*A9*	6.09	3.79	149	....	69	....	.7
6117	*A9*	8.75	6.45	292	19	97	.041	.7
6829	*A9*	9.35	7.05	334	....	77	....	....
7632	*A9*	8.32	6.02	83	....	112	....	.3
10448	*A9*	6.78	4.48	284	....	63	....	.7
15392	*A9*	7.98	5.68	508	....	82	....	.7
683	*Fo*	6.29	3.79	52	34	63	.40	.7
875	*Fo*	6.18	3.68	185	....	158	....	.5
3064	*Fo*	5.96	3.46	128	108	103	.66	.7
3730	*Fo*	6.13	3.63	80	....	106	....	.5
4229	*Fo*	6.06	3.56	43	....	139	....	.7
4256	*Fo*	6.97	4.47	243	....	78	....	.7
4396	Fo	8.85	6.35	195	446	....	.016	....
6828	*Fo*	6.99	4.49	55	....	103	....	.7
7050[b]	*Fo*	6.0	3.5	105	....	95	....	.7
7115	*Fo*	4.27	1.77	99	....	91	....	.7
8690	*Fo*	7.68	5.18	294	....	54	....	.5
9254	*Fo*	6.70	4.20	131	....	104	....	.7
14[h]63	Fo	3.42	0.92	156	83	110	.80	.7
10279	Fo	6.97	4.47	137	204	94	.11	.7
13277	*Fo*	5.89	3.39	45	....	81	....	.7
15828	Fo	6.25	3.75	124	....	124	....	.5
11131[a]	*F1*	7.5	4.8	237	....	73	....	.3
1394	*F1*	5.42	2.72	193	116	88	.33	.5
2963	*F1*	5.59	2.89	84	15	137	.40	.7
6811	*F1*	7.19	4.49	238	....	127	....	.7
9375	*F1*	5.20	2.5	117	....	70	....	.3
9517	*F1*	7.82	5.12	370	....	105	....	.5
10094	*F1*	7.94	5.24	280	....	101	....	.5
10723	*F1*	5.58	2.88	170	121	121	.54	.7
11698[a]	*F1*	7.8	5.1	178	199	84	.16	.7
13442	*F1*	6.47	3.77	137	193	114	.50	.7
13887	*F1*	5.06	2.36	86	136	104	.71	.7
14054	*F1*	7.99	5.29	275	....	137	....	.5
14702	F1	4.76	2.06	52	57	26	.55	.5
16345	*F1*	5.78	3.08	128	66	103	0.36	0.7

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h_1'	t'	s'	Tr. Wt. p	Sp. Wt. p
616[1]	*F2*	7.88	5.08	156	....	120	....	0.5
1339	*F2*	6.06	3.26	157	99	99	0.62	.7
2111	F2	7.99	5.19	229	22	....	.14	....
2465	*F2*	5.91	3.11	193	....	109	....	.5
6^{h}36	F2	6.02	3.22	167	101	....	.25	....
7704	*F2*	7.22	4.42	168	....	99	....	.7
8519	F2	7.00	4.20	104	....	118	....	.3
9966	F2	6.41	3.61	95	....	132	....	.7
11262	*F2*	6.61	3.81	214	....	81	....	.7
12962	*F2*	6.12	3.32	45	....	148	....	.7
13196	*F2*	7.58	4.78	44	....	127	....	.7
14355	*F2*	6.80	4.00	95	....	82	....	.5
14412	F2	7.43	4.63	346	84	....	.061	....
15971	*F2*	4.41	1.61	135	34	93	.92	.7
16057	F2	6.55	3.75	68	17	79	0.13	0.7
2409[13]	*F3*	8.95	6.00	522	....	111	....	0.5
3353	*F3*	7 24	4.29	217	....	115	....	.7
3589	*F3*	6.04	3.09	200	37	108	0.56	.7
7^{h}83	*F3*	5.05	2.10	87	184	121	.26	.3
7187	*F3*	6.83	3.88	144	18	84	.38	.5
7303	*F3*	6.15	3.20	188	....	140	....	.7
7307	*F3*	6.52	3.57	181	....	103	....	.7
7712	*F3*	5.86	2.91	145	230	168	.51	.5
7846	*F3*	6.54	3.59	147	....	110	....	.7
9306	*F3*	7.85	4.90	381	....	124	....	.5
11123	*F3*	6.50	3.55	46	....	154	....	.7
12808	*F3*	5.61	2.66	6.1	6.8	99	.52	.7
12913	*F3*	5.10	2.15	156	121	129	.86	.7
14556	*F3*	7.39	4.44	77	....	101	....	.5
15791	*F3*	9.11	6.16	340	....	85	....	.3
1631	*F4*	5.82	2.72	119	84	91	.44	.7
2612	*F4*	6.84	3.74	174	....	101	....	.7
3243	*F4*	6.42	3.32	111	....	102	....	.5
3866[a]	*F4*	7.5	4.4	242	....	121	....	.5
5368	*F4*	8.27	5.17	184	....	98	....	.5
5436[d]	*F4*	6.27	3.17	34	134	112	.26	.3
6126	*F4*	6.16	3.06	184	119	98	.48	.7
6190	*F4*	5.86	2.76	157	....	86	....	.7
8022	*F4*	9.32	6.22	821	....	105	....	.3
8505	*F4*	6.66	3.56	129	....	93	....	.5
8814[b]	*F4*	7.5	4.4	243	....	106	....	.7
9192	*F4*	6.57	3.47	203	....	114	....	.7
9329	*F4*	7.40	4.30	55	....	80	....	.7
9357	*F4*	6.31	3.21	149	79	97	.54	.7
10229[1]	*F4*	6.98	3.88	113	− 6	113	.26	.7
10425	*F4*	8.51	5.41	127	....	121	....	.7
12029	*F4*	7.05	3.95	204	....	99	....	.7
13256[b]	*F4*	7.8	4.7	157	....	113	....	.5
14916	*F4*	7.77	4.67	163	....	112	....	.7
15076	*F4*	7.57	4.47	266	117	110	.14	.7
15896	*F4*	6.19	3.09	141	145	125	0.64	.7
16291	*F4*	7.09	3.99	204	....	88	....	0.7

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h'_1	t'	s'	Tr. Wt. p	Sp. Wt. p
806	F5	7.63	4 43	131	231	....	0.19	....
1780	F5	9.25	6 05	292	244	....	.020	....
1904	F5	5.88	2 68	110	....	103	....	0.7
2081	F5	4 22	1 02	198	125	119	.11	.7
2402	F5	4.02	0 82	112	105	127	.94	.7
2999[4]	F5	6.02	2 82	48	95	88	.29	.3
3188	F5	5.89	2.69	188	....	83	....	.7
3224	F5	7 91	4.71	241	− 26	123	.049	.7
3880	F5	9.26	6.06	277	489	....	.017	....
6^h3	F5	5.90	2.70	203	111	....	.45	....
5228	F5	9.25	6.05	84	226	....	.024	....
6381	F5	5.63	2.43	67	....	121	....	.7
6623	F5	7.13	3.93	208	....	73	....	.7
6977	F5	6.53	3.33	46	....	70	....	.5
7896	F5	6 87	3.67	81	....	103	....	.5
8344	F5	8.00	4.80	192	164	....	.062	....
11^h72[1]	F5	7.25	4.05	310	238	....	.13	....
8575	F5	8.51	5.31	184	....	81	....	.7
13^h102	F5	6.44	3.24	227	....	116	....	.7
9389	F5	6.27	3.07	70	....	120	....	.7
9580	F5	6.97	3.77	210	....	113	....	.7
12201	F5	7.93	4.73	150	....	124	....	.7
12803	F5	7.83	4.63	110	8.4	....	.23	....
13403	F5	6.53	3.33	111	....	116	....	.5
13560	F5	7.17	3 97	248	74	124	.29	.7
13611	F5	8.85	5.65	88	14	....	.025	....
15571[o]	F5	7.1	3.9	283	42	133	.58	.7
15935	F5	5.85	2.65	88	136	119	.27	.3
16111	F5	8.21	5.01	121	362	....	.083	....
23^h3	F5	4.39	1.19	75	− 1.7	....	.68	....
17054	F5	7.80	4.60	250	....	100	....	.7
2046	F6	5.79	2.44	135	71	123	.78	.7
5166[d]	F6	7.2	3.85	71	....	47	....	.5
6483[1]	F6	7.10	3.75	99	....	118	....	.7
6886[b]	F6	6.3	2.95	66	160	148	.29	.3
7871	F6	8.06	4.71	149	....	87	....	.7
8131	F6	6.95	3.60	129	....	110	....	.5
8202	F6	5.77	2.42	73	....	104	....	.3
8627[bd]	F6	6.3	2.95	109	....	55	....	.7
9025	F6	4.51	1.16	195	104	108	.85	.7
9237	F6	7.55	4.20	215	....	118	....	.7
9406[o]	F6	6.1	2.75	71	....	75	....	.7
9584	F6	5.19	1.84	112	89	98	.80	.7
9728	F6	6.52	3.17	82	....	112	....	.5
9727	F6	8.77	5.42	304	....	109	....	.5
10345	F6	5.81	2.46	227	143	100	.68	.7
14878	F6	7.6	4.25	149	7.1	121	.24	.7
15270	F6	4.71	1.36	163	86	81	.78	.7
41	F6	7.96	4.61	151	67	117	0.060	0.5

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h_i'	t'	s'	Tr. Wt. p	Sp. Wt. p
2122	*F7*	7.34	3.84	264	211	111	0.48	0.7
2316	*F7*	7.64	4.14	213	....	108	....	.5
4200	*F7*	7.10	3.60	158	89	89	.54	.7
8128	*F7*	6.96	3.46	128	....	148	....	.7
8196	*F7*	6.31	2.81	212	102	154	.71	.7
8531	*F7*	6.54	3.04	207	85	118	.66	.7
9053	*F7*	6.51	3.01	148	84	92	.38	.7
9115	*F7*	7.78	4.28	287	....	101	....	.7
9182	*F7*	7.74	4.24	282	628	127	.074	.7
9842	*F7*	7.06	3.56	257	....	119	....	.7
$0^{h}29$	*F8*	6.73	3.08	39	203	95	.16	.3
918	F8	8.05	4.40	160	190	....	.057	....
$1^{h}14$	F8	5.10	1.45	131	94	....	.70	....
2406	*F8*	5.14	1.49	124	112	110	.10	.5
2416	*F8*	8.85	5.20	242	275	99	.037	.7
3389	F8	8.75	5.10	178	42	....	.035	....
4371	F8	7.96	4.31	197	161	....	.080	....
7655	*F8*	7.28	3.63	91	117	117	.17	.5
8434	F8	7.97	4.32	270	....	110	....	.7
8561	*F8*	7.61	3.96	168	....	118	....	.5
8573	*F8*	6.44	2.79	166	163	109	.42	.7
8757	*F8*	6.38	2.73	25	....	123	....	.5
9273	*F8*	4.99	1.34	59	80	96	.87	.7
1703	*F9*	5.91	2.11	130	127	93	.82	.7
7198	*F9*	8.45	4.65	384	....	85	....	.5
8065	*F9*	7.63	3.83	193	....	117	....	.7
9425	*F9*	6.89	3.09	133	....	83	....	.7
14738	*F9*	8.16	4.36	97	....	105	....	.7
$0^{h}2$	G0	5.77	1.82	28	104	....	.55	....
475	G0	7.38	3.43	43	....	88	....	.7
716	*G0*	7.60	3.65	151	....	124	....	.7
1158	*G0*	7.94	3.99	46	....	107	....	.7
2373[1]	G0	9.35	5.40	253	72	....	.035	....
$3^{h}60$	G0	7.24	3.29	159	100	....	.21	....
$5^{h}67$[1]	G0	7.89	3.94	258	221	....	.16	....
8606	*G0*	9.05	5.10	231	....	105	....	.7
$14^{h}28^{b}$	G0	5.36	1.41	65	38	....	.30	....
$14^{h}92$	G0	8.06	4.11	279	20	....	.12	....
9418	*G0*	8.48	4.53	153	....	105	....	.7
9423	*G0*	8.21	4.26	178	43	114	.11	.7
$15^{h}37$	G0	7.71	3.76	238	248	....	.18	....
11257	G0	8.09	4.14	169	− 6.7	....	.17	....
11483	G0	6.74	2.79	127	69	....	.29	....
$22^{h}57$	G0	7.55	3.60	79	131	....	.27	....
16557	*G0*	6.63	2.68	138	....	151	....	.5
16649	*G0*	8.56	4.61	251	243	....	.073	....
16713	*G0*	6.65	2.70	104	205	125	.22	.7
17149	*G0*	6.58	2.63	195	....	121	....	0.7
9	G0	7.80	3.85	159	112	....	0.32	....

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h'_1	t'	s'	Tr. Wt. p	Sp. Wt. p
2204	*G1*	8.05	3.95	179	....	105	....	0.7
7082	*G1*	7.01	2.91	111	57	122	0.27	.7
8250[bd]	*G1*	6.6	2.5	206	171	111	.68	.7
8506	*G1*	9.36	5.26	209	....	68	....	.5
10781	*G1*	8.46	4.36	246	....	104	....	.5
12101	*G1*	6.83	2.73	191	....	106	....	.7
16417[1]	*G1*	7.03	2.93	69	104	100	.60	.7
7477	*G2*	7.25	3.05	118	....	102	....	.7
7744	*G2*	7.41	3.21	215	....	132	....	.7
8440	*G2*	6.88	2.68	209	103	103	.35	.7
8959	*G2*	8.93	4.73	159	26	106	.088	.5
8981	*G2*	7.92	3.72	117	....	94	....	.7
10775	*G2*	9.49	5.29	376	....	114	....	.5
12040	*G2*	8.51	4.31	123	....	87	....	.7
12557	*G2*	9.27	5.07	351	....	113	....	.5
14736	*G2*	7.81	3.41	216	....	72	....	.7
15934	*G2*	6.36	2.16	268	159	114	.68	.7
566	*G3*	7.09	2.69	207	....	142	....	.7
2173	*G3*	9.44	5.04	244	81	132	.13	.7
9696	*G3*	6.93	2.53	183	103	106	.68	.7
10904	*G3*	8.21	3.81	191	....	92	....	.7
12169	*G3*	6.63	2.23	204	115	112	.92	.7
12815	*G3*	6.26	1.86	212	83	106	.85	.7
16270	*G3*	7.28	2.88	166	105	113	.68	.7
17107	*G3*	9.03	4.63	321	219	110	0.31	0.7
237	*G4*	8.76	4.26	256	....	128	....	0.5
497	*G4*	9.02	4.52	248	209	112	0.20	.7
8094	*G4*	7.69	3.19	65	....	83	....	.7
9291	*G4*	8.25	3.75	79	220	124	.18	.3
9564	*G4*	8.19	3.69	203	....	104	....	.7
11275	*G4*	7.14	2.64	115	....	128	....	.5
14847	*G4*	6.60	2.10	155	131	131	.71	.7
16672[b]	*G4*	5.6	1.1	71	63	76	.88	.7
974	G5	8.44	3.79	80	34	....	.14	....
2630	G5	8.97	4.32	197	117	....	.092	....
2959	*G5*	7.37	2.72	136	60	91	.25	.7
3596	G5	8.19	3.54	112	117	....	.15	....
5752[1]	G5	9.26	4.61	293	400	....	.073	....
7778	*G5*	8.7	4.05	194	....	90	....	.7
8477[b]	*G5*	8.33	3.68	278	....	114	....	.7
9535	*G5*	6.83	2.18	137	52	87	.68	.7
9763[1]	*G5*	5.80	1.15	160	78	88	.53	.5
11568	G5	8.63	3.98	144	50	....	.35	....
20h22	G5	8.66	4.01	203	234	....	.18	....
15610	G5	9.06	4.41	99	0	....	.085	....
17131	G5	8.46	3.81	116	174	....	.33	....
6914	*G6*	5.44	0.64	79	83	102	.54	.7
9910	*G6*	7.39	2.59	165	....	96	....	.7
11971[2]	*G6*	9.4	4.6	234	75	100	0.086	0.7

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h'_1	i'	s'	Tr. Wt. p	Sp. Wt. p
2757	*G7*	8.2	3.25	206	188	121	0.71	0.7
3417	*G7*	8 91	3.96	279	130	130	.56	.7
9387	*G7*	7 39	2.44	96	....	89	....	.7
12469	*G7*	7.97	3.02	141	....	101	....	.7
14528	*G7*	8 24	3.29	214	....	77	....	.5
8917	*G8*	8.60	3.5	161	....	105	....	.5
13110	*G8*	3.90	−1.2	41	44	60	.66	.5
15966	*G8*	8.83	3.73	167	....	123	....	.7
16145	*G8*	8.85	3.75	281	281	73	.23	.7
2644[a]	*G9*	6.6	1.3	136	47	75	.90	.7
13434	*G9*	7.94	2.64	118	61	142	.47	.7
14270	*G9*	7.50	2.20	118	44	88	.76	.7
2756	K0	7.93	2.43	172	107	....	.37	....
7^h25	K0	7 96	2.46	68	227	....	.40	....
8140	*K0*	8.16	2.66	133	....	92	....	.7
8162	*K0*	6.55	1.05	71	88	86	.92	.7
9346	*K0*	7.42	1.92	82	....	53	....	.7
9969	*K0*	7.54	2.04	115	133	105	.88	.7
10871[1]	K0	8.72	3.22	101	75	....	.19	....
7654[2]	*K1*	7.65	1.95	106	101	93	.30	.3
8949	*K1*	7.70	2.00	108	....	116	....	.7
14638	*K1*	7.27	1.57	76	110	114	.40	.5
2218	*K2*	7.59	1.69	128	129	120	.66	.7
2995	*K2*	7.31	1.41	123	56	84	.77	.7
3514	*K2*	8.60	2.70	76	104	83	0.30	.3
8450	*K2*	8.16	2.26	198	....	108	....	0.7
8841	*K3*	6.61	0.46	122	84	98	0.91	0.7
10417	*K3*	5.31	−0.84	51	123	118	0.97	.7
3900	*K5*	7.9	1.2	155	112	104	0.60	.5
7721	*K5*	9.26	2.56	85	....	98	....	.7
8100[12]	*K5*	7.80	1.1	123	132	86	0.88	.7
8553	*K5*	9.13	2.43	101	....	95	....	.7
9167	*K5*	8.81	2.11	98	....	97	....	.7
9446	*K5*	6.00	−0.70	166	124	126	0.99	.7
10329	*K5*	8.5	1.8	138	94	126	0.71	.7
12664	*K5*	8.72	2.02	127	124	107	0.82	.7
12889	*K5*	8.3	1.6	123	100	92	0.88	.7
17062[b]	*K5*	6.9	0.2	75	101	91	0.66	.5
7139	*K6*	9.30	2.0	121	....	111	....	.7
8486	*K6*	10.24	2.94	166	....	116	....	.7
14636	*K6*	5.58	−1.72	166	136	125	1.00	.7
16242[2]	*K6*	9.84	2.54	122	....	93	....	.5
16633[2]	*K6*	9.8	2.5	70	164	66	0.50	0.7
5^h97[1,8]	K8	8.51	1.21	131	93	....	0.68	....

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h'_1	t'	s'	Tr. Wt. p	Sp. Wt. p
48	*Mo*	9.28	....	113	89	....	1.00	....
7251	*Mo*	7.90	....	215	162	....	1.00	....
8887	*Mo*	9.52	....	23	67	....	0.67	....
7067	*M1*	9.30	....	90	96	....	1.00	....
246	*M3*	7.80	....	230	284	....	1.00	....
433[14]	*M3*	10.37	....	139	103	....	0.67	....
11632	*M4*	8.8	....	202	282	....	1.00	....

III. Giant Stars—Physical Pairs

ADS	Sp.	m_b	Mod.	h'_1	t'	s'	Tr. Wt. p	Sp. Wt. p
5514	*Go*	5.75	5.05	143	− 20	82	0.050	0.7
3841[9,15]	*G1*	0.87	0.17	133	77	82	6.25	2.0
4066	*G1*	2.96	2.26	156	26	91	0.53	0.5
16538[b]	*G1*	5.0	4.30	225	14	94	0.29	0.7
3954	*G2*	5.38	4.68	194	− 17	121	0.10	0.7
1081	*G3*	6.37	5.67	245	....	109	....	0.7
1199	*G3*	3.72	3.02	33	40	88	0.40	0.5
10993[16]	*G3*	4.91	4.21	111	28	90	0.39	0.7
13007	*G3*	4.03	3.33	120	4.6	111	0.5	0.7
16317	*G3*	6.09	5.39	227	....	120	....	0.7
1697[ac]	*G4*	6.2	5.50	226	25	151	0.090	0.7
2850	*G4*	4.84	4.14	101	17	98	0.15	0.7
12850[d]	*G4*	7.2	6.50	180	−240	100	0.020	0.5
14592	*G4*	5.89	5.19	75	....	98	....	0.3
1971	*G5*	5.05	4.35	82	− 30	96	0.14	0.5
3072	*G5*	5.08	4.38	66	....	75	....	0.3
10[h]74	*G5*	2.86	2.16	124	....	89	....	0.7
11468	*G5*	6.17	5.47	161	37	87	0.13	0.7
15753	*G5*	5.61	4.91	202	77	125	0.11	0.5
16407	*G5*	7.99	7.29	371	−200	....	....	....
1	*G5*	5.98	5.28	216	....	91	....	0.5
3409[b]	*G6*	7.0	6.35	318	....	167	....	0.5
4432	*G6*	5.62	4.97	187	....	109	....	0.3
5586	*G6*	4.88	4.23	120	77	120	0.31	0.7
6988	*G6*	4.16	3.51	226	116	86	0.43	0.5
8695	*G6*	5.18	4.53	266	....	105	....	0.7
10058	*G6*	2.89	2.24	87	112	76	0.73	0.7
15881[d]	*G6*	7.10	6.45	113	....	78	....	0.3
17137	*G6*	5.08	4.43	115	154	92	0.24	0.5

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h'_1	t'	s'	Tr. Wt. p	Sp. Wt. p
558	G7	5.63	4.98	148	59	89	0.090	0.5
6321	G7	3.70	3.05	147	94	98	.58	.7
7071	G7	6.13	5.48	137	....	94	....	.7
8035[1]	G7	1.95	1.30	100	40	75	.82	.7
8576	G7	7.54	6.89	620	....	119	....	....
10052	G7	5.95	5.30	85	....	92	....	.7
14158	G7	5.83	5.18	95	....	87	....	.5
14259	G7	4.35	3.70	209	66	104	.52	.7
16365	G7	6.03	5.38	166	....	119	....	.5
16666	G7	5.00	4.35	252	192	104	.20	.7
16957[b]	G7	5.4	4.75	196	160	89	.20	.7
11479	G8	6.37	5.77	184	....	99	....	.7
11979	G8	6.51	5.91	320	....	76	....	.3
15967	G8	7.73	7.13	480	....	80	....	....
8600[a]	G9	5.18	4.63	287	....	93	....	.5
10277	G9	8.13	7.58	130	....	130	....	....
1^h1	Ko	4.08	3.58	172	94	120	.20	.5
1457	Ko	6.20	5.70	180	235	110	.097	.5
2459[17]	Ko	5.72	5.22	255	166	122	.21	.7
5^h3[b]	Ko	4.65	4.15	102	54	108	.11	.3
6^h45	Ko	5.00	4.50	175	63	....	.090	....
7^h16[d]	Ko	3.87	3.37	194	....	127	....	.5
6185	Ko	5.83	5.33	87	....	93	....	.7
12^h19[b]	Ko	5.90	5.40	144	36	192	.056	.3
13^h41	Ko	4.57	4.07	72	52	....	.15	....
9372	Ko	2.64	2.14	54	40	64	.60	.7
18^h6	Ko	5.89	5.39	117	....	96	....	.5
12289	Ko	5.35	4.85	94	37	112	.14	.3
12540[a]	Ko	3.8	3.3	128	11	59	.30	.3
13149	Ko	4.03	3.53	300	51	91	.30	.5
14864	Ko	5.80	5.30	126	46	....	.13	....
22^h60	Ko	4.33	3.83	169	99	111	.14	.3
7724	K1	2.61	2.21	122	41	89	.70	.7
10442[1]	K1	5.46	5.06	134	92	134	.10	.3
10875	K1	5.15	4.75	67	....	98	....	.5
11325	K1	5.03	4.63	118	67	93	.080	.5
14279[17]	K1	4.48	4.08	412	164	151	.50	.7
726[b]	K2	5.9	5.55	219	....	103	....	.3
2004	K2	7.51	7.16	297	....	108	....	....
3079	K2	5.20	4.85	290	93	103	.083	.7
3338	K2	7.41	7.06	257	....	129	....	....
2008	K3	7.42	7.12	318	....	106	....	....
3797[b]	K3	4.9	4.6	34	−133	75	.10	.3
8123	K3	3.71	3.41	63	34	82	.20	.3
8470	K3	6.88	6.58	538	....	124	....	....
9000[17]	K3	5.73	5.43	171	36	122	.25	.7
12506	K3	5.61	5.31	266	474	115	0.065	0.5

TABLE 50—*Continued*

ADS	Sp.	m_b	Mod.	h'_t	t'	s'	Tr. Wt. p	Sp. Wt. p
2867	*K4*	5 24	5.04	93	132	71	0.088	0.3
7902	*K4*	6.35	6.15	124	....	93	....	.3
9596	*K4*	5.50	5.30	287	57	149	.13	.5
10429	*K4*	4.89	4.69	199	78	95	.16	.5
11916	*K4*	7.28	7.08	390	....	104	....	....
16836	*K4*	5.96	5.76	227	85	99	.045	.7
3006	*K5*	6.19	6.09	231	....	115	....	.3
3321	*K5*	1.06	0.96	165	72	93	.30	.3
5816	*K5*	8.30	8 20	830	....	131	....	....
7873	*K5*	8.53	8.43	465	....	97	....	....
18^h37[7b]	K5	4.58	4.48	165	....	150	....	.3
16228	*K5*	6.24	6.14	202	51	68	.068	.5
3158[a]	*Mo*	6.8	6.95	223	221	98	....	....
8489	*Mo*	5.92	6.07	160	....	131	....	.5
2294	*M1*	5.72	5.92	290	....	107	....	.3
15208	*M1*	5.36	5.56	219	39	129	.095	.3
9480	*M2*	6.03	6.28	70	....	72	....	.5
2472	*M3*	3.95	4.25	184	−113	85	.11	.3
4841	*M3*	3.7	4.0	139	94	82	.35	.7
8979	*M3*	6.83	7.13	194	....	53	....	....
10418[d]	*M5*	3.48	3.88	132	− 24	72	0.60	0.7
11524	*M6e*	6.72	7.12	194	−106	....	....	....

IV. Subgiants

ADS	Sp.	m_b	Mod.	h'_t	t'	s'	Tr. Wt. p	Sp. Wt. p
14778	*G5*	7.74	5.24	108	....	89	....	0.7
14430	*G6*	6.28	3.83	44	....	100	....	0.5
8060	*G7*	5.66	3.26	121	162	108	0.52	0.7
11661	*G7*	6.31	3.91	79	....	97	....	0.3
7779	*G9*	7.39	5.09	146	....	94	....	0.7
755[1,9]	*K1*	6.30	4.20	270	....	118	....	2.0
20^h36[9]	*K1*	6.42	4.32	241	336	124	.12	0.5
2459[17]	*Ko*	5.72	3.52	116	76	110	.55	0.7
14279[17]	*K1*	4.48	2.38	188	75	87	.78	0.7
9000[17]	*K3*	5.73	3.73	78	17	100	0.62	0.7

V. Supergiants

ADS	Sp.	m_b	Mod.	h'_t	t'	s'	Tr. Wt. p	Sp. Wt. p
1477[18b]	*cF7*	2.4	1.72	36	6.6	20	0.63	0.5
10074[19]	*M1*	1.25	1.45	29	55	17	0.60	0.5

NOTES TO TABLE 50

a. Brighter component spectroscopic binary; two spectra visible.
b. Brighter component spectroscopic binary; one spectrum visible.
c. Fainter component spectroscopic binary; two spectra visible.
d. Fainter component spectroscopic binary; one spectrum visible.

1. See Table 51.
2. Pair BC.
3. o Eridani, white dwarf. Not included in computations.
4. Cluster parallax used in sixth column.
5. Mean of cluster and trigonometric parallax used in sixth column.
6. Burnham's *General Catalogue* number.
7. Spectrum = Harvard K2. Treated as K5 in work on masses.
8. Spectrum = Harvard K5. Treated as K8 in work on masses.
9. Value of h'_1 derived from orbital elements; in work on masses this star not handled with "orbit" group.
10. π Centauri. α 11^h16^m4, δ $-53°56'$ (1900).
11. Pair CD.
12. Pair AC.
13. See Table 53, n. 13.
14. Also ADS 440.
15. Capella.
16. Star A = *A1*; B = *G3*; Δm = 0.08. In work on masses, star A incorrectly treated as G3.
17. Treated as normal giants in work on masses; subsequently found to be subgiants.
18. Polaris, excluded.
19. Antares, excluded.

With the aid of Table 50 anyone may add additional material as it becomes available—or may rediscuss the data, using such weights, etc., as he sees fit.

For a number of stars the data used in the discussion of the masses were found to be capable of improvement by the time (almost two years later) when the final catalogue of dynamical parallaxes was prepared. The data used in the work on the masses are listed in Table 51. In the first part, containing stars with orbits, the older values of a'' and P are given, with the adopted h_1 and a reference to the source; in the second, for physical pairs, the earlier-adopted data for the relative motion, with explanatory notes.

The revised values of the data for these stars may be found in Tables 52 and 53. Comparing these with the older values, we find that the changes in the first part of the table (corresponding to improved orbits) are small. For 30 stars the mean change in h_1 is

TABLE 51

DATA USED IN STUDY OF MASSES BUT REPLACED BY IMPROVED VALUES IN TABLES 52 AND 53

I. BINARIES WITH KNOWN ORBITS

ADS	Sp.	P (Years)	a	h_1	Grade	Note
520	*G7*	25.0	0″.66	0″.077	g	1
671	*F9*	478.7	11.92	.195	g	2
755	*K1*	124.2	0.97	.039	g	3
1538	*F9*	136	1.15	.043	f	1
1630	*B9*	55	0.35	.024	g	1, 4
2200	*A6*	33.3	0.16	.015	g	1
3135	*F6*	92	0.54	.026	g	5
3210	*G5*	28.9	0.25	.027	g	1
3264	*A6*	148.3	1.04	.037	f	1
3475	*F7*	16.6	0.19	.029	g	1
3483	*F5*	86	0.56	.029	g	1
6420	*G2*	23.34	0.69	.084	g	1
7284	*K4*	35.4	0.62	.058	g	6
8148	*F4*	183.3	1.92	.060	p	7
8539	*A6*	690	1.30	.016	f	1
8630	*F0*	182	3.74	.116	g	1
9413	*G5*	151.4	4.87	.171	g	1
9494	*G1*	204.7	3.58	.102	g	1
9578	*F8*	165	1.02	.034	p	8
9757	*A0*	101	0.62	.028	g	1
9979	*F6*	1679	9.02	.064	g	9
9982	*K6*	215	1.53	.043	f	1
11046	*K1*	89.09	4.56	.229	g	10
11111	*A7*	423	1.33	.023	f	1
11950	*A4*	21.2	0.56	0.074	g	1

II. PHYSICAL PAIRS WITHOUT ORBITS

ADS	Sp.	Δm	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	100w	h_1	Grade	Note
616	*F2*	1.0	80	18°	− 13°	1″.39	−0″.20	0″.38	0″.013	f	11
1392	A5	1.5	15	34.0	+ 95.0	0.26	0.00	0.43	.0071	p	12
2336	B8	4.8	80	355.5	− 0.8	5.21	−0.15	0.19	.011	p	12, 13
2373	G0	0.0	16	315	− 325	0.34	+0.45	1.99	.021	f	12
2799	A3	0.3	97	202	+ 30	0.53	+2 40	2.42	.028	g	14
3358	*A5*	2.3	75	293	− 35	1.60	−0.20	1.03	023	g	12, 13
5^h97	K8	0.2	80	122	− 40	3.25	−3.50	4.22	.075	g	12, 13
5752	G5	0.2	19	95	−2600	0.14	0 00	6 39	.035	g	12, 14
6483	*F6*	0.2	15	40	+ 205	0.32	−1.00	1.52	.018	g	15
8035	*G7*	9.0	95	303	− 350	0.75	−2 90	5 44	.055	g	14
8347	*A1*	2.5	80	37.0	+ 3.2	3.77	−0.15	0.26	.012	f	12, 13
11^h72	F5	0.3	25	183	− 510	0.57	+1.25	5.2	.048	f	14, 16
13^h79	B5	1.45	80	110.2	− 3.6	8.12	−0.55	0.78	.033	p	17
9763	*G5*	6.2	18	95.8	− 90.0	3.60	0 00	5.66	.094	f	12
15^h106	A2	0.1	30	227	− 620	0.46	−6.8	8.4	.062	f	14, 16
10229	*F4*	1.7	85	313	− 57	0.50	−0.89	1.02	.019	g	11
10442	*K1*	1.51	70	355.9	− 0.7	10.50	−0.12	0.12	.013	p	11
10871	K0	0.2	31	277	+1150	0.16	0.00	3.23	.023	g	12, 14
13728	*A0*	2.2	80	288.2	− 8.0	3.30	+0.22	0.48	.018	f	12, 13
16417	*G1*	0.5	20	165	0	0.20	+2.00	2.00	0.018	g	12, 14

TABLE 51—*Continued*

I. Binaries with Known Orbits							II. Physical Pairs without Orbits											
ADS	Sp.	P (Years)	a	h_1	Grade	Note	ADS	Sp.	Δm	E	θ	$100\frac{d\theta}{dt}$	s	$100\frac{ds}{dt}$	$100w$	h_1	Grade	Note
12973	*A0*	25.2	0.32	0.037	g	1												
14073	*F3*	26.79	.48	.053	g	1												
14773	*F3*	5.70	.27	.084	g	1												
15281	*F2*	11.53	.21	.043	g	11												
16665	*K1*	86	0.79	0.040	g	1												

NOTES TO TABLE 51

1. See catalogue by van den Bos, *B.A.N.*, **3**, No. 101, p. 149, 1926.
2. Volet, *J. d. observateurs*, **16**, 107, 1933.
3. Jackson, *M.N.*, **80**, 546, 1920.
4. Pair BC.
5. Lauritzen, *A.N.*, **237**, No. 5676, p. 209, 1929.
6. Doberck, *A.N.*, **174**, No. 4169, p. 257, 1907.
7. Voronov, *Pub. Tashkent Astr. Obs.*, **4**, No. 2, p. 45, 1933.
8. Voronov, *Bull. Tashkent Astr. Obs.*, No. 6, 1935.
9. Doberck, *A.N.*, **169**, No. 4051, p. 289, 1905.
10. Tschilschke, *A.N.*, **236**, No. 5664, p. 409, 1929.
11. Error in h_1 discovered after work on masses was completed:

ADS....................	15281	616	10229	10442
Correct h_1..............	0″.041	0″.011	0″.016	0″.010

12. Data from *A.J.*, No. 930, or *Lick Bull.*, No. 451.
13. Pair AB and C; wide pair rejected. Pair AB listed in Table 52 or 53.
14. Star transferred to "orbit" list (Table 52).
15. Orbit listed in *A.J.*, No. 930, rejected; new orbit adopted after work on masses was completed.
16. Data from Finsen, *Union Obs. Circ.*, No. 93, 1935.
17. Value of w in *A.J.*, No. 930, used in work on masses erroneous; correct $w = 0''.72$.
18. (5^h67) Henry Draper photographic magnitude, corrected for color index, used in calculation of d, Table 53. Henry Draper photometric magnitude used to calculate m_b, Table 50.

$-0\overset{''}{.}001$; and the average, without regard to sign, $\pm 0\overset{''}{.}004$. The mean percentage change is -0.1; and the numerical average, ± 11.

These include one remarkable instance of a large change, ADS 12973 (ζ Sagittae), for which the old value of h_1 is $0\overset{''}{.}037$ and the new $0\overset{''}{.}018$. Both orbits have $e = 0.85$; but the older one has $\lambda = 65°$ and $i = 78°$, so that the major axis of the apparent orbit is but little greater than the minor axis of the true orbit, while the second has $\lambda = 180°$, so that the apparent and true major axes agree. Other differences in the rather ill-defined apparent orbits account for the rest of the change.

For 8 of the 20 physical pairs in Table 51 orbits have recently become available, so that they now appear in Table 52. For 5 others, which are triple systems, a dynamical parallax derived from the wide pair was inadvertently used in the earlier discussion, while the close pair gives the better value. For the remaining 7 stars the improved data for the motion change h_1 in the means by $-0\overset{''}{.}001$ or without regard to sign by $\pm 0\overset{''}{.}002$.

Finally there are 11 stars included in the earlier lists for which upon re-examination, the data were not judged good enough to justify even a "poor" dynamical parallax. The number of these stars in Burnham's *General Catalogue* (by which they are identified in the previous lists) are as follows:

447	3422	10404
1021	5124	10775
1577	9439	11130
2126	9695	

Two stars which appeared with orbital parallaxes in the *Astronomical Journal*, 39, No. 930, p. 168, have been transferred to the list of physical pairs. These are ADS 2959 and 10425 (=βGC 2007 and 7936).

B. THE CATALOGUE

73. *Description of the Tables.*—The dynamical parallaxes here given have been derived by the strict application of the formulae and methods given in chapter iv. Table 52 gives the results for 166 stars for which fairly reliable orbits have been calculated, while Table 53 contains 2363 slow-moving physical pairs, discussed by the

statistical process. There are no duplications, so that the whole number of dynamical parallaxes is 2529.

The first six columns are the same in both tables. The leading column contains the ADS number, for all stars which appear in this catalogue. For southern stars the number in Innes' *Reference Catalogue* is used by preference. For the few remaining stars the DM or CPD number is given, or occasionally that in Burnham's catalogue, the latter in parentheses and also indicated by a footnote.

The next two columns list the position of the star for 1900; the fourth gives the combined visual magnitude. Photometric (visual) magnitudes are indicated by two decimal places.

The sources adopted, in order of preference, are:

1. Photometric measures of double stars (*Harvard Ann.*, 56, No. 7, p. 231, 1912, and 64, No. 6, p. 165, 1913.

2. The two-place "photometric" magnitudes of the *Henry Draper Catalogue*.

3. The photometric magnitudes in Boss's *New General Catalogue* would have been given at least equal consideration; but, unfortunately, a great part of the present work was done before this catalogue became available, so that it could be used only in special cases (see below).

4. The magnitudes given by Finsen,[7] which evidently represent the result of a somewhat critical selection.

5. For stars north of $-19°$ the one-place "photometric" magnitudes of the *Henry Draper Catalogue*, which represent the visual estimates of the *Durchmusterungen* with systematic corrections to the Harvard scale.

6. For stars south of $-19°$, the Henry Draper "photographic" magnitudes were corrected for color index (*Harvard Ann.*, 91, 13, 1918) and adopted. These are based upon the *Cape Photographic Durchmusterung* and are probably more reliable than the visual *Durchmusterung* estimates.

7. For the remaining stars, magnitudes were obtained in some cases from Boss's *New General Catalogue*—and, in default of better, from the corrected *Bonner Durchmusterung*.

[7] *Union Obs. Circ.*, No. 93, p. 139, 1935.

The fifth column gives the spectra—of both components when known; otherwise, of the brighter component or the combined light. The Mount Wilson determinations, which have been given the preference, are printed in italics. Most of the rest are from the *Henry Draper Catalogue*, with additions from Boss's *New General Catalogue*, representing Harvard material not previously published. In calculating the dynamical parallax d, the Henry Draper spectrum Oe5 has been treated as B0; A as A3; G as G3; Ma as M0; and Mb as M5.

The sixth column contains the difference in visual magnitude Δm. Values given to two places represent photometric measures, the rest are collected from various sources. Many of these are of low accuracy, but the reader will at least know what values we have used.

The remaining columns of Table 52 contain: columns 7 and 8, the adopted orbital elements P and a''; column 9, the hypothetical parallax $h_1 = a''P^{-2/3}$; column 10, the dynamical parallax d, derived as described in chapter iv; column 11, the grade assigned to the determination (good [g], fair [f], and poor [p]); and column 12, a reference to the source of the orbit.

In Table 53 the seventh column contains the hypothetical parallax $h_1 = 0.418 \sqrt[3]{sw^2}$; and the eighth column, the dynamical parallax d, derived as above. The last column contains the grade. Here "F" denotes a determination by Finsen, and "F:" a determination marked by him with a colon. In the discussion these have been grouped respectively with grades "f" and "p."

A discussion of the probable errors of these parallaxes and the corrections which should be made when they are used for calculating distances, space motions, or absolute magnitudes will be found in chapter iii, section A (pp. 90–100).

TABLE 52

DYNAMICAL PARALLAXES OF BINARIES WHOSE ORBITS HAVE BEEN DETERMINED

ADS	α1900	δ1900	Mag.	Sp.	Δm	P (Years)	a	h	d	Grade	Ref.
61	0h 1m.0	+57° 53′	6.10	G3 G8	1.1	107.2	1″.44	0″.064	0″.052	g	1
102	0 3.8	+79 10	6.22	A3	0.3	275	0 72	.017	.010	f	1
221	0 11.5	+35 56	7.70	F7	0 7	108	0 41	.018	.013	g	1
−21°30*	0 23 3	−20 53	6.41	G0	0 0	5.56	0.14	.046	.036	f	1
450	0 27.0	− 5 44	8.9	G5	0 0	10 5	0 14	.030	.027	g	1
0h27	0 27 0	−63 31	4.46	A2	1.0	41.3	0 48	.040	.024	f	1
490b	0 30.1	− 4 9	5.24	F7	0.8	6 91	0.24	.067	.045	g	1
520	0 32.2	−25 19	5.71	G7	0.2	25 0	0 67	.078	.063	g	1
588	0 37 2	+ 3 37	7.61	F6	2 1	357 0	1 67	.033	.027	p	1
671	0 43.0	+57 17	3 64	F9 M0	3.74	526.0	12.53	.193	.160	g	1
684	0 44.8	+50 5	8.02	F5	0.5	91.2	0 39	.019	.014	f	1
746	0 49.3	+18 38	5.76	A0	1.1	163 4	0.57	.019	.011	p	2
755g	0 49 6	+23 5	5.60	K1	0 1	169.3	1.01	.033	.017	g	1
1h13	1 11.6	−69 21	7.4	G5	0.6	128	1.47	.058	.052	f	1
1h34	1 30 4	−30 26	7.19	G5	0 4	4.56	0 17	.063	.056	f	3
1h37	1 36.0	−56 42	5.26	G5 G5	0.03	251	8.31	.208	.202	f	1
1538b	1 50.7	+ 1 21	6.18	F9	0 0	158.4	1.00	.034	.022	f	1
1598	1 53.8	+70 25	4.61	A6s	2.5	63.3	0.66	.042	.027	g	1
1630†	1 57.8	+41 51	5.08	B9n	1.2	56.0	0.32	.022	.012	g	1
1709	2 7.6	+47 1	6.03	F2	0 9	149.6	0.92	.033	.023	g	1
2h11	2 15 6	−35 54	8.2	G5	0.1	191.6	1 36	.041	.037	f	1
1865	2 22.5	+ 3 59	8 6	K5	0.2	25.2	0 55	.064	.063	f	1
2028	2 34 7	− 0 17	8.2	G0	0.0	19	0 17	.024	.019	g	1
2200	2 47.4	+37 56	5.35	A6s	1.0	31.6	0.22	.022	.013	g	1
3h10*b	3 8.9	−44 48	5.92	F2	0.6	39.95	0 52	.044	.029	f	1
2616	3 28.5	+24 7	5 92	A2	0.1	600	0.72	.010	.005	f	1
2799	3 44.3	+25 17	5.38	A3	0.3	63.5	0.47	.030	.019	f	1
3082	4 9.6	+31 27	7.40	G0	0.0	170.2	0.52	.017	.011	f	1
3093†‡	4 10.7	− 7 49	9.48	A2n M5e	1.8	247.9	6.89	.173	——	g	1
3135	4 14.2	+16 17	6.86	F6	1.8	90 0	0.57	.028	.021	g	1
3169b	4 17.1	+14 49	7.09	F8 G1	2.0	487	1.72	.028	.019	f	1
3159	4 17.4	−25 58	5.88	F2	0.0	84 20	0.53	.027	.018	f	1
3210	4 20.0	+18 39	7.72	G5	0.6	30 4	0.24	.024	.018	g	1
3248*	4 23.3	+15 56	6.58	F8	0 8	40	0.42	.036	.027	f	1
3264a	4 24.4	+15 25	5.70	A6n	2.5	170	1.03	.034	.020	f	1
3475	4 45.7	+10 54	6.96	F7	0.0	16.34	0.19	.030	.022	g	1
3483	4 46.2	+13 29	6.70	F5	3.0	116.2	0.79	.033	.026	f	1
3588*	4 54.6	−16 32	5.54	F2	0.3	56.0	0.56	.038	.026	f	1
3711a	5 2.5	+ 8 22	5.47	F2	0.8	524	1.75	.027	.015	f	1
3841g	5 9.3	+45 54	0.21	G1	0.0	0.285	0.054	.123	.053	g	1
4299	5 38.0	− 6 51	5.98	F5	0.7	20.6	0.21	.028	.018	g	1
4617b	5 56.9	+ 9 39	4.19	A2	2.3	17.5	0.27	.040	.021	f	1
4929	6 13.6	+28 28	7.16	A3	0.0	45.7	0.26	.020	.014	f	1
5234	6 30.2	+27 22	6.89	G2	2.5	103	0.77	.035	.027	f	1
5423	6 40.7	−16 35	−1.58	A2s A5	10.02	49.94	7.62	.563	.396	g	1
6h83*	6 53.7	−35 22	6.19	F5	0.2	16.5	0.32	.049	.037	f	1
5752	6 59.4	− 2 54	8.6	G5	0.2	36.7	0 26	.023	.019	g	1
5871	7 06.6	+27 24	6.44	F6	0.0	120	0.87	.036	.026	f	1
6175bd	7 28.2	+32 06	1.58	A2s A8s	0.86	340	5 84	.120	.058	g	1
6251	7 34.1	+ 5 29	0.48	F3	10.3	40.23	4.26	.362	.278	g	1
6420	7 47.1	−13 38	5.34	G2	0.6	23.18	0.58	.072	.054	g	1
6483	7 52.1	+ 1 24	6.44	F6	0.2	62.5	0.34	.021	.014	f	1
6554	7 58.8	+12 35	7.9	K2	0.0	44.0	0.38	.030	.024	g	1
6650	8 06.5	+17 57	5.10	F7	0.70	58.00	0.91	.060	.043	g	4
6993	8 41.5	+ 6 47	3.53	F8	1.5	15.3	0 23	.037	.020	g	3
7203	9 1.6	+67 32	4.87	F4	3.28	10850	25.50	.052	.036	f	1
7284	9 12.0	+29 00	7.26	K4	0.3	34.2	0.66	.063	.054	g	1
7390	9 23.1	+ 9 30	5.52	F8	0.8	116.7	0.84	.035	.023	g	1
9h40	9 26.8	−40 2	3.64	A7n	2.0	34.9	0.91	.085	.059	g	1
7545	9 45.3	+54 32	4.54	A3s	0.6	112.7	0.34	0.015	0.007	g	1

ADS	α1900	δ1900	Mag.	Sp.	Δm	P (Years)	a	h_1	d	Grade	Ref.
7555	$9^h47^m.5$	−7°38′	5.16	*A0n*	0 3	78 8	0″.37	0″.020	0″.011	g	1
8035[a]	10 57 6	+62 17	1.95	*G7*	9 0	44 0	0 63	.051	.024	p	1
8048†	10 59.6	− 3 41	9.25	——	0 0	23 5	0 23	028	.024	p	1
8119§[bd]	11 12 9	+32 6	3 86	*G0 G0*	0 46	59.86	2.54	.160	.118	g	1
8148	11 18 7	+11 5	4 03	*F4 F5*	2.67	414	3 60	065	.044	f	1
11^h22	11 20 3	−61 6	7 4	K5	0 6	233 5	4.05	107	.105	f	1
8189[a]	11 25.4	+41 51	6.99	*F1*	0 4	84 7	0 35	018	.011	g	1
8197	11 26.7	+61 38	5 47	*F4*	0 3	71.9	0.78	.045	.031	g	1
8239	11 31 8	−11 48	9 3	G5	0 3	437	0 99	.017	.014	f	3
8337	11 48.3	+74 19	6.78	*F7*	1.3	77.1	0.38	.021	014	g	1
11^h72	11 58.5	−38 27	6 64	F5	0 4	115	0.74	.031	.022	p	1
8419	12 1 0	+69 15	7.14	*F5*	0 0	103 3	0 32	014	.009	f	1
8539	12 19.4	+26 8	6.31	*A6n*	1 2	361	1 00	020	.012	f	1
12^h61	12 36 0	−48 25	2.38	A0	0 0	84.5	0.93	048	.024	g	1
8630	12 36 6	− 0 54	2 90	*F0 F0*	0 03	171.4	3.75	.122	.081	g	1
8739	12 56 4	+56 54	4 89	*A6n*	3 6	89.6	1.27	064	.048	f	1
8804	13 5 1	+18 4	4 47	*F4*	0 00	25.87	0 66	.076	.054	g	1
8891	13 19.9	+55 27	2 40	*A2s*	0 0	0 056	0 012	.078	.044	f	1
13^h50*	13 30.4	−61 11	5.59	F5	0 3	34.8	0.50	.047	.033	f	1
8974	13 33.0	+36 48	4.92	*A3n*	3.5	220	1.20	.033	.022	g	1
8987	13 34.7	+11 15	5.54	*A6n*	0 0	22.6	0 22	.028	.017	g	1
9031	13 44.6	+27 29	7.26	*K6 K6*	0 3	151.0	2.47	.087	.078	g	5
9094	13 58.8	+ 8 58	7.8	*F5*	0.1	38.4	0.20	.018	.013	g	1
9229	14 16.7	+48 58	7 23	F8	0.1	321	0.90	.019	.013	f	1
9247†[b]	14 18.5	+ 8 54	6.64	*F2*	1.7	40 0	0.24	.020	.012	g	1
9301	14 27.9	+27 7	5.90	*A3n*	0 2	29.4	0.21	.022	.014	f	1
14^h59	14 32.8	−60 25	0.06	*G4 K5*	1.37	80.09	17.67	.949	.748	g	1
9343	14 36.4	+14 9	3.86	*A2n*	0.40	126	0.60	.024	.012	f	1
9352	14 38.1	+19 56	9.4	*M0*	0 5	50.6	0.66	.048	.044	f	1
9380	14 41.4	+10 5	7.17	*G1*	0.8	178	0.79	.025	.018	g	1
9378	14 41.7	+42 48	7.24	*F5*	0.5	87.7	0.32	.016	.011	g	1
9413	14 46.8	+19 31	4.64	*G5 K5*	2.02	150.0	4.88	.173	.153	g	1
9494	15 0.5	+48 3	4.86	*G1*	0 80	219.5	3.61	.099	.078	g	1
9578	15 14.0	+27 12	6.55	*F8*	0.50	199	1.26	.037	.027	p	6
9617	15 19.1	+30 39	5.05	*F9*	0.50	41.62	0.91	.076	056	g	1
9626†	15 20.7	+37 42	6.67	*G0*	0.6	224	1 30	.035	.026	g	1
9689*†	15 27.2	−24 9	7.06	A3	0.2	54.6	0.25	.017	.011	p	1
15^h55	15 28.5	−40 50	2.95	B3	0.2	104	0.78	.035	.016	f	1
9716	15 32.5	+40 8	6.78	*K4*	0.3	55.11	0.80	.055	.044	f	3
9744*[a]	15 37.1	+20 0	4.49	A2	0 0	10.6 21.3	0.12 0.23	.024 .030	.011 .014	p	7
9757[a]	15 38.5	+26 37	3.93	*A0n*	3.0	90.0	0.68	.034	.017	g	1
9769	15 45.1	+80 17	6.93	*F0p*	1.0	140	0.40	.015	.009	f	1
15^h106*	15 55.4	−57 30	4 94	A2	0.1	26.7	0.38	.042	.027	f	1
9909	15 58.9	−11 6	4.16	*F4*	0.30	44.70	0.72	.057	.036	g	1
9979[a]	16 10.9	+34 7	5.36	*F6 G1*	0.90	1162	7.27	.066	.044	g	1
9982	16 11.1	+ 7 37	8.6	*K6*	0.5	680	3.04	.039	.034	f	1
10075	16 24.5	+18 37	7.04	*K2*	0 0	217.1	2.20	.061	.052	g	1
10087[b]	16 25.9	+ 2 12	3.85	*A1n*	2.1	132.9	0.97	.037	.019	f	1
10157	16 37.5	+31 47	3.00	*G0*	3.5	34.46	1.35	.127	.080	g	8
10188	16 40.8	+43 40	8.3	*K6*	0.5	120	0.91	.037	.030	g	1
10235	16 47.9	+28 50	6.52	*F5*	1.5	262	1.01	.025	.017	f	1
−8°4352*	16 50.1	− 8 9	9.20	*M3e*	0.20	1.66	0.185	.132	.134	f	1
10360	17 4.5	+36 4	5.38	A5	0.0	16.08	0.17	.026	.016	g	1
17^h31	17 11.4	−46 32	5.58	K0	2.8	242	4.94	.127	.113	f	1
(7929)‖	17 12.1	−34 53	5.89	*K5*	2.0	42.2	1.83	.150	.137	g	1
10561	17 22.3	−20 53	8.0	F8	0.2	36.7	0.23	.021	.016	f	1
10598	17 25.2	− 0 59	5.34	*G6*	0.2	46.0	1.06	.082	.064	g	1
10585	17 25.5	+29 29	9.1	K2	0.4	76.0	0.64	.036	.034	f	1
10660	17 33.9	+61 58	5.31	*G1*	4.7	76.06	1.55	.086	.072	f	9
10786†	17 42.5	+27 47	9.68	*M3*	0.5	43.23	1.30	.105	.108	g	8
10871	17 49.2	+25 1	8.06	K0	0.2	38.4	0.28	.025	.019	f	1
11005[b]	17 57.6	− 8 11	4.88	*F3*	0.70	224	1.31	.035	.019	f	1
17^h129	17 59.6	−43 26	5.02	A3	0.00	154.0	1.11	.039	.025	f	1
11046	18 0.4	+ 2 31	4.07	*K1 K6*	1.70	87 85	4.56	.231	.198	g	1
11060[a]	18 1.6	+21 26	6.92	*G1*	1 3	19.75	0.30	0.041	0.028	g	1

ADS	α_{1900}	δ_{1900}	Mag.	Sp.	Δm	P (Years)	a	h_1	d	Grade	Ref.
11077	18^h 3^m2	+30°33′	5.21	*F5*	5 3	54 7	1″13	0″078	0″066	g	1
11111[b]	18 4.6	+ 3 50	5 67	*A7n*	1.5	391	1.33	025	.014	f	1
11186	18 9.4	+ 0 9	7.88	F2	0.3	323	0.79	.017	.012	g	1
11520	18 33.1	− 3 17	6.47	*F8*	0 2	12 12	0 18	.033	.024	g	1
11871	18 53.3	+32 47	5.21	*G0*	3 5	57.0	1 24	083	.068	g	3
11897	18 55.8	+58 5	6 31	A2	0 6	233	0.53	.014	.008	f	1
11950	18 56 3	−30 1	2.71	*A4n*	1 0	20 8	0 52	.069	040	g	1
18^h113	18 59.7	−37 12	4.26	*F7 F7*	0.00	119.3	2.07	085	.060	g	1
12145†	19 7.7	+38 37	8 2	*K0*	0 0	58	0 40	026	.021	g	1
12447	19 22.5	+27 7	7 8	*F9*	0.2	474	1 35	.022	.017	g	1
19^h54	19 40.2	−62 3	7.48	G0	0 5	72	0.45	.026	.020	f	1
12880	19 41.8	+44 53	2.97	*A1n*	4.9	321	2.12	.045	.026	f	1
12973	19 44.5	+18 53	4.99	*A0n*	1.0	22.8	0 15	.018	.010	f	1
12972	19 45.0	+35 4	6 52	*F2*	1 0	128	0.57	.022	.014	p	1
13125	19 51.6	+41 36	7.36	K0	0.0	25.7	0.25	.029	.021	g	1
13461	20 6 9	+43 39	7 14	*G4*	1.0	84	0.43	022	.015	g	1
13723	20 16.6	+45 3	7.02	F5	0.9	95.0	0.33	.016	.010	f	1
20^h36[‖g]	20 20.4	−37 44	6.26	*K1*	1.7	108	0 75	.033	.019	p	1
14073	20 32.9	+14 15	3 72	*F3*	1.3	26.60	0 48	.054	.033	g	1
14238	20 40 2	+12 23	8.5	G	0.2	107.7	0.49	.022	.017	f	1
14360	20 46.1	− 6 0	5 99	*F3*	1.3	152	0.70	.024	.016	g	1
14424	20 50 8	+27 43	8.0	G5	0.4	133	0 41	.016	.011	p	1
14499	20 54.1	+ 3 55	5.29	*F0*	0 5	101.4	0.66	.030	.019	g	1
14773	21 9.6	+ 9 36	4.61	*F3*	0.5	5.70	0.26	.082	.060	g	1
14787	21 10.8	+37 37	3 82	*F0*	2.50	49.8	0 94	.070	.046	g	1
14839	21 13.8	+11 9	6.97	G0	2.5	63	0.52	.033	.025	p	1
15176	21 34.4	− 0 30	6.80	*F7*	0.1	46.6	0.41	.032	.023	g	3
15267	21 39.4	+27 24	8.0	F5	0.0	119.0	0 35	.014	.010	p	1
−58°7893*	21 39.5	−58 8	8.6	Ma	0 1	6 75	0.21	.060	.051	p	1
15281[b]	21 40.1	+25 11	4.27	*F2*	0.5	11.53	0 21	.041	.022	g	1
15972	22 24.5	+57 11	9 43	*M3*	1.7	44 52	2.36	.188	.218	g	1
15988	22 24.9	+ 3 55	5.47	*F2*	1.4	150	0.81	.028	.018	f	1
16173	22 35.9	+14 1	5.81	*G3*	0.0	21.62	0.31	.040	.028	g	1
16326	22 48 0	+57 12	7.8	K0	0.8	90.0	1.00	.050	043	p	1
16417	22 53.5	+ 8 50	6.50	*G1*	0.5	27.0	0.41	.046	.035	g	1
16497	22 59.9	− 8 14	5.56	F0	0.0	22.0	0 20	.026	.018	g	1
16665	23 13.8	+ 4 52	8.6	*K1*	1 0	108 0	0.83	.037	.033	g	1
16800[a]	23 25.5	+30 17	7.26	*F5*	0.00	49.4	0 20	.015	.009	g	1
16819*	23 27 1	+ 6 32	6.84	F5	0.60	30	0.24	.025	.018	f	1
17178	23 56.3	+39 4	8.6	*K1*	0.4	144	0.64	.023	.019	f	1
17175	23 56.9	+26 34	5.85	*G1*	5 2	26.46	0.82	0.092	0.085	g	1

NOTES TO TABLE 52

a. Brighter component spectroscopic binary; two spectra visible.
b. Brighter component spectroscopic binary; one spectrum visible.
c. Fainter component spectroscopic binary; two spectra visible.
d. Fainter component spectroscopic binary; one spectrum visible.
g. Treated as giant in calculation of *d*.

* Entirely new—no dynamical parallax previously published for this star.
† Pair BC.
‡ White dwarf; *o* Eridani.
§ ξ Ursae Majoris. Both components are spectroscopic binaries. No spectroscopic binary correction applied for star B, since its mass function is excessively small.
‖ Burnham's *General Catalogue* number.

1. See catalogue by Finsen, *Union Obs. Circ.*, No. 100, 1938.
2. Janssen, *A.N.*, **236**, No. 5646, p. 93, 1929.
3. See catalogue by Finsen, *Union Obs. Circ.*, No. 91, p. 23, 1934.
4. Voronov, *Bull. Tashkent Astr. Obs.*, No. 2, p. 38, 1934.
5. Van Biesbroeck, *Pub. Yerkes Obs.*, **5**, 12, 1925.
6. Aller, *A.N.*, **268**, No. 6410, p. 23, 1938.
7. Van den Bos, *Union Obs. Circ.*, No. 98, 1937.
8. See catalogue by van den Bos, *B.A.N.*, **3**, No. 101, p. 149, 1926.
9. Rudnick, *A.J.*, **43**, No. 1012, p. 164, 1934.

TABLE 53

Dynamical Parallaxes of Physical Pairs

ADS	α_{1900}	δ_{1900}	Mag.	Sp.	Δm	h_1	d	Grade
48	$0^h\ 0^m.3$	$+45^\circ\ 16'$	8.62	*Mo* K2	0.2	0″.11	0″.12	g
56	0 0.6	+17 17	7.9	F2	0.0	.0090	.005	f
0^h1	0 0.8	−30 53	9.0	F8	2.0	.031	.028	F:
60	0 0.9	+17 32	8.8	K0	0.0	.028	.023	g
0^h2[g]	0 1.1	−49 38	5.77	G0	5.9	.012	.006	F
84	0 2.5	− 5 6	9.0	G0	1.2	.018	.014	g
——[g]	0 4.0	−54 33	6.34	G0	1.5	.017	.009	F
111	0 4.3	−28 33	5.46	F2	0.1	.020	.011	F
118	0 4.7	+ 7 54	8.5	G5	0.1	.0097	.006	f
0^h6	0 4.8	−34 20	9.0	G5	1.4	.015	.011	F
119	0 4.9	+45 50	7.7	A3	1.5	.014	.010	g
122	0 4.9	+10 36	5.51	*B9n*	4.5	.027	.017	f
143	0 6.4	+55 24	7.41	B8	0.5	.0050	.002	f
144	0 6.4	− 3 38	7.56	F8	1.6	.020	.014	f
147	0 6.7	+27 52	7.9	F5	0.3	.0091	.005	f
149	0 7.0	+46 13	8.8	F8	0.3	.0059	.003	f
161	0 8.2	+26 25	6.30	F5	1.4	.0088	.004	g
188	0 9.4	+77 28	8.2	K0	2.5	.027	.022	f
191	0 9.8	+ 8 16	5.87	*A9s A9s*	1.6	.026	.017	g
197	0 10.0	+43 39	6.64	B9	0.2	.0048	.002	g
207	0 10.6	+76 24	6.23	B9	0.5	.014	.008	g
205[g]	0 10.7	− 6 9	7.7	G5	2.5	.011	.006	f
218[b]	0 11.4	+54 6	7.47	*A6n A9n*	1.3	.020	.013	f
220	0 11.5	+36 5	6.95	A0	2.5	.013	.008	g
237	0 12.2	+15 57	8.4	*G4*	1.0	.036	.032	f
246	0 12.5	+43 27	7.73	*M3 M5*	3.0	.23	.23	g
243	0 12.6	+72 23	7.50	A3	0.3	.0064	.003	f
251	0 13.2	−21 42	6.74	A0	1.0	.0051	.002	F
252	0 13.3	+25 36	7.21	A2	0.8	.0091	.005	p
257	0 13.5	+15 26	8.8	F5	0.0	.011	.008	f
277	0 15.2	+44 57	7.02	F5	2.0	.018	.012	g
283	0 15.4	+67 7	8.1	G5	0.6	.020	.015	f
285[g]	0 15.5	+32 21	5.97	K5	0.5	.027	.013	g
287	0 15.8	+10 25	6.55	A0	0.9	.012	.006	g
296	0 16.5	+65 54	7.7	A5	0.8	.0082	.005	f
302	0 16.8	−23 34	7.40	G0	1.3	.022	.015	F
305	0 17.2	+44 22	8.8	....	1.0	.0057	.003	p
341	0 20.1	+31 57	9.3	F8	0.2	.013	.010	f
347[1]	0 20.5	−19 22	8.3	F5	2.3	.043	.039	F:
363[2]	0 22.0	− 8 26	8.3	G5	0.0	.016	.012	g
382	0 23.4	+36 45	8.2	A5	0.0	.010	.006	g
0^h22[g]	0 23.4	−55 11	8.25	G5	0.7	.0035	.001	F:
397	0 24.0	+68 31	8.5	A2	0.3	.0055	.003	g
421	0 25.7	+33 34	8.0	F5	0.1	.0099	.006	f
426[3]	0 26.0	−10 38	8.9	G0	0.0	0.011	0.008	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_t	d	Grade
433[4]	0h 26m.2	+66° 42′	10.3	*M3*	3 0	0″.14	0″.16	f
441	0 26.2	+77 34	8.9	G5	0.1	.018	.014	f
434	0 26.2	+53 59	4.88	B8	0.3	.0093	.004	g
439	0 26.5	+36 25	8.1	G0	3.1	.036	.032	f
466[1]	0 28.6	−26 39	7.47	K0	2.6	.084	.080	F:
0h29[5]	0 28.8	−35 32	6.57	*F8*	1.52	.0094	.005	F:
0h28[5]	0 28.8	−55 53	7.65	A2	0.5	.014	.009	F
475[g]	0 29.4	− 5 6	6.96	G0	0.8	.0088	.004	g
486[g]	0 29.8	+36 17	6.82	K0	2.0	.017	.009	p
497	0 30.7	+29 28	8.6	*G4 G6*	0.8	.031	.028	g
504	0 31.0	+55 35	8.0	F5	0.0	.0074	.004	g
515	0 31.6	+57 28	7.40	B9	0.0	.013	.008	p
527	0 32.5	+29 57	9.4	....	0.3	.0068	.005	g
532	0 32.9	+ 2 01	9.4	F8	0.9	.0086	.006	f
558[g]	0 34.6	+20 54	5.57	*G7 F0*	3.24	.015	.007	f
562	0 35.1	+23 30	7.18	A3	1.9	.024	.017	g
566	0 35.7	− 7 46	7.05	*G3 M1*	3.5	.060	.053	g
582	0 36.4	+70 50	7.30	*A1s*	0.26	.0056	.003	f
591	0 37.4	− 6 16	10.0	G0	0.5	.011	.009	g
597	0 37.6	+20 14	9.2	....	0.0	.0087	.006	f
601	0 37.8	+46 53	9.0	....	1.0	.014	.010	g
616	0 38.7	+45 42	7.52	*F2*	1.0	.011	.007	f
618	0 39.0	+33 5	7.70	A3	0.8	.0074	.004	f
——[6]	0 41.0	−42 27	7.31	K0	0.5	.092	.088	F
652	0 41.9	− 1 48	8.6	F8	0.0	.0052	.003	f
659	0 42.3	+50 54	6.76	*A2n*	0.9	.0029	.001	f
662	0 42.8	+25 1	9.2	G5	0.3	.013	.010	f
674	0 43.3	+56 47	8.8	....	0.5	.0062	.004	g
673	0 43.5	+18 8	7.62	G0	0.0	.013	.008	g
683	0 44.5	+27 10	5.54	*F0 F2*	0.00	.0091	.004	g
701[g]	0 46.3	+22 5	7.29	K0	0.0	.0047	.002	g
710	0 46.4	+68 20	7.12	A2	0.0	.0070	.004	f
705	0 46.6	+31 21	9.2	....	0.1	.0054	.003	g
709	0 46.9	+10 4	8.4	F8	0.7	.018	.014	g
0h48	0 47.2	−44 15	6.64	F2	0.5	.013	.007	F:
716	0 47.3	−23 9	7.24	*G0*	1.0	.028	.021	g
726[7, b, g]	0 47.8	−24 33	5.59	*K2*	5.2	.017	.007	F:
732	0 48.0	+ 3 33	7.02	F0	1.0	.0081	.004	g
735	0 48.1	+52 9	6.22	A0	3.0	.020	.013	f
729	0 48.1	+ 4 28	9.0	F2	1.3	.029	.026	f
733	0 48.3	−25 19	6.44	F2	1.9	.012	.006	F:
754	0 49.5	+ 8 54	7.8	F0	1.0	.0091	.005	f
784[b]	0 50.8	+59 50	5.54	B9	0.7	.013	.006	g
780	0 50.8	−16 54	9.3	G0	0.2	.024	.021	f
799	0 52.7	−19 32	7.08	A0	3.0	0.016	0.010	F:

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
805	$0^h 53^m.0$	+20° 52′	6.41	A0	1.4	0″.015	0″.009	g
806	0 53.1	−16 14	6.97	F5	0.2	.017	.011	f
815	0 53.7	− 8 17	9 5	G5	0.3	.012	.009	g
816	0 53.9	+ 4 54	9.8	....	0.5	.017	.014	f
819	0 54.2	− 1 13	7.7	F5	0.4	.011	.007	g
824[a, d]	0 54.4	+44 10	5.62	*A1n A0n*	0 80	.012	.005	f
822	0 54.5	− 1 44	8.67	G5	0.4	.020	.015	g
829	0 54.9	+46 47	7.60	A2	0.5	.011	.006	f
836	0 54.9	+68 49	6.67	B9	0.0	.0087	.004	f
832	0 55.0	+59 49	8.0	F5	0.4	.0088	.005	g
835	0 55.4	+ 8 57	8.2	F0	1.0	.0028	.001	f
0^h60	0 56.3	−53 7	7.37	F0	0.4	.0067	.004	F
854	0 57.1	+ 5 20	8.76	G0	0.0	.016	.012	f
862[a]	0 57.3	+46 50	6.36	*A3n*	1.3	.011	.005	f
864	0 57.6	+ 1 41	9.0	....	0.1	.035	.029	f
873	0 58.5	+34 55	7.22	A3	0.0	.0063	.003	g
875	0 58.7	+ 0 50	6.07	*F0*	2.4	.034	.024	f
0^h63[5]	0 59.2	−60 38	6.9	F5	0.27	.011	.006	F:
0^h64	0 59.3	−41 11	7.22	A2	1.0	.0084	.005	F
883	0 59.4	+36 17	9.2	....	0.3	.0079	.005	f
884	0 59.7	+ 1 3	8.8	F2	0.0	.015	.011	p
0^h65	0 59.8	−34 4	6.54	G5	4.0	.027	.018	F
895	1 0.2	+14 53	9.1	K2	0.5	.023	.019	f
899	1 0.4	+20 56	4.92	A2 A0	0.27	.028	.016	f
897	1 0.4	+58 9	9.0	....	0.2	.0035	.002	f
918	1 1.5	+38 7	7.30	F8	0.0	.021	.014	g
1^h1[g]	1 1.6	−47 15	3.35	K0	0.0	.033	.013	f
923	1 2.1	− 2 16	7.04	F5	0.8	.027	.020	g
940	1 3.7	+46 42	4.28	*B8s*	1.6	.012	.005	g
1^h6	1 4.1	−30 9	7.97	F5	1.0	.011	.007	F
1^h7	1 4.1	−47 12	7.22	A5	2.2	.012	.007	F:
1^h8[8, a]	1 4.2	−55 47	4.13	B8	3.6	.028	.013	F:
955	1 4.3	+23 16	6.65	F0	0.2	.0068	.003	f
963	1 4.6	+50 28	6.88	F5	0.4	.021	.014	g
974	1 5.6	+40 41	7.87	G5	0.4	.014	.009	g
978	1 6.0	+47 16	8.0	A2	0.1	.014	.010	f
981	1 6.3	+50 59	8.2	A0	0.2	.0048	.003	f
988	1 7.4	+31 32	6.57	*A1s A8n*	1.0	.028	.020	f
996[b, d]	1 8.5	+ 7 3	5.18	A5 *F6*	0.92	.025	.012	f
1002	1 9.1	+27 53	9.1	....	1.5	.013	.009	f
1030	1 9.7	+80 20	7.20	F0	1.5	.010	.006	f
1016	1 10.6	+ 9 15	7.10	F2	3.0	.011	.006	g
1036	1 11.0	+72 51	8.6	A0	0.2	.0053	.003	g
1023	1 11.3	− 7 41	9.1	G0	0.1	.0080	.005	f
1040	1 11.8	+48 29	6.82	A0	1.2	0.0039	0.002	f

TABLE 53—*Continued*

ADS	α_{1900}	δ_{1900}	Mag.	Sp.	Δm	h_1	d	Grade
1037	$1^h11^m.9$	$+39°46'$	9.4	....	0.0	0″.018	0″.014	p
1038	1 12.3	−23 46	8 7	G0	1.0	.016	.012	F:
1^h14	1 12.4	−69 24	4.96	F8	2.19	.067	.049	g
1049	1 12.7	−12 52	8.3	F8	0.5	.016	.011	f
1055[b]	1 13 1	+36 52	6.34	A3	2.8	.016	.008	f
1^h16	1 13.6	−66 55	6.30	A0	3.0	.016	.010	F
1066	1 14.0	−27 2	8.4	G0	0.5	.020	.015	F:
1077[2]	1 14.6	− 5 51	8.3	F8	0.4	.010	.007	g
1081[g]	1 14.7	− 1 2	6.01	*G3*	1.0	.018	.009	g
1^h19	1 14.7	−35 1	7.83	F0	0.2	.0061	.003	F
1087	1 15.0	−16 20	6.75	G0	0.2	.027	.018	g
1097	1 16.1	+11 1	6.89	F0	0.5	.0089	.005	f
1^h22	1 16.5	−57 52	7.10	F8	2.5	.027	.019	F:
1107	1 16.6	+72 19	7.16	A0	3.2	.0080	.004	p
1102	1 17.0	−24 39	8.4	G0	0.4	.019	.014	F
1105	1 17.0	+57 37	6.45	F5	0.2	.0094	.005	g
1106	1 17.7	−19 36	6.44	F5	2.3	.014	.008	F:
1113	1 18.8	−24 53	6.66	A5	1.86	.011	.006	F:
1120	1 18.8	+42 39	10.2	....	0.0	.013	.011	f
1^h26[6]	1 19.0	−70 14	6.96	F2	0.3	.038	.029	F
1126	1 19.2	+45 5	8.22	F0	0.0	.0094	.006	g
1128	1 19.4	+42 46	8.8	B9	0.0	.0034	.002	g
1131[a]	1 20.0	− 6 28	6.78	*F1*	3.8	.026	.016	p
1148	1 21.7	+ 3 1	6.42	B8	1.1	.024	.016	g
1158[g]	1 22 4	+ 4 51	7.33	*G0*	0.3	.0072	.003	g
1477[b]	1 22.6	+88 46	2.12	*cF7 F1*	6.67	.016	.006	f
1183	1 24.2	+22 19	6.75	A0	0.2	.012	.007	f
1187	1 25.0	+12 8	9.1	F0	0.5	.022	.019	g
1200	1 25.8	+57 54	8.9	B8	0.9	.022	.019	g
1199[g]	1 26.1	+14 50	3.72	*G3*	7.0	.0083	.003	f
1217	1 27.2	+69 23	9.2	K0	1.0	.0073	.004	p
1^h31[6]	1 27.6	−53 53	7.92	F5	0.3	.021	.016	F
1224	1 28.5	+35 40	8.3	A2	2.7	.0095	.006	p
1227	1 28.8	+34 9	9.4	....	0.0	.0054	.003	p
1232	1 29.3	− 8 58	8.20	F5	0.3	.0042	.002	f
1238	1 29.6	+12 3	7.02	A3	1.1	.025	.018	f
1254	1 30.8	+ 7 8	6.94	F8	0.0	.017	.011	g
1^h35	1 31.6	−30 25	5.68	F0	1.12	.037	.025	g
1287	1 33.1	+40 35	8.7	F8	0.7	.011	.007	f
1294	1 33.6	+26 27	9.3	F8	0.5	.0097	.007	p
1297	1 34.0	− 1 19	9.4	G5	1.3	.0095	.007	g
1309	1 34.2	+54 26	7.50	A0	0.6	.019	.013	f
1321	1 35.6	+10 47	8.9	G5	0.0	.017	.013	f
1323	1 35.7	+ 1 7	8.49	F8	0.0	.011	.007	f
1339	1 36.8	−11 49	5.84	*F2 F3*	1.6	0.035	0.024	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
1345[2]	1h37m4	− 7°16′	8.01	F2	0.5	0″013	0″009	g
1359	1 37.7	+57 2	6.14	A2	1.4	.024	.016	f
1371	1 38.4	+56 37	9.5	....	0.3	.021	.017	f
1360	1 38.5	+ 9 4	8.9	Ko	0.3	.022	.018	g
1368	1 38.6	+39 28	8.1	F5	1.5	.012	.008	f
1411	1 38.8	+80 23	7.23	Ao	0.2	.0098	.006	g
1380	1 39.0	+63 19	8.5	G5	0.6	.011	.007	f
1369	1 39.0	+ 8 59	8.3	F2	0.4	.012	.008	g
1392[9]	1 40.2	+44 9	8.6	A5	1.5	.0082	.005	g
1394	1 41.0	−25 33	5.39	*F1*	4.0	.055	.042	f
1406	1 41.0	+32 41	8.1	F8	0.5	.016	.011	g
1415	1 41.5	+52 47	9.9	....	0.5	.0073	.005	g
1437	1 42.9	+56 15	8.9	....	0.5	.0093	.006	g
1438[0]	1 43.3	+47 24	5.99	A2	0.5	.015	.008	g
1h48	1 43.5	−44 28	7.9	F5	0.3	.014	.009	F:
1449	1 44.0	+32 34	9.2	....	0.0	.0074	.005	f
1447	1 44.1	−18 52	8.3	G5	3.5	.012	.008	p
1457[R]	1 44.6	+21 47	5.89	*Ko*	1.2	.013	.006	f
1458	1 45.0	− 4 37	9.3	....	0.0	.012	.009	f
1473	1 45.7	+24 10	6.89	A5	0.2	.0080	.004	g
1504	1 46.2	+75 44	7.02	A5	0.8	.011	.006	f
1487	1 46.8	+10 19	7.67	Fo	0.0	.014	.009	g
1500	1 47.3	+36 49	7.04	F5	1.0	.0068	.003	f
1502	1 47.3	+44 8	8.8	Go	0.0	.0090	.006	p
1503	1 47.8	+14 55	8.3	F5	0.7	.016	.011	g
1509	1 47.8	+59 26	8.0	F5	0.0	.011	.007	f
1507	1 48.0	+18 48	4.04	Aop	0.08	.035	.019	g
1505	1 48.4	−17 14	8.5	G5	1.0	.012	.008	f
1531	1 49.4	+60 47	7.34	Ao	0.0	.017	.012	g
1522	1 49.4	+28 18	7.27	F2	0.7	.0085	.005	g
1530	1 50.2	+ 2 28	9.4	....	2.0	.024	.021	g
1533	1 50.4	+ 1 17	9.4	....	0.3	.020	.017	g
1548	1 51.3	+30 32	7.71	F5	1.5	.012	.008	g
1562	1 52.1	+32 41	8.3	....	0.3	.015	.011	f
1h56[8]	1 52.1	−60 48	6.55	Fo	0.04	.027	.018	F
1588[b]	1 52.9	+75 0	6.64	Ao	1.5	.012	.006	g
1567	1 52.9	− 2 33	6.57	Ao	5.0	.017	.011	p
1587	1 53.7	+58 3	8.0	Fo	2.6	.019	.014	p
1579	1 53.7	+24 21	7.8	A3	0.3	.0070	.004	g
1582	1 54.0	+20 31	9.3	Go	2.5	.012	.010	g
1606	1 54.4	+73 21	6.24	A3	2.3	.0082	.004	g
1610	1 56.0	+23 37	9.0	Go	0.5	.016	.012	f
1613	1 56.2	+36 14	8.0	G5	0.2	.032	.027	g
1h61[6]	1 56.8	−44 19	8.01	F8	1.2	.012	.008	F:
1615[b, d]	1 56.9	+ 2 17	3.94	*A2n A3n*	0.90	0.043	0.021	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
1632	1h 57m.7	+60° 2'	8.06	G0	2.1	0".012	0".008	g
1631	1 58 0	+25 28	5.68	*F4*	2.2	.034	.023	g
1655	2 1.3	−22 38	8.4	F5	0.5	.0066	.004	F:
1664	2 2.1	+15 7	8.49	A0	1.8	.010	.007	p
——[1]	2 2.3	−71 44	9.3	G0	0.2	.088	.10	F:
1673	2 3.7	− 0 54	7.8	F0	1.0	.0092	.005	p
1682	2 3.9	+61 53	7.7	F0	0.5	.0031	.001	f
1678	2 4.2	+19 52	8.00	A3	1.2	.016	.012	f
1681	2 4.4	+32 53	7.70	A0	0.8	.021	.015	f
1683	2 4.8	+38 34	5.58	A0 *A0n*	0.66	.018	.010	f
1689	2 5.4	+13 14	7.9	G5	0.5	.026	.020	g
1697[a, c, g]	2 6.6	+29 50	5.20	*G4 F2*	1.56	.018	.007	g
1696	2 6.6	+23 30	7.61	G5	1.9	.015	.010	f
1703	2 7.7	− 2 52	5.72	*F9 G4*	1.89	.049	.036	g
2h5	2 7.8	−49 48	8.07	F0	0.6	.015	.010	F:
1711	2 8.1	+34 3	8.7	A	1.4	.010	.007	f
1723	2 8.9	+29 57	7.21	B8	0.05	.0072	.004	p
2h6	2 9.5	−62 7	7.9	F5	0.5	.025	.019	F
1737	2 10.0	+60 53	8.08	G0	0.9	.021	.016	g
1738	2 10.3	+55 27	8.5	F8	0.5	.0082	.005	f
1729[9]	2 10.6	+ 6 10	9.0	K2	0.0	.017	.012	f
1733	2 11.0	−18 41	7.92	*K3 K3*	1.0	.068	.065	g
1749	2 11.5	+23 26	7.9	F0	0.5	.0048	.002	f
1752	2 11.6	+28 18	6.61	F5	1.5	.025	.017	f
1758	2 11.9	+21 46	8.6	F5	0.2	.0072	.004	f
1762	2 12.0	+38 52	9.4	....	0.1	.013	.010	g
1763	2 12.4	+39 49	7.12	F2	1.6	.011	.006	p
1769	2 12.5	+53 11	8.8	A0	1.5	.0054	.003	f
2h9	2 12.6	−33 28	8.6	G0	0.1	.011	.007	F:
2h10	2 13.1	−31 11	7.64	F5	1.0	.021	.015	F:
1780	2 14.3	+29 21	8.5	F5	0.0	.018	.014	g
1786	2 14.8	+42 20	8.8	K0	0.0	.032	.028	g
1798[g]	2 15.5	+38 56	8.9	G5	0.8	.0060	.003	p
1801	2 16.1	+ 8 25	7.8	F0	0.9	.011	.007	f
1824[10]	2 18.0	+33 3	10.2	....	0.3	.024	.020	p
1833	2 18.2	+61 4	7.11	B8	0.5	.014	.008	g
2h15	2 18.9	−30 19	6.94	F8	0.3	.017	.011	F
1839	2 19.8	− 2 34	9.0	F2	0.5	.015	.012	f
2h17	2 20.4	−30 19	7.9	F5	0.3	.0062	.003	F
1860[b]	2 20.8	+66 57	4.64	*A3sp F5*	2.26	.027	.014	g
1850	2 20.8	+31 40	9.8	....	0.5	.0079	.006	f
1851	2 21.1	+30 50	8.1	F5	2.5	.018	.014	f
1868[g]	2 22.3	+29 26	7.8	G0	2.3	.0033	.001	f
1874	2 22.8	+16 55	8.6	F5	0.0	.022	.018	g
1882	2 23.4	+ 8 13	9.1	G0	0.0	0.0084	0.006	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
1904	2h 24m.8	+24° 48′	5.86	F5	4.5	0″.032	0″.022	g
1913	2 25.1	+42 7	7.7	A0	0.1	.0064	.003	f
1932	2 25.8	+59 33	7.46	A0	4.0	.0080	.005	f
1933	2 25.8	+58 0	7.46	A2	0.0	.013	.008	f
1924	2 26.3	+ 0 40	6.75	A2 A	0.5	.023	.016	g
1938	2 26.4	+51 52	6.51	*A2s*	0.5	.0061	.003	f
1941	2 27.4	+ 3 18	8.0	F0	0.1	.0037	.002	f
1940	2 27.4	+ 5 54	8.8	G5	0.0	.014	.010	f
1959[9]	2 29.0	+39 51	8.7	F8	0.2	016	.012	f
1953[g]	2 29.1	− 6 4	7.29	K2	0.2	013	.006	f
1985	2 30.1	+68 52	8.02	A2	0.3	.0066	.004	f
1971[g]	2 30.6	+ 5 9	5.02	*G5*	4.68	.011	.005	f
1992	2 31.7	+45 38	8.2	F5	0.1	.0035	.002	f
1993	2 32.3	+19 37	8.6	F0	0.0	.0086	.006	g
2017[g]	2 32.8	+65 12	8.2	G5 G5	0.0	.0044	.002	f
2014[g]	2 32.8	+61 3	7.75	G5	0.5	.0063	.003	f
2004[11, g]	2 32.9	+32 59	7.05	*K2*	0.7	.011	.005	g
2018	2 33.1	+60 51	7.8	F0	2.0	.032	.026	f
2008[g]	2 33.5	+14 26	7.30	*K3*	2.3	.012	.006	f
2020	2 33.8	+33 32	7.75	G5	2.3	.013	.008	f
2040	2 34.5	+55 3	7.56	A2	1.2	.0081	.005	g
2034	2 34.9	+26 12	8.1	F5	1.6	.025	.020	g
2042	2 35.5	+18 23	6.89	B9	0.3	.0090	.005	f
2043	2 35.7	+ 4 26	7.8	F0	2.2	.016	.011	f
2044	2 36.0	−24 34	7.45	F5	0.5	.0061	.003	F
2046	2 36.1	− 1 8	5.73	*F6*	3.2	.044	.032	g
2081	2 37.3	+48 48	4.22	*F5 M3*	5.63	.12	.099	g
2093[g]	2 38.0	+57 13	8.6	G5	0.6	.0052	.002	p
2080	2 38.1	+ 2 49	3.58	*A2n F4*	2.47	.019	.009	g
2091	2 38.6	+29 3	7.06	F0	0.2	.018	.012	g
2092[g]	2 39.4	−28 19	7.63	G5	2.0	.0047	.002	F
2h28	2 39.5	−40 58	6.34	A0	0.22	.015	.009	f
2096	2 39.6	−28 52	8.5	F8	0.3	.012	.008	F:
2115	2 40.5	+53 32	7.44	A0	1.0	.012	.008	f
2110	2 40.6	+19 32	9.4	....	0.2	.012	.009	f
2117	2 40.9	+35 9	6.34	F2	2.1	.021	.013	f
2111	2 41.0	− 5 22	7.9	F2	2.6	.020	.016	f
2122	2 41.8	+18 58	6.95	*F7*	0.9	.045	.036	g
2128	2 41.8	+30 39	8.06	G0	2.0	.011	.007	f
2h37	2 44.6	−37 49	6.72	F2	1.14	.030	.021	F:
2173	2 44.9	+45 35	9.2	*G3*	1.5	.024	.021	g
2185	2 45.8	+52 35	6.42	B9	0.2	.011	.006	g
2180	2 45.9	+36 53	9.4	F8	0.2	.040	.041	g
2204	2 46.2	+72 29	7.66	*G1*	0.9	.029	.023	g
2186[5, g]	2 47.2	−21 42	8.3	G0	0.0	0.0061	0.003	F:

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h₁	d	Grade
2^h43	$2^h47^m.2$	−45° 39′	8.5	F8	0.2	0″.012	0″.008	F:
2193	2 47.4	+ 6 3	7.9	B9	0.0	.010	.006	f
2218	2 49.7	+26 28	7.45	*K2 M1*	2.2	.058	.051	g
2^h45	2 50.0	−60 20	8.9	F8	1.3	.027	.023	F:
2236	2 52.1	+ 1 29	7.8	G0	0.1	.024	.018	g
2237[g]	2 52.1	− 0 59	7.08	G5	2.0	.0076	.004	f
2^h47	2 52.7	−39 51	7.98	G0	0.4	.021	.015	F:
2242[6]	2 52.8	−25 22	8.0	G5	0.2	.027	.020	F:
2294[g]	2 52.8	+79 1	5.66	*M1*	3.2	.019	.008	p
2246[g]	2 52.8	+23 45	7.62	K0	0.1	.0053	.002	g
2253	2 53.2	+21 13	6.68	A3	0.0	.013	.008	g
2257	2 53.5	+20 56	4.64	*A4s A4s*	0.30	.023	.012	g
2261	2 54.1	+ 6 15	7.38	F0	0.5	.011	.006	g
2273	2 54.1	+39 10	8.7	F0	0.4	.0063	.004	f
2^h50[a]	2 54.5	−40 42	3.06	A2	1.00	.037	.016	g
2279	2 54.9	+17 36	6.94	A0	3.0	.014	.008	f
2286[g]	2 55.4	+32 1	6.71	G5 A5	1.5	.014	.007	f
2283	2 55.5	−11 41	8.9	G5	0.0	.0073	.004	f
2301	2 56.6	+17 55	9.8	....	0.7	.018	.015	g
2316	2 58.0	− 2 29	7.48	*F7*	2.0	.032	.025	f
2336	2 59.6	+24 52	5.36	B8	0.00	.0090	.004	g
2343	3 0.4	+39 59	9.5	....	3.4	.027	.025	f
2358	3 2.0	+ 8 1	8.5	F5	0.8	.011	.007	f
2371	3 2.5	+63 25	7.38	F8	0.7	.0066	.003	f
2377	3 2.7	+71 11	7.68	F8	0.0	.027	.021	g
2403	3 4.0	+78 8	7.80	K0	1.8	.022	.016	f
2373[9]	3 4.3	+ 4 49	8.6	G0	0.0	.022	.017	g
2375	3 4.4	+21 23	7.8	F0	0.0	.013	.009	g
2390	3 5.8	+36 49	7.43	G0	0.2	.024	.017	g
2394	3 5.9	+36 36	8.8	A0	2.7	.033	.032	f
2397	3 6.2	+43 55	8.1	G0	0.2	.010	.006	f
2409[12]	3 7.1	+38 46	8.2	*F3*	0.0	.033	.028	f
2406	3 7.7	− 1 34	5.14	*F8*	6.4	.062	.048	f
———	3 7.7	−30 39	8.8	A5	1.5	.011	.008	F:
2402	3 7.8	−29 23	3.95	*F5*	3.0	.077	.053	g
2436	3 8.8	+65 17	6.35	*A0n*	0.6	.012	.006	g
2416	3 8.9	+ 0 23	8.1	*F8*	0.0	.022	.017	g
2443	3 10.6	+40 7	6.44	A0	0.8	.0086	.004	p
2440	3 11.0	− 6 17	6.02	*B8n*	0.2	.0077	.003	p
2446	3 11.3	+38 16	7.30	G0	0.8	.019	.013	g
2454	3 11.8	+46 40	8.4	F0	2.1	.017	.012	p
2459[g]	3 13.3	− 1 17	5.62	*K0*	2.5	.023	.012	g
2461	3 13.6	−19 26	8.0	F8	0.6	.020	.014	F:
2464	3 13.8	− 1 23	10.0	G5	0.0	.013	.011	g
2463[5,7,g]	3 14.0	−22 53	5.05	*G6*	2.6	0.0092	0.004	F

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
2465	3h 14m.1	−18° 55′	5.83	*F2*	2.9	0″.046	0″.034	f
——	3 14.5	−63 39	8.6	F5	0.3	.011	.007	F:
2475	3 14.7	+19 21	7.67	A2	0.1	.0098	.006	f
2472[?, K]	3 15 1	−22 7	3.95	*M3*	6 0	.026	.010	F:
2491	3 16.3	+ 8 24	8.3	Go	1.0	.018	.013	g
2499	3 16.8	+29 27	7.8	*A5s*	0.0	.040	.035	f
2504[K]	3 17.6	+20 37	7.35	G5	1.7	.0063	.003	g
2507	3 18.4	− 8 9	6.28	Go	5.6	.052	.043	f
2538	3 20.2	+59 54	6.48	B8	1.4	.011	.006	g
2537	3 20.4	+54 50	8.7	A2	0.0	.0053	.003	p
2531	3 20.7	+12 8	8.1	Go	1.5	.014	.010	g
2564	3 21.1	+71 31	8.5	Ao	2.8	.022	.019	g
3h25	3 21.2	−46 1	7.19	A2	1.6	.0052	.003	F
2563	3 22.2	+59 2	6.06	Ao	1.0	.0087	.004	f
2546	3 22.3	+20 7	6.68	A3	1.0	.014	.008	g
2559[b]	3 22.4	+44 42	7.37	B3	1.3	.0074	.003	f
2554	3 22.5	+28 43	8.7	F8	1.5	.011	.008	g
2577	3 25.2	−11 29	8.3	F5	2.5	.022	.017	f
2582	3 25.3	+27 14	5.93	Ao	0.31	.016	.009	f
2584	3 25.5	+19 26	7.9	F8	0.0	.012	.007	f
2580	3 25.6	+ 4 50	7.9	Fo	2.0	.013	.009	f
2581	3 25.6	− 4 38	7.55	A3	0.2	.011	.007	g
2612	3 26.8	+59 43	6.48	*F4*	1.0	.031	.022	g
2613	3 28.3	+ 6 44	9.3	A2	0.2	.0045	.003	f
2618	3 28.7	+19 27	8.1	Ao	0.0	.020	.015	f
2625	3 29.1	+33 19	7.8	Fo	0.0	.0078	.004	f
2628	3 29.4	+31 21	6.83	Fo	0.0	.017	.011	g
2630	3 29.4	+42 1	8.4	G5	0.4	.027	.023	g
2639	3 31.1	+ 9 51	9.3	Go	0.2	.013	.010	g
2644[a]	3 31.7	+ 0 16	6.12	*G9 K5*	2.72	.075	.055	g
2678	3 32.6	+69 32	7.13	A3	0.0	.0098	.005	p
2668	3 33.8	+33 48	6.85	F5	0.0	.030	.022	g
2680	3 34.3	+45 35	8.4	F8	0.2	.0095	.006	f
2679	3 34.5	+28 27	6.86	Ao	0.48	.014	.008	p
2710	3 36.5	+52 9	9.2	A	0.1	.0084	.006	p
2737	3 37.8	+57 15	9.2	A	0.3	.0047	.003	f
2726[a]	3 38.0	+31 58	3.94	*B1s*	4.5	.013	.005	f
2730	3 38.3	+31 51	8.4	*B3*	0.2	.010	.006	f
2740	3 39.1	+22 25	9.4	G5	0.5	.011	.008	f
2765	3 39.9	+64 27	8.8	F5	0.2	.013	.010	g
2757	3 40.1	+41 10	7.7	*G7 K2*	0.6	.046	.040	g
2755	3 40.4	+23 53	8.2	A3	1.0	.0089	.006	g
2760	3 41.1	+24 31	8.6	A2	2.0	.0065	.004	p
2756	3 41.2	−28 11	7.9	Ko	3.9	.056	.052	f
2772[a]	3 41.5	+33 18	6.36	B3	2.5	0.010	0.005	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
2791	$3^h42^m.3$	$+59°49'$	8.6	B5	1.0	0″.013	0″.009	f
2778	3 42.8	+10 50	5.03	B3	4.08	.020	.011	f
2785	3 43.4	+11 24	8.6	F0	0.8	.0066	.004	f
2793	3 43.9	+18 0	9.3	....	0.3	.018	.014	f
2801	3 44.4	+22 23	8.7	F5	0.0	.0065	.004	f
3^h44	3 44.9	−37 55	4.35	B8 A0	0.56	.052	.032	f
2815	3 45.4	+40 30	7.52	A2	0.5	.011	.007	f
2828[9]	3 46.1	+52 59	7.9	G0	0.4	.016	.011	g
2818	3 46.5	+ 0 2	9.03	K0	2.8	.054	.054	p
2845[g]	3 47.4	+56 12	8.2	G5	2.6	.0074	.004	f
2832	3 47.7	− 5 39	5.49	*B7n*	4.8	.028	.018	f
2843	3 47.8	+31 35	2.91	*B1n*	6.39	.028	.012	p
2867[g]	3 48.6	+60 49	5.22	*K4* A0	3.2	.0091	.003	p
2849[3]	3 49.2	+ 4 54	9.3	....	0.0	.0061	.004	f
2850[g]	3 49.2	− 3 14	4.68	*G4 A1n*	1.38	.015	.006	g
2894	3 52.5	− 1 26	8.7	K2	2.0	.086	.095	f
2910	3 53.1	+38 31	6.38	A0	2.7	.0099	.005	p
3^h55	3 53.1	−40 12	7.74	A3	0.75	.0083	.005	F:
2963	3 53.3	+80 25	5.25	*F1* A2	0.9	.022	.013	g
2923[3]	3 54.8	+ 8 1	9.8	....	0.1	.012	.009	f
2926	3 55.1	+22 57	6.54	B9	0.9	.012	.006	p
2959	3 57.4	+39 14	7.18	*G5*	1.8	.039	.031	g
3^h60	3 58.2	−34 46	6.67	G0	0.4	.035	.026	f
2952	3 58.3	−28 48	7.55	F8	0.4	.019	.013	F
2980	3 59.5	+43 9	7.46	G5	0.0	.014	.009	g
4^h2	4 0.9	−85 34	6.46	B9	1.5	.015	.008	F:
2995	4 1.0	+37 49	6.81	*K2*	1.29	.064	.053	g
4^h4[g]	4 1.2	−35 43	7.3	G0	0.3	.0040	.002	F:
2993	4 1.4	+ 5 26	8.4	F8	0.5	.011	.007	f
3007	4 1.8	+45 58	7.07	A0	0.3	.0074	.004	g
3014	4 1.9	+52 19	9.9	....	0.8	.0067	.005	f
2999	4 2.0	+14 54	5.94	*F5*	2.8	.013	.007	p
3006[g]	4 2.3	+17 4	6.13	*K5*	3.1	.014	.006	p
3015	4 2.4	+39 53	7.47	F8	2.5	.026	.020	g
3017	4 2.7	+28 56	8.6	G0	0.9	.013	.009	f
3021	4 3.0	+28 24	9.0	G0	0.3	.012	.009	g
3019	4 3.0	+22 52	6.81	A3	0.0	.0088	.005	f
3032	4 4.5	− 8 12	7.28	A2	0.0	.0051	.003	g
3038	4 4.6	+41 36	8.0	F8	0.0	.0095	.006	f
3044	4 5.0	+53 32	9.7	....	0.7	.012	.009	f
3041[g]	4 5.8	− 5 8	7.50	G0	0.0	.012	.006	f
3054	4 7.0	+ 0 29	6.76	G5	3.0	.032	.024	f
3062	4 7.0	+42 20	8.0	F0	1.8	.013	.008	f
3065	4 8.1	+ 2 37	8.4	F5	0.0	.0094	.006	f
3064	4 8.2	+ 7 28	5.35	*F0*	0.3	0.026	0.016	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
3072[g]	4h 8m.5	+ 9° 1′	4.98	*G5*	2.5	0″.0088	0″.003	p
3080	4 8.6	+45 9	8.02	G5	0.0	.018	.012	f
3069	4 9 0	−28 48	7.44	F8	0.1	.020	.014	F
3098	4 9.6	+58 32	6.91	A0	0.5	.013	.008	g
3079[g]	4 9.6	−10 30	5.13	*K2*	3.74	.031	.016	g
3105	4 9.9	+60 14	7.41	B9	0.4	.0060	.003	f
3095	4 10.9	+ 0 12	7.18	A0	1 7	.017	.011	p
3102	4 11.2	+19 25	7.6	F5	0.0	.0076	.004	p
3108	4 11.7	+18 57	9.4	....	2.2	.011	.008	f
3114	4 12.3	+22 34	7.48	F5	0.0	.0088	.005	g
3127	4 14.2	− 7 40	8.3	F0	2.8	.028	.024	f
3141[g]	4 14.3	+49 48	7.37	G0	2.1	.0084	.004	f
3147	4 14.8	+51 22	8.0	A0	0.0	.0079	.005	f
4h13	4 14.8	−61 12	6.32	A0	1.0	.023	.015	F
4h14	4 15.3	−34 9	6.34	A2	1.8	.013	.007	F:
3158[a, g]	4 16.5	+20 35	6.11	*M0*	3.1	.0091	.003	p
3172	4 16.6	+42 12	5.98	B9	0.5	.0053	.002	g
3163	4 17.2	− 4 54	7.51	A5	0.6	.013	.009	g
3189	4 17.3	+60 2	8.26	F0	0.5	.0062	.004	f
3174	4 17.8	+11 9	6.90	A2	1.5	.027	.020	g
3188	4 18.1	+33 44	5.81	*F5*	2.8	.054	.042	g
3187	4 18.1	+29 54	9.0	A2	0.0	.0050	.003	f
3182	4 18.4	+ 9 14	5.06	*A3n*	0.0	.0084	.004	g
4h21[6, g]	4 18.5	−43 2	7.62	G5	2.6	.010	.005	F
3207	4 18.7	+55 26	7.26	F0	1.2	.0097	.006	g
3197	4 19.9	− 8 59	8.3	G0	0.9	.022	.017	g
3227	4 20.7	+53 42	8.5	F5	0.0	.0080	.005	p
3258	4 21.1	+72 26	8.8	....	0.6	.010	.007	f
3224	4 21.2	+18 52	7.72	*F5*	1.8	.028	.022	g
3228	4 22.0	+10 59	5.84	B8	3.9	.010	.005	f
4h27	4 22.3	−57 17	6.54	G0	0.30	.055	.044	g
3243	4 22.5	+30 9	6.26	*F4*	1.68	.024	.016	f
3230	4 22.8	−24 18	6.14	A2	0.5	.028	.019	g
4h30	4 23.3	−40 45	6.78	A0	1.8	.0081	.004	F:
3241	4 23.3	−24 41	8.8	G0	0.6	.021	.017	F:
3247[5]	4 23.6	−21 44	6.73	F5	0.7	.019	.012	F
3274[a]	4 24.1	+53 42	5.42	B1	0.75	.015	.006	f
3294	4 24.7	+70 31	8.6	F2	1.0	.0066	.004	g
4h32[g]	4 27.6	−35 59	7.24	K0	0.0	.0072	.003	F
3297	4 27.8	+17 48	6.24	B8	0.0	.0053	.002	f
4h33	4 27.9	−36 7	7.7	F0	0.8	.013	.008	F
3324	4 29.7	+40 53	7.8	A2	1.7	.019	.014	f
3321[g]	4 30.2	+16 19	1.06	*K5*	11.8	.11	.048	p
3321[10]	4 30.2	+16 19	10.5	....	2.4	.028	.026	f
3326	4 30.7	+ 8 1	8.0	A0	0.0	0.0090	0.005	f

TABLE 53—*Continued*

ADS	α_{1900}	δ_{1900}	Mag.	Sp.	Δm	h_1	d	Grade
3329	$4^h30^m.7$	$+19°34'$	7.4	A2	0.0	0″.0082	0″.005	g
3330	4 30.8	+19 17	8.4	G0	0.5	.023	.019	g
3336	4 30.9	+44 15	9.0	....	1.3	.011	.008	f
3338[g]	4 31.1	+41 56	7.25	*K2*	2.0	.010	.005	g
3358	4 32.0	+53 17	5.44	*A5n*	2.0	.0078	.004	g
3341	4 32.4	−17 45	9.3	....	0.0	.011	.008	f
3353	4 32.4	+26 45	6.49	*F3 F0*	0.0	.030	.021	g
3355	4 33.4	−13 14	6.77	A0	0.45	.016	.010	p
3389	4 34.6	+57 0	8.0	F8	0.0	.017	.012	g
3391	4 34.6	+59 20	6.53	A3	0.0	.0089	.005	g
3376	4 35.4	−20 5	9.3	G0	0.5	.017	.013	F:
3390	4 35.5	+37 20	7.8	F8	0.0	.024	.019	g
3387	4 35.8	+16 31	8.2	A2	0.0	.0059	.003	g
3395	4 37.0	+ 2 48	7.8	A0	0.0	.038	.032	f
3399	4 37.1	+20 6	9.1	....	0.3	.042	.036	f
3429	4 37.4	+57 5	9.2	....	2.0	.036	.033	g
4^h43[6]	4 38.1	−45 54	8.73	A5	0.2	.0087	.006	F:
4^h44	4 38.6	−59 8	6.38	G0	0.3	.075	.063	g
3409[b, g]	4 38.8	− 8 59	5.96	*G6 F3*	0.08	.017	.008	f
3417	4 39.5	+ 5 7	8.16	*G7* K1	0.0	.045	.041	g
3438	4 40.4	+43 13	8.7	F0	0.0	.011	.008	g
3430	4 41.2	− 7 11	8.1	G0	2.8	.018	.014	f
3459	4 43.5	−20 59	8.6	A0	1.3	.0028	.001	F
3465	4 44.4	+ 2 2	7.6	A0	0.0	.0072	.004	f
3484	4 44.7	+ 1 1	9.1	....	0.2	.022	.017	f
3472	4 45.3	+ 0 53	8.6	F0	0.8	.0094	.006	f
4^h49[1]	4 46.0	−56 11	9.1	F0	0.7	.052	.056	F
——[6]	4 47.4	−54 4	7.61	G5	0.4	.048	.041	F
3497	4 47.8	− 5 27	8.1	F8	0.1	.0028	.001	f
3514	4 48.8	+ 7 13	7.9	*K2 K1*	0.1	.022	.015	p
3551	4 49.0	+69 56	8.49	G0	0.8	.0089	.006	f
3517	4 49.5	+ 8 27	6.77	A0	2.0	.012	.007	f
3832	4 50.8	+86 44	8.8	G	1.5	.015	.011	p
3542	4 51.0	+ 3 2	8.0	B9	0.5	.0069	.004	f
3573	4 51.9	+53 18	8.0	F5	0.0	.014	.009	f
3572[b]	4 52.5	+37 44	4.99	*A1s F9*	2.67	.037	.021	g
3567	4 52.6	+13 48	8.2	G0	1.0	.020	.015	f
3568	4 52.9	+ 1 31	8.2	F5	0.0	.018	.013	g
3589	4 53.4	+39 15	6.00	*F3*	3.7	.048	.037	g
3596	4 55.2	+ 4 57	7.95	G5	1.5	.022	.017	g
3600	4 55.2	+14 20	8.8	A5	0.4	.0095	.007	f
3608	4 55.6	+26 31	6.86	F5	2.6	.033	.024	p
3658	4 55.6	+73 27	8.0	A0	1.8	.013	.009	g
3623	4 56.8	+ 1 28	6.21	B8 A	1.2	.021	.013	f
3618	4 57.1	−23 51	7.51	F2	2.5	0.022	0.016	F:

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
3662	$4^h59^m.4$	−6° 10′	6.72	B9	1.0	0″.0096	0″.005	g
3672	4 59.6	+19 40	6.46	A2	0.6	.013	.008	g
3689	4 59.7	+54 52	7.86	B9	0.0	.0089	.005	f
5^h3[b, g]	5 0.8	−35 37	4.62	K0	3.8	.015	.005	F
3712	5 2.5	+ 8 15	9.0	K2	0.0	.012	.009	f
3734	5 3.5	+37 11	6.17	B2	0.30	.0040	.001	p
3730	5 3.5	+27 55	5.97	*F0*	2.0	.015	.008	f
3728	5 3.7	+ 3 5	6.54	A0	0.5	.0058	.003	f
5^h5	5 3.8	−74 29	6.97	A0	0.3	.014	.008	F:
3744	5 4.5	+31 55	7.9	G5	0.7	.024	.019	g
3737	5 4.5	+ 8 3	7.09	F8	2.8	.022	.016	g
3768	5 5.9	+41 28	8.7	A0	0.0	.0052	.003	f
3764	5 6.6	+ 0 55	6.07	F5	1.5	.0050	.002	g
3812[g]	5 7.8	+47 3	6.97	G5	3.3	.0094	.005	f
3797[b, g]	5 8.1	+ 2 45	4.64	*K3*	3.80	.0040	.001	p
3799	5 8.3	+ 1 51	6.25	A2 G	0.2	.0054	.002	f
3824[13, b, d]	5 8.8	+32 35	5.14	*A7s F4*	2.89	.017	.007	f
3823	5 9.7	− 8 19	0.34	*cB8*	6.32	.016	.005	g
3837	5 9.9	+19 36	8.1	F0	0.9	.0087	.005	f
3853	5 10.5	+33 13	7.6	A3	0.0	.0076	.004	g
5^h17[6]	5 10.8	−47 59	8.07	F0	0.3	.018	.013	F:
3870	5 10.8	+52 43	8.2	F5	0.5	.013	.009	g
3854	5 10.9	+18 20	7.5	B3	0.5	.011	.006	g
(2615)[f, 14]	5 11.2	+25 58	8.8	....	0.5	.046	.039	f
3856	5 11.2	− 3 37	8.6	B9	0.4	.012	.009	f
3857	5 11.4	− 1 45	8.5	B8	0.3	.0058	.003	g
3866[a]	5 11.6	+20 1	6.84	*F4 K2*	3.0	.032	.021	f
3850	5 11.7	−29 38	8.2	A2	0.6	.012	.008	F
3880	5 12.8	+ 3 35	8.8	F5	0.7	.017	.013	f
3900[a]	5 14.1	− 3 11	7.87	*K5*	4.0	.089	.090	f
3955	5 15.0	+64 38	7.65	F8	1.0′	.0099	.006	f
5^h23	5 15.2	−41 9	7.69	F5	0.6	.019	.013	F:
3956	5 15.3	+63 17	7.27	G0	0.3	.030	.023	g
3936	5 16.0	+12 34	7.8	A3	0.0	.0078	.004	f
3959	5 17.4	+ 2 31	8.3	G5	2.5	.047	.043	p
3954[g]	5 17.7	−24 52	5.14	*G2* A3	1.22	.022	.011	g
3981	5 17.9	+24 52	8.2	A2	0.0	.0075	.004	f
3978	5 18.5	− 8 30	5.83	A0	1.5	.022	.013	f
3991[1, 2, 15]	5 18.8	− 0 58	6.11	F5	0.3	.031	.021	g
4002[u]	5 19.4	− 2 29	3.44	*B1s*	1.0	.014	.005	g
5^h34	5 19.9	−36 46	8.3	G5	0.6	.037	.032	F
4011	5 20.0	− 9 19	9.2	A2	0.4	.0061	.004	p
4012	5 20.0	+ 1 50	8.5	B9	0.9	.011	.008	p
4020	5 20.4	− 0 38	6.25	B9	0.6	.022	.013	g
4032	5 20.7	+27 32	7.8	F8	0.0	0.015	0.010	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
4076	5ʰ 20ᵐ.8	+70° 44′	8.0	G5	0.5	0″.015	0″.010	g
4042	5 22.1	−20 48	7.34	A3	3.0	.014	.009	F:
——	5 22.3	−35 26	7.5	F5	0.6	.013	.008	F:
——	5 22.5	−52 24	6 32	A0	0.3	0090	.004	F:
4022	5 22.8	− 4 5	9.9	G5	0.5	.010	.008	g
4068	5 23.1	+25 4	5.44	*B9n*	0.77	.024	.014	g
4083	5 23.2	+41 13	7.6	A0	0.7	.0046	.002	f
4078	5 23.9	− 3 24	6.17	B9	0.3	0065	.003	p
4066[7, g]	5 24.0	−20 50	2.96	*G1*	6.6	.055	.029	F
4115	5 25.4	+ 5 52	4.32	B3	1.5	.020	.010	g
4123	5 26.0	+ 3 13	5.52	*B2s*	1.3	.0037	.001	p
4131[b, d]	5 26.4	+16 59	5.49	B9	0.37	.020	.009	f
5ʰ41	5 27.5	−68 42	6.15	F0	0.3	.020	.012	F
4150	5 28.1	− 1 48	6.46	B3	1.6	.0053	.002	f
5ʰ46	5 28.1	−42 23	6.98	A5	0.6	.020	.013	F
4166	5 28.7	+30 51	8.0	F5	0.0	.013	.008	g
4153	5 28.9	−24 19	8.5	K0	0.3	.028	.023	F
4179[d]	5 29.6	+ 9 52	3.49	*O8n B2s*	1.90	.0045	.001	p
4176	5 29.7	− 4 28	8.0	B8	1.1	.0071	.004	f
4204	5 30.0	+41 46	7.10	F8	1.0	.017	.011	[illegible]
4187[b]	5 30.4	− 4 54	4 65	*B2s*	4.0	.020	.009	f
4200	5 30.4	+21 56	6.74	*F7 F6*	1.0	.030	.022	g
4193[b]	5 30.5	− 5 59	2.87	O9n B9s	4 44	.015	.005	f
4208[b]	5 30 9	+26 52	5.70	B8	0.1	.015	.007	g
——	5 31.2	−51 8	8.7	K0	0.2	.048	.046	F
4219[g]	5 31.2	+42 57	8.5	G0	0.0	.0054	.003	g
4223	5 32.0	+26 51	9.1	G0	0.4	.0073	.005	f
4236	5 32.1	+43 39	7.9	F5	0.0	.0060	.003	g
4229	5 32.2	+30 26	5.56	*F0*	0.4	.0084	.004	g
4224	5 32.4	+ 8 53	8.3	G0	0.8	.014	.010	f
4222	5 32.6	− 1 30	9.1	B8	1.0	.013	.010	f
4234	5 33.0	− 0 14	7.9	B9	0.2	.0043	.002	p
4243	5 33.1	+37 55	7.33	B9	0.7	.0087	.005	g
4241[b]	5 33.7	− 2 39	3.78	*O9s*	1.5	.0091	.003	g
4256	5 34 6	+15 18	6.66	*F0*	1.2	.031	.022	g
5ʰ67[16]	5 35.4	−46 9	7.8	G0	1.3	.042	.037	f
4258	5 35.5	−25 12	8.4	B9	1.5	.0064	.004	F:
4265[b]	5 35.5	+16 29	4.87	B3	0.2	.0082	.003	g
4263	5 35.7	− 2 0	1.91	*B0n B1n*	2.16	.013	.004	g
4271	5 35.9	+16 11	8.3	A0	2.2	.013	.009	f
5ʰ71	5 36.6	−50 11	7.9	G5	0.6	.016	.011	F
4289	5 36.9	+25 20	8.2	A0	1.0	.015	.010	f
4376	5 39.6	+62 46	6.13	A2	1.0	.017	.010	g
4349	5 40.1	+21 16	7.79	F2	0.4	.011	.007	g
4371[17]	5 41.0	+29 37	7.21	F8	0.0	0.027	0.019	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_t	d	Grade
4302[b]	$5^h 42^m.4$	+20° 50′	5.94	B9	1.5	0″.0041	0″.001	f
4388	5 42.5	+ 7 55	7.5	F8	0.8	.0080	.004	g
4390	5 42.6	+ 6 25	5.27	*A5n*	0.0	.011	.005	g
4396	5 43.1	+ 1 35	8.1	F0	0.0	.010	.007	g
4421	5 43.4	+31 45	6.72	A3	1.1	.0024	.0009	p
4432[k]	5 45.0	−14 30	5.57	*G6*	3.4	.019	.009	p
4452	5 45.3	+38 32	6.82	B8	1.1	.0093	.005	g
5^h86[6]	5 45.7	−48 57	7.25	A0	1.6	.019	.013	F:
4463	5 46.2	+34 25	7.7	F8	2.2	.012	.008	f
4490	5 48.5	+18 54	7.7	A0	0.0	.0085	.005	p
5^h90[k]	5 49.4	−38 33	6.74	K0	3.0	.0065	.003	F:
4499	5 49.6	+ 5 51	6.73	B9	2.5	.0080	.004	f
4516	5 50.3	+ 8 58	8.5	G0	0.5	.014	.010	f
4544	5 52.1	+29 37	7.81	B9	1.8	.012	.008	g
5^h91	5 52.3	−61 51	6.94	A0	0.4	.017	.011	F
4566	5 52.9	+37 12	2.71	*A1sp*	4.5	.056	.032	g
4557	5 53.4	− 4 39	6.85	G0	6.8	.033	.026	p
4762	5 53.8	+84 12	8.9	F8	0.0	.010	.007	g
4570	5 53.9	− 1 20	8.5	B9	1.0	.014	.010	g
5^h93	5 54.0	−41 46	7.4	F2	0.4	.023	.016	F:
4604	5 54.4	+58 13	9.2	....	0.0	.011	.008	g
4592	5 54.8	+36 31	7.8	A0	0.8	.012	.008	f
5^h97[18]	5 56.6	−31 3	7.85	K5	0.5	.080	.076	F
4615[b]	5 57.1	−10 36	4.97	B8	4.5	.0074	.003	p
4687[3]	6 0.9	+10 45	9.0	....	0.2	.0093	.006	g
6^h1	6 1.1	−41 9	7.7	F5	0.0	.011	.007	F
4676	6 1.2	−25 1	7.80	A0	0.2	.0044	.002	F
4682	6 1.5	−28 40	7.7	F2	0.1	.0083	.005	F:
6^h3	6 1.8	−45 5	5.82	F5	2.9	.058	.046	g
6^h4	6 2.2	−48 27	6.44	G5	0.2	.066	.055	g
4728	6 2.9	+13 59	7.4	B2	0.9	.0019	.0007	p
4730[a]	6 2.9	+17 25	8.4	G0	0.4	.0034	.002	f
4773	6 3.9	+48 44	5.64	A0	0.73	.018	.010	f
4768	6 4.4	+23 1	6.68	*B9n*	0.3	.0056	.003	g
– —	6 6.6	−52 7	7.7	F8	0.9	.011	.007	F
4814[t]	6 7.6	− 1 16	8.5	A	0.5	.032	.029	f
4811	6 7.6	−13 7	9.0	K0	0.5	.040	.039	g
4841[k]	6 8.8	+22 32	3.7	*M3*	5.8	.022	.008	g
4840	6 9.0	+14 38	6.82	B9	0.5	.0098	.005	p
4866	6 10.7	− 9 0	6.03	B9	0.0	.0075	.003	g
4957	6 11.4	+73 57	9.4	F8	0.5	.021	.019	f
6^h17	6 11.5	−61 27	6.75	B9	0.6	.0086	.004	F:
4950	6 13.2	+59 24	6.02	*A1n*	1.5	.0097	.005	g
4971	6 16.2	+ 2 19	6.25	A5	0.3	.030	.021	f
4984	6 16.2	+25 17	9.5	....	1.0	0.021	0.018	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_t	d	Grade
4991	$6^h 17^m.0$	+17° 37′	6.77	A0	1.0	0″.0071	0″.004	f
5023[5]	6 19 5	−19 44	6.56	B8	0.30	.0066	.003	F:
5062	6 20.9	+15 35	6.71	B9	2.5	.017	.011	f
5067[1]	6 21.6	+ 4 8	8.3	K0	0.0	.048	.044	f
5069[3, g]	6 21.6	+ 0 30	8.9	G5	0.7	.0047	.002	f
5088	6 21.9	+40 11	7.77	A0	2.1	.012	.008	f
6^h30	6 22.0	−34 59	7.15	A3	0.9	.012	.007	F:
6^h31	6 23.1	−48 7	5.94	B9	2.3	.0073	.003	F:
5107[3]	6 24.0	− 6 58	4.64	B2p	0.38	.017	008	g
6^h33	6 24.9	−32 18	5.80	B3	1.6	.011	.006	F
5157	0 25.3	+28 28	8.0	A0	1.3	.013	.009	f
5153[b]	6 25.6	+11 19	5.83	B0	2.2	.020	.009	f
5178	6 26.0	+52 33	6.82	A3	0.9	.024	.017	g
5166[d]	6 26.5	+17 51	6.72	*F6 F6*	0.54	.012	.006	f
6^h36[5]	6 27.4	−50 10	5.39	F2	1.0	.038	.025	F
6^h36[6, 10, 19]	6 27.4	−50 10	8.96	....	0.1	.031	.026	F
6^h37	6 27.4	−40 23	6.72	B8	0 3	.0094	.005	F:
5197	6 28.7	+14 50	7.44	F5	0.1	.023	.017	g
5228	6 30.0	+30 11	8.5	F5	0.1	.0052	.003	g
6^h40	6 31.1	−33 56	6.72	F0	0.2	.010	.005	F:
5276	6 31.1	+58 10	8.1	G5	1.7	.012	.008	g
5269	6 31.6	+41 40	6.79	B9	1.0	.0038	.002	p
5258	6 31.6	+23 44	9.0	F0	0.2	.013	.010	f
6^h42[5]	6 31.9	−36 42	5.60	B9	0.5	.019	.011	F
6^h43	6 32.0	−38 44	7.18	B8	0.1	.011	.006	F
——	6 32.4	−36 0	6.28	F5	0.4	.025	.016	F
5260	6 32.5	−22 32	6.23	B8	2.9	.019	.012	F
5289	6 33.3	+28 21	5.84	B8	1.8	.0041	.002	g
5296	6 33.4	+41 4	6.86	F2	0.9	.014	.008	g
5332	6 35.4	+30 47	8.0	F5	0.0	.0079	.005	f
5322	6 35.5	+ 9 59	4.68	*O8n*	2.8	.012	.005	f
5334	6 36.0	+ 9 55	9.1	....	0.5	.011	.007	p
6^h45[5, g]	6 36.0	−48 8	5.00	K0 A0	2.3	.022	.010	F
5368	6 36.1	+59 33	8.1	*F4*	1.9	.017	.013	f
5336	6 36.4	− 7 54	8.1	B9 B9	0.7	.0053	.003	p
5392	6 37.4	+54 27	8.6	F2	1.5	.021	.017	f
5400	6 37.4	+59 33	4.89	*A2n*	0.9	.029	.017	g
6^h52	6 38.9	−38 18	6.31	A3	1.3	.0060	.003	F:
5408	6 39.1	+29 28	8.6	G0	0.5	.0079	.005	f
5436[d]	6 39.9	+55 49	5.55	*F4 F6*	0.05	.0078	.003	p
5447	6 41.5	+18 18	6.16	A0	0.5	.014	.008	g
6^h58	6 41.7	−30 51	5.91	B3	2.5	.0098	.005	F
5442	6 42.0	−20 39	8.3	A0	0.0	.013	.009	F
5455	6 42.6	+ 0 27	6.90	A2	0.5	.012	.007	g
5452	6 42.7	−20 34	8.1	F8	1.6	0.049	0.045	F:

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	π	d	Grade
5488	$6^h43^m.2$	+43° 50′	8.9	A0	0.7	0″.010	0″.007	p
5472	6 43.8	−13 19	8.3	G0	0.3	.015	.010	g
5514[g]	6 44.2	+59 34	5.44	*G0* A2	1.2	.014	.007	g
5487[b]	6 44.4	−15 2	5.29	*B6n*	1.5	.0096	.004	f
6^h64[g]	6 44.6	−54 35	6.35	G5	3.0	.0081	.004	F:
5522	6 44.7	+57 57	8.9	F2	0.0	.0082	.005	f
5515	6 45.6	+25 10	9.1	....	0.3	.0078	.005	p
5498	6 45.6	−23 58	6.24	A0	1.6	.0051	.002	F
5535	6 46.6	+25 5	8.91	G5	0.1	.019	.015	f
5555	6 47.8	+48 41	8.0	F2	1.5	.0090	.006	f
5539[6]	6 47.8	−26 28	7.62	B5	1.0	.010	.006	F:
5586[g]	6 48.6	+58 33	4.54	*G6*	1.1	.017	.007	g
5559	6 49.0	+13 18	4.70	*A7n G4*	2.71	.064	.046	g
5570	6 49.1	+30 18	8.1	G0	0.0	.030	.025	g
5557	6 49.2	− 5 44	6.35	A3	0.1	.0071	.003	f
5588	6 49.8	+32 35	8.7	G5	1.9	.046	.044	f
5608	6 51.0	+25 5	7.59	A0	1.0	.016	.011	f
5645	6 53.3	+18 52	8.3	A3	0.0	.0046	.002	p
5654	6 54.7	−28 50	1.63	B1	6.0	.0052	.001	F
5706[3]	6 55.0	+54 19	9.3	....	0.3	.0080	.005	f
5703	6 56.5	− 8 58	9.1	....	0.1	.0082	.005	f
5699	6 56.7	−25 30	7.27	B3	2.5	.024	.016	F:
5707	6 56.8	− 9 34	7.9	F2	0.5	.013	.008	p
5720	6 56.9	+16 7	8.7	F8	0.0	.022	.018	f
5712	6 57.1	−10 45	6.77	F8	0.5	.014	.008	f
5746	6 57.7	+52 53	6.19	A2	0.10	.026	.017	g
5769	7 0.1	− 0 43	7.6	F5	0.3	.012	.008	f
7^h2[5]	7 1.7	−59 2	5.69	B9	0.8	.023	.014	F
7^h3	7 1.9	−34 37	6.32	F0	1.1	.045	.034	f
5816[g]	7 3.0	+17 4	7.6	*K5 K4*	0.1	.019	.010	p
5813	7 3.0	− 4 31	7.7	F0	0.7	.0088	.005	p
5824	7 4.0	−13 50	8.5	A2	0.5	.013	.010	f
5858	7 6.1	+22 27	7.35	F5	0.16	.020	.014	f
5879	7 6.4	+48 40	7.6	A0	3.2	.016	.011	p
5857	7 6.5	−10 23	8.0	B8	0.0	.0050	.003	f
5896	7 6.9	+52 43	7.16	F0	0.6	.0057	.003	f
5893	7 7.5	+28 40	7.7	A0	2.0	.0085	.005	f
5882	7 7.7	− 3 0	8.1	F5	1.2	.023	.018	f
7^h13[g]	7 8.0	−60 13	6.91	K0	0.5	.014	.007	F:
7^h14[g]	7 8.9	−36 23	5.94	B5	1.50	.0087	.004	F:
5918	7 9.0	+26 4	8.2	F8	0.1	.0071	.004	f
7^h16[d, g]	7 9.6	−70 20	3.70	K0 G0	1.94	.041	.018	F:
5925	7 10.2	−15 18	7.05	F8	0.0	.016	.010	f
———	7 11.1	−63 1	6.10	A0	0.4	.010	.005	F
7^h21	7 11.3	−44 29	9.2	A5	1.5	0.018	0.015	F:

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h	d	Grade
5958	$7^h 12^m.2$	+9°28′	6.99	G0	0.0	0″.026	0″.018	g
5961	7 12.3	+16 43	3.65	*A2n*	6.10	.020	.010	f
5956	7 12.4	−11 51	6.70	F5	0.0	.0077	.004	f
5983[h]	7 14.2	+22 10	3.51	*A8n K6*	4.51	.058	.032	g
5980	7 14.2	+13 33	8.3	A2	0.7	.019	.014	g
6028	7 14.5	+73 16	7.51	*A7n*	2.1	.011	.007	g
$7^h 25$[6]	7 14.6	−46 49	6.76	K0	1.0	.022	.014	F
6004[b, d]	7 14.6	+50 20	6.62	*A5n A5n*	0.10	.0080	.003	p
$7^h 26$	7 14.7	−30 37	6 88	B3	0.4	.011	.006	F:
6012[a]	7 14.7	+55 28	5.22	*B8n B9n*	0.92	.014	.006	f
$7^h 27$	7 15.0	−36 35	7.9	A0	0.6	.0056	.003	F:
5986	7 15.1	−21 52	6.84	A2	0.1	.012	.007	F
5996	7 15.4	+0 35	6.83	B9	0.4	.0099	.005	g
6006	7 15.8	+4 15	8.7	A0	0.0	.013	.009	f
5998	7 15.9	−24 14	8.0	B8	0.8	.011	.007	F:
$7^h 32$	7 17.9	−52 8	5.88	F2	0.63	.039	.027	f
6038	7 18.2	+21 39	8.0	B9	0.7	.012	.008	g
6032	7 18.4	−5 26	8.9	A0	0.1	.0068	.004	p
6060	7 19.7	+20 43	6.89	A5	1.0	.0094	.005	f
6065	7 20.8	−20 59	6.73	B2	1.0	.0040	.002	F
$7^h 41$	7 21.2	−37 6	6.28	A3	0.08	.0057	.003	F
6076	7 21.5	−20 46	7.5	....	2.3	.019	.013	F:
6117	7 22.7	+50 11	8.0	*A9n*	0.0	.015	.010	g
6126	7 24.8	−14 47	5.94	*F4*	1.6	.045	.033	g
6140	7 25.3	+5 28	7.5	B9	1.5	.013	.008	p
6155	7 26.8	−0 18	9.0	A0	0.0	.013	.010	f
6185[g]	7 28.8	+31 11	5.34	*K0*	0.6	.0075	.003	g
6191	7 28.9	+43 15	6.30	F0	1.6	.0097	.005	f
6180	7 29.0	+12 31	7.4	B8	0.7	.0080	.004	g
6184	7 29.8	−23 29	7.4	B9	0.6	.0082	.004	F:
6190	7 30.1	−23 15	5.18	*F4 F5*	0.15	.044	.029	g
6216	7 32.0	−14 16	7.15	B9	0.0	.013	.008	f
6232	7 32.8	−2 22	7.7	A5	2.6	.014	.009	f
6245	7 33.8	−4 46	7.9	F0	0.1	.0054	.003	g
6254[18]	7 34.6	−20 3	7.23	A3	0.5	.0078	.004	F
6255	7 34.7	−26 34	3.81	B8 B3	0.12	.022	.010	F
6263[b]	7 34.8	+5 28	5.81	A0	0.3	.013	.006	g
6276	7 35.0	+37 40	7.6	A3	1.0	.0075	.004	f
6279	7 36.0	−1 10	9.0	G0	0.1	.012	.009	f
$7^h 62$	7 36.2	−37 55	5.74	B5	2.8	.012	.006	F:
6291	7 36.3	+9 57	8.17	G0	0.5	.026	.021	g
———[6]	7 36.5	−43 3	7.34	A0	0.4	.0060	.003	F:
$7^h 65$[5]	7 37.4	−74 3	6.46	B9	0.02	.017	.010	F
6321[a]	7 38.4	+24 38	3.68	*G7*	5.69	.036	.018	g
6347	7 40.3	+21 22	7.47	F2	0.8	0.012	0.007	f

TABLE 53—*Continued*

ADS	α_{1900}	δ_{1900}	Mag.	Sp.	Δm	h_1	d	Grade
$7^{h}67$[6]	$7^{h}40^{m}.5$	−58° 26′	7.50	B9	2.5	0″.0079	0″.004	F:
———	7 41.0	−33 30	7.8	G5	0.1	.030	.023	F
6381	7 43.3	−11 57	5.52	*F5*	2.40	.022	.013	g
6383	7 43.4	− 4 32	8.5	A2	1.0	.0047	.003	p
6394	7 44.3	+24 47	8.76	G0	0.0	.0088	.006	f
$7^{h}76$	7 45.4	−30 18	7.58	F5	0.2	.026	.020	F
$7^{h}82$	7 47.4	−54 49	7.22	A0	1.5	.015	.009	F:
6425	7 47.5	+ 3 38	6.99	A2	0.5	.012	.007	g
$7^{h}83$[7]	7 48.5	−34 27	5.02	*F3*	4.0	.034	.021	F:
6454	7 49.6	− 2 32	7.15	F0	0.0	.018	.012	g
6484	7 52.6	− 9 33	8.7	F8	0.7	.012	.008	g
6487	7 52.9	− 0 33	8.3	K2	5.0	.074	.080	g
6499	7 54.2	+13 58	7.7	F5	0.0	.0070	.004	f
6516	7 54.3	+54 53	8.11	A0	1.8	.0096	.006	f
$7^{h}93$	7 54.4	−47 37	6.08	B5	0.9	.014	.007	F
6513[8]	7 55.1	+23 53	6.42	K0	4.6	.018	.010	f
———	7 56.7	−60 35	6.7	B8	0.9	.010	.005	F:
6548	7 56.7	+58 42	8.6	F8	0.3	.010	.007	g
6538	7 57.2	+26 33	6.96	A2	0.7	.0026	.001	f
6532	7 57.2	+ 4 26	8.3	G5	1.9	.030	.025	g
6552	7 57.6	+47 35	7.7	F5	0.5	.015	.010	f
6535	7 57.8	−26 56	6.52	B9	2.6	.023	.016	F
6549	7 57.8	+33 19	6.61	A0	0.6	.0087	.005	g
6547[9]	7 58.2	− 6 8	9.3	....	0.0	.022	.018	g
6578	7 59.3	+54 24	8.0	A0	0.0	.0075	.004	p
6569	7 59.5	+27 49	6.16	B9	0.90	.012	.007	g
6582[9]	8 1.0	− 0 29	8.4	G5	0.1	.023	.018	g
6620	8 2.3	+56 6	7.9	A2	3.3	.025	.021	f
6606	8 2.8	−11 55	9.3	F8	0.5	.012	.009	g
6623	8 3.2	+32 30	6.74	*F5 F8*	0.9	.034	.025	g
6614	8 3.8	−21 51	7.2	G5	1.9	.017	.011	F
6636	8 5.5	−26 50	8.6	A	0.6	.0085	.006	F
$8^{h}13$	8 6.4	−42 21	6.40	A0	1.1	.026	.017	F:
6659	8 7.5	+ 9 53	7.57	A5	1.7	.020	.014	f
6663	8 8.1	+11 9	7.30	F8	2.1	.027	.021	g
6671	8 8.5	+ 2 19	8.0	F8	0.2	.012	.008	f
6727	8 11.4	+56 46	9.0	G5	0.3	.013	.009	f
6721	8 11.7	+39 18	8.6	G5	0.5	.023	.019	g
6722	8 11.8	+31 8	8.1	F2	1.5	.014	.010	f
$8^{h}26$[8]	8 11.9	−30 37	6.30	G5	2.0	.018	.009	F
6719	8 12.3	− 5 4	7.75	F2	0.3	.013	.008	f
6729	8 13.3	−26 58	8.5	F5	0.2	.0063	.004	F
$8^{h}33$	8 13.8	−62 36	5.26	A2	2.7	.0089	.004	F
6746[20]	8 14.0	+47 43	9.7	....	0.7	.019	.016	f
$8^{h}34$	8 14.5	−37 4	6.54	A0	0.5	0.016	0.009	F:

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_i	d	Grade
8h37	$8^h 15^m.9$	$-73° 30'$	6.65	A5	0.1	0".015	0".009	F:
6762	8 16.3	− 1 17	6.35	A0	0 7	.014	.008	g
8h40	8 18.6	−40 40	7.1	F5	0.4	.025	.018	F
6815	8 20.7	+27 16	5.56	*A3n A2n*	0 02	.018	.010	g
6811	8 20.7	+24 52	6.58	*F1 F6*	0.54	.030	.021	g
6829	8 22.4	+51 31	8.6	*A9n*	0.0	.013	.009	f
8h42	8 22.6	−38 44	6.19	A0	0.60	.010	.005	F
6828	8 23.4	− 2 11	6.29	*F0*	0.1	.0070	.003	f
———	8 25.9	−38 16	8.2	A0	0.6	.012	.008	F:
8h44[5, b]	8 26.1	−44 23	5.06	B5	1.44	.0020	.0005	F
6867	8 26.4	+57 16	7.8	A0	2.0	.0053	.003	p
6862	8 27.0	−19 14	5.38	A0	0.6	.018	.010	F
6871	8 28.8	−24 16	6.15	*A7s*	0.0	.034	.024	g
6886[b]	8 30.5	+ 6 58	5.71	*F6 G5*	1.11	.017	.008	p
6887[19, g]	8 31.1	−24 46	7.51	G0	0.5	.0040	.002	F
6903[g]	8 34.2	−19 23	6.53	K5	2.7	.011	.005	F
6914	8 34.8	−22 19	5.13	*G6*	1.2	.059	.040	g
6929	8 35.9	−11 49	7.8	A2	0.3	.012	.008	f
6945	8 36.3	+49 15	6.88	F0	0.26	.017	.010	p
8h61	8 37.1	−59 58	6.40	A0	0.5	.021	.013	f
8h63	8 38.3	−57 11	6.44	A2	2.0	.031	.022	F
6960	8 38.8	+ 3 7	9.1	....	0.8	.0062	.004	f
6964	8 39.1	− 3 50	7.8	A2	1.0	.0067	.004	f
6977	8 40.3	− 2 14	6.17	*F5 F3*	1.0	.0098	.005	f
6988[g]	8 40.6	+29 8	4.09	*G6* A5	2.41	.045	.023	f
8h65[5]	8 41.9	−54 20	2.01	A0	3.1	.067	.034	F:
7016	8 42.5	+56 34	8.6	F5	1.1	.010	.007	g
8h66	8 42.7	−58 21	6.30	B8	0.40	.011	.006	F:
6999[g]	8 42.8	−16 41	6.62	G0 A2	2.5	.0064	.003	f
7010	8 43.4	+ 2 59	9.1	F8	0.7	.0076	.005	f
7012[9]	8 43.5	+ 1 19	7.7	F5	0.0	.011	.007	g
7021	8 44.1	+ 9 14	7.6	A5	1.1	.0091	.005	f
7034	8 44.4	+35 26	6.73	F8	0.00	.012	.007	g
7042	8 44.9	+31 47	9.6	....	1.0	.017	.014	f
7028	8 45.0	−26 3	8.5	A0	0.6	.030	.026	F
7054	8 45.6	+55 20	7.35	G0	0.0	.025	.018	g
7046[g]	8 45.9	+ 8 42	8.0	K0	1.2	.0041	.002	p
7067	8 46.0	+71 11	8.6	*M1 M1*	0.1	.090	.083	g
7049	8 46.0	+12 29	7.8	F0	2.3	.019	.014	f
7050[b]	8 46.6	− 6 49	5.60	*F0*	2.0	.021	.011	g
7071[g]	8 48.1	+30 57	5.60	*G7 K1*	0.5	.011	.005	g
7082	8 49.0	+26 36	6.67	*G1*	1.1	.029	.020	g
7077[9]	8 49.1	− 2 6	9.17	F8	0.1	.011	.008	g
7093	8 50.6	− 7 36	6.03	A3	0.24	.0086	.004	f
7103	8 52.3	−17 3	6.62	F2	0.40	0.024	0.016	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
7114	8h 52m.4	+48° 26′	3.12	*A4n M1*	7.07	0″.11	0″.084	g
7118	8 53.0	+35 21	8 6	G5	0.5	.018	.014	f
7115	8 53.0	+12 15	4 27	*F0*	6.9	.044	.029	g
8h89b	8 53.3	−52 20	4.77	B5	2.8	.037	.020	F
7121g	8 54.1	−22 27	8.7	K2	0.7	.0091	.005	F
7139	8 55.8	+15 41	8.6	*K6*	0.1	.048	.042	g
7142	8 56.3	− 2 9	8 6	....	4.2	.045	.043	g
7152	8 56.7	+ 3 4	7.01	A2	2.5	.0091	.005	f
7158	8 56.8	+47 33	3.68	*B9n*	0.2	.023	.010	g
8h96	8 58.7	−51 48	5.42	B9	1.6	.014	.008	g
7182	9 1.4	+ 3 13	7.9	F5	0.3	.024	.018	f
7187	9 1.7	+23 23	6.30	*F3 F4*	0 5	.024	.015	f
9h46	9 2.4	−49 45	7.77	A0	0.3	.0080	.004	F:
7198	9 2.9	− 6 44	8.4	*F9*	3.3	.045	.044	f
7215	9 4.3	+ 3 21	7.7	F0	1.6	.0066	.004	p
7229	9 4.5	+70 23	8.48	G5	0.2	.012	.008	g
7217	9 4.9	−28 25	9.3	F8	0.0	.017	.013	F:
7251	9 7.7	+53 7	7.20	*M0 M0*	0.11	.22	.22	g
9h13	9 8.8	−43 12	5.74	B8	0.8	.010	.005	F
7263	9 10.4	−19 42	7.34	A0	1.3	.020	.014	F:
7270	9 11.2	− 7 56	7.08	A5	0.5	.018	.012	g
7281	9 11.5	+24 4	7.20	F5	0.30	.026	.019	g
7276	9 11.5	+ 1 9	6.54	F5	4.5	.033	.025	f
9h19	9 11.6	−45 8	6.34	A0	1.1	.012	.006	F:
7277	9 11.8	−22 43	8.4	A5	1.0	.0033	.002	F
7286	9 12.3	+35 47	5.76	*A4n*	0.3	.011	.005	g
7292b	9 12.6	+37 14	3.82	*B9n*	1.85	.016	.006	g
7294	9 13.0	+21 14	8.6	F5	0.5	.012	.009	g
7303	9 13.8	+51 42	6.12	*F3*	4.1	.043	.034	g
7307	9 14.7	+38 37	5.86	*F3 F2*	0.2	.035	.024	g
7312	9 15.8	−23 3	7.58	F0	1.8	.0040	.002	F
7344	9 18.1	+28 20	8.1	F5	1.5	.014	.010	g
7354b	9 18.6	+54 27	7.36	A2	1.0	.0051	.002	f
7341g	9 18.9	+18 34	7.12	G0	1.5	.010	.005	f
7360	9 19.0	+54 10	9.0	K0	0.5	.0088	.006	f
7352	9 19.2	+ 6 47	6.81	F5	0.1	.023	.016	g
7350	9 19.3	−23 14	7.7	F2	2.8	.0082	.005	F:
7356	9 19.8	−23 23	9.4	....	0.1	.0068	.004	F:
7365	9 20.2	+29 5	8.1	F8	0.2	.011	.007	f
7378	9 21.2	+45 22	9.1	A3	0.0	.0071	.005	f
7384	9 21.9	+31 55	8.7	G0	0.2	.011	.007	f
7380	9 22.0	+ 6 41	6.71	F5	0.0	.020	.013	g
7379	9 22.4	−28 21	6.02	B8	0.8	.010	.005	F:
7398	9 23.6	+42 42	7.81	F5	0.0	.019	.014	f
7402	9 23.7	+63 30	3.75	*A4n*	5.34	0.058	0.038	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
7425	$9^h 26^m 0$	$+67° 14'$	7.37	F5	0.01	0".0045	0".002	f
7412	9 26.3	+ 1 54	6.95	F8	1.0	.016	.010	f
$9^h 39$	9 26.5	−31 27	5.94	A0	0.86	.0023	.0008	F
7446	9 28.4	+73 32	6.43	F0	0.07	.026	.017	f
7435	9 29.6	+ 5 42	8.7	K0	1.0	.012	.008	f
7441	9 29.7	+36 16	5.48	*K0*	8.5	.15	.14	g
$9^h 47$	9 30.2	−48 34	5.35	B3	0.60	.026	.015	g
7456	9 31.6	+16 42	7.7	F8	0.1	.0058	.003	p
$9^h 50^6$	9 31.7	−30 47	7.42	A3 G	0.9	.024	.018	F
$9^h 51$	9 32.8	−48 18	6.48	F0p	2.8	.030	.021	F:
7477	9 35.2	+39 25	6.96	*G2*	1.3	.029	.021	g
7500	9 38.2	+ 3 5	7.32	F8	3.2	.023	.017	g
7503	9 38.7	+43 42	8.1	F5	0.4	.020	.015	f
$9^h 65$	9 44.5	−34 33	7.31	A3	0.0	.010	.006	F:
7530	9 44.5	+17 2	8.5	G0	2.2	.011	.008	f
7527	9 44.6	−10 5	9.1	....	0.2	.0069	.004	p
$9^h 66$	9 44.6	−64 36	3.08	F0	2.88	.0085	.003	F
7541	9 45.1	+36 58	7.81	F2	0.0	.0071	.004	f
7551	9 46.7	+27 27	8.1	*G9*	1.0	.030	.024	g
7566	9 46.8	+69 23	8.7	F5	0.0	.0082	.005	f
7570	9 49.6	−27 32	6.88	B9	2.6	.0069	.004	F
7588	9 50.9	+46 23	8.6	G0	1.0	.022	.018	f
7598	9 52.4	+53 44	8.5	F8	0.7	.0094	.006	g
7611	9 55.0	+69 16	7.9	F5	3.2	.022	.017	g
7613[g]	9 56.3	+46 51	8.17	G5	1.0	.0077	.004	f
7616	9 56.6	+58 1	8.9	G5	0.7	.010	.007	g
7629	9 59.8	−27 54	7.29	F8	0.3	.024	.017	F
7632	9 59.9	+31 35	7.86	*A9n*	0.7	.0052	.003	p
$10^h 1$	10 0.5	−61 24	6.34	B8	1.4	.014	.008	F:
7644	10 2.3	−24 13	7.16	F8	0.1	.026	.018	f
7647	10 2.6	−19 13	7.30	A0	0.5	.0082	.004	f
7664	10 2.7	+73 31	8.9	F2	0.8	.016	.012	f
7654[3]	10 2.9	+12 29	7.64	*K1*	5.4	.043	.037	p
7655[7]	10 3.6	−19 15	7.24	*F8*	3.5	.017	.012	F
7662	10 3.8	+20 49	6.65	F5 A2	0.0	.016	.010	f
7677	10 6.6	+21 48	9.0	F8	0.0	.016	.012	f
7679	10 6.9	+16 50	9.0	G0	0.0	.012	.009	f
$10^h 14$	10 7.0	−68 12	6.06	A0	0.1	.014	.007	F:
7685	10 7.5	+27 55	8.2	G5	1.7	.013	.009	g
7695	10 9.7	+19 37	8.0	F2	0.1	.0055	.003	f
7705	10 9.8	+71 34	6.09	A3 A3	0.62	.012	.007	f
$10^h 18$	10 10.4	−83 35	7.31	G5	0.5	.034	.026	f
7704	10 10.8	+18 14	6.56	*F2*	0.2	.022	.014	g
7712	10 11.8	+23 37	5.85	*F3*	5.6	.038	.028	f
7715	10 12.5	+28 2	7.9	F2	1.0	0.0078	0.004	p

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_t	d	Grade
——	10ʰ 12ᵐ.6	−59° 24′	6.44	A2	0.0	0″.019	0″.012	F
7721	10 13.7	+21 4	8.8	*K5*	0.7	.026	.020	g
7722	10 14.4	−22 40	8.2	F5	0.4	.012	.008	F:
7724[g]	10 14.5	+20 21	2.30	*K1 G5*	1.19	.044	.017	g
7730	10 15.4	+ 6 56	7.9	F5	0.5	.011	.007	g
7737	10 15.9	+44 26	7.04	F5	0.5	.0040	.002	f
7738	10 16.7	− 9 16	7.7	G5	0.6	.025	.019	g
7739	10 16.9	−22 1	6.45	A0	1.7	.011	.006	F
10ʰ31[s]	10 17.2	−55 32	4.70	B5p	4.78	.015	.008	F
7744	10 17.4	+15 51	7.37	*G2*	3.5	.049	.044	g
7758	10 19.5	+25 7	8.2	G5	0.0	.022	.017	g
7762	10 19.7	+53 8	7.36	F5	0.3	.018	.012	f
7764	10 20.3	+ 9 17	7.7	F0	1.7	.014	.009	f
7775	10 21.4	+17 44	7.42	F8	0.5	.0076	.004	f
7778	10 21.7	+18 34	8.7	*G5 G5*	0.0	.030	.026	g
7779[g]	10 22.3	+ 4 4	7.22	*G9*	1.9	.014	.008	g
7785	10 22.7	+46 28	8.5	G0	0.8	.012	.008	f
7792	10 23.9	+10 40	7.6	F8	1.9	.010	.006	f
7802	10 24.7	+21 20	8.4	G5	0.5	.023	.018	g
10ʰ40[s, b]	10 25.0	−30 6	5.65	B9	3.80	.0058	.002	F
7817	10 26.5	+22 33	7.45	F0	0.6	.0074	.004	f
7827	10 27.5	+38 11	9.2	G0	0.0	.020	.017	f
7833	10 28.3	+23 52	7.10	A2	1.8	.0086	.005	p
7837[b]	10 29.8	+ 9 10	5.70	A0	2.7	.011	.005	f
7839	10 29.9	+26 48	9.6	....	1.0	.022	.019	f
7846	10 31.4	−26 10	6.25	*F3*	1.3	.028	.019	g
7854	10 32.0	− 8 19	7.12	G0	3.2	.045	.038	f
7860	10 32.6	+27 7	8.7	K0	2.7	.030	.026	f
10ʰ59[s, b]	10 33.1	−47 42	4.06	F2 A3	0.5	.044	.022	F
7864	10 33.5	+ 6 15	7.6	F2	0.9	.021	.015	g
7873[g]	10 34.5	+38 55	8.0	*K5 K3*	0.5	.0096	.004	p
7871	10 34.5	+ 9 22	7.9	*F6*	2.0	.017	.012	g
7878	10 34.8	+42 40	7.8	F2	0.0	.011	.007	f
7888	10 36.4	+11 16	7.6	A2	1.0	.0083	.005	g
7898	10 37.3	+45 7	9.0	....	0.3	.011	.007	p
7896	10 37.5	+ 4 6	6.63	F5	1.5	.015	.009	f
7902[g]	10 38.1	+ 5 16	5.99	*K4 G7*	1.04	.0073	.003	p
7901	10 38.3	−20 30	9.6	....	0.1	.010	.007	F:
7912	10 39.4	+ 4 34	8.65	F0	3.2	.012	.009	f
10ʰ70[6]	10 40.3	−59 3	7.60	A3 B	0.07	.0038	.002	F:
7926	10 41.9	+23 6	8.2	F0	0.9	.0073	.004	g
7929	10 42.3	+41 38	6.85	A3	0.4	.015	.009	g
10ʰ74[g]	10 42.5	−48 54	2.84	G5	4.5	.046	.022	g
7930[3]	10 42.7	−14 43	6.89	F5	0.3	.018	.012	f
7936	10 44.3	− 3 30	6.49	A2	0.8	0.018	0.011	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
$10^h77^{5,g}$	10^h44^m3	−79° 56′	5.48	Ko	0.2	0″.0081	0″.003	F:
10^h79	10 45.5	−58 48	6.10	Ao	1.0	.018	.010	f
7952^9	10 46.7	+16 38	7.9	F5	0 0	.0067	.004	g
10^h82	10 48.4	−58 13	8 2	B3	2.0	.0055	.003	F
7979	10 50.2	+25 17	4.32	*B9n*	1.79	.027	.015	g
10^h83^5	10 50.4	−70 11	6.11	B8	0.8	.0082	.004	F
8007	10 55.0	− 2 56	7.28	F8	0.6	.019	.013	g
8022	10 56.8	+15 9	8.9	*F4*	0.8	.047	.046	p
8032	10 57.6	+55 4	8.7	Go	0.5	.049	.048	f
10^h93	10 58.1	−81 1	6.67	F5	0.1	.017	.010	F
8038	10 58.5	−26 59	7.52	Ao	1.5	.011	.007	F
8043	10 58.8	+ 4 12	7.11	Fo	0.1	.0096	.005	g
8047	10 59.4	+38 58	7.56	F2	0.2	.0098	.006	p
8060^g	11 1.8	+ 2 30	5.66	*G7*	6.0	.027	.015	g
8061	11 2.0	+11 27	9.3	Ko	0.3	.012	.009	f
8065	11 2.3	+53 21	7.34	*F9*	1.3	.033	.025	g
11^h2	11 2.7	−42 6	5.34	*A5sp*	3.7	.013	.007	f
8083	11 5.6	+31 0	8.8	*M1 M2*	1.0	.094	.090	g
8092	11 7.8	+55 58	7.48	F2	0.8	.013	.009	f
11^h8	11 8.5	−46 31	6.8	F8	0.2	.021	.013	F:
8094^g	11 8.5	+20 41	6.94	*G4*	0.0	.015	.008	g
8100^{13}	11 8.7	+74 1	7.80	*K5*	5.0	.074	.073	g
8096	11 9.0	−14 54	8.1	Fo	0.9	.0077	.004	p
8102^g	11 9.6	+38 7	8.1	G5	0.8	.0044	.002	f
8105	11 10.0	+28 7	7.13	A5	0.3	.0081	.004	g
11^h12	11 11.9	−45 20	6.51	F2	0.28	.024	.016	f
8123^g	11 13.1	+33 38	3.71	*K3*	5.54	.013	.005	p
8128	11 13.7	+14 49	6.65	*F7*	1.2	.026	.018	g
8131	11 14.3	− 1 6	6.59	*F6 G6*	1.0	.024	.016	f
8138^1	11 16.4	−19 54	8.8	K5	2.5	.073	.077	F
——	11 16.4	−53 57	4.26	B5	0.3	.030	.015	F
8140	11 16.6	+18 45	8.1	*Ko*	3.2	.039	.035	g
8147	11 18.8	− 9 53	8.3	Ko	3.0	.012	.008	p
11^h19^5	11 19.0	−64 24	5.34	B5	1.18	.019	.010	F
8151	11 19.4	− 3 51	9.6	G5	0.2	.037	.037	f
8153	11 19.9	−17 8	4.14	A5	6.3	.030	.017	p
8155	11 20.4	− 7 18	8.5	F8	0.3	.0067	.004	g
8162	11 21.7	+ 3 33	6.19	*Ko K5*	1.04	.044	.032	g
8175^b	11 23.7	+39 54	5.26	*B9n*	3.14	.036	.021	g
11^h26^b	11 23.8	−42 7	5.34	B9	2.58	.015	.007	F
8179^3	11 24.2	−16 48	9.1	Go	1.0	.012	.009	f
8183	11 24.6	−23 55	5.73	Ao *F2n*	2.0	.033	.021	p
8190	11 25.8	− 6 10	7.6	F8	3.0	.027	.021	f
8196	11 26.6	+14 55	6.15	*F7 K6*	2.0	.058	.046	g
8200	11 27.0	− 2 59	8.7	Ko	1.0	0.023	0.018	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
8202[5, 7]	11h27m3	−28°43′	5.07	*F6 F7*	0.09	0″.024	0″.013	F:
11h31[5]	11 28.7	−40 2	5.50	A2	0.1	.015	.008	F
8220[d]	11 29.5	+17 21	5.76	*B3n*	1.4	.0069	.003	f
8231	11 31 0	+28 20	5.82	*A1n*	0.4	.020	.012	g
8236	11 31.2	+56 43	7.40	G5	0 5	.029	.022	g
8240	11 33.2	+64 54	6.44	A2	1.0	.0076	.004	f
8250[b, d]	11 33 5	+45 40	6.34	*G1 K5*	1.63	.065	.044	g
8252	11 33 7	+41 43	8.2	F8	1.6	.021	.016	g
8255	11 34.3	+25 51	8.0	G0	3.5	.027	.022	g
8302	11 41.7	+15 33	7.9	A5	2.5	.0080	.005	f
8311	11 43.5	+14 51	5.90	*A6n*	4.0	.031	.022	g
11h55	11 47.8	−33 21	4.40	B9	0.4	.020	.010	g
8332	11 48.1	−15 7	8.06	F0	0.2	.014	.010	f
8336	11 48.3	−24 2	8.0	G5	0.8	.027	.020	F:
8344	11 48.7	+72 29	7.54	F5	0.7	.021	.015	g
11h59	11 49.8	−41 50	8.0	F0	0.1	.014	.009	F:
8347	11 49.9	+47 2	6.46	*A1n*	1.8	.0078	.004	f
11h60	11 50.0	−55 32	6.92	G0	0.2	.054	.044	F
11h61[6]	11 50.3	−41 21	7.27	A2	0.0	.015	.009	F:
8355	11 51.1	+36 1	6.55	F2	1.9	.017	.011	g
8360	11 51.9	−24 55	8.5	F8	1.0	.016	.012	F
8361	11 52.2	−21 59	7.32	F5	0.2	.021	.015	F
8378	11 54.6	+53 57	8.6	F8	1.0	.010	.007	f
11h66[5]	11 54.6	−77 40	5.05	B9	0.7	.022	.012	F:
8394	11 56.3	−20 58	8.1	F8	1.5	.016	.011	F:
11h69	11 56.7	−34 6	6.90	G0	0.8	.034	.025	F
8403	11 58.4	− 1 53	7.9	F0	0.0	.0061	.003	f
8404	11 58.6	+25 40	9.0	G5	0.5	.011	.008	f
8405	11 59.1	+24 41	8.41	F5	1.2	.0095	.006	g
8406	11 59.2	+22 1	5.77	*A8s F2*	1.5	.0093	.005	g
8414	12 0.5	+52 29	7.00	F5	1.0	.010	.006	f
12h1[g]	12 1.0	−32 24	6.60	G0	2.4	.018	.010	F:
8421	12 1.0	+39 22	8.8	....	1.2	.016	.012	g
8433	12 2.9	+43 15	8.8	F8	0.1	.0056	.003	f
8434	12 3.2	+56 1	7.40	F8 *G1*	0.42	.037	.030	g
8436	12 3.2	−18 1	7.9	F2	0.5	.020	.014	p
8440	12 4.3	−11 17	6.78	*G2*	2.5	.061	.053	g
12h9	12 4.9	−34 9	6.14	A0	2.3	.012	.006	F:
8446	12 5.8	+40 27	6.82	A3	0.7	.018	.012	g
8450	12 6.6	+53 59	7.50	*K2 K1*	0.2	.070	.063	g
8459	12 7.1	−21 58	9.4	G5	0.5	.015	.011	F:
8460	12 7.5	+36 19	8.50	F5	0.3	.011	.007	f
12h19[b, g]	12 8.8	−45 10	5.29	K0	1.21	.012	.004	p
8470[g]	12 9.1	+33 20	6.78	*K3*	1.83	.026	.014	p
12h22	12 9.8	−35 40	7.42	A2	2.0	0.0094	0.006	F

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
8477[b]	12h10m.0	− 6° 42′	7.42	*G5 G8*	0.3	0″.051	0″.040	g
8480	12 10.4	+40 41	8.8	K2	2.0	.014	.010	p
8481	12 10.6	−22 47	6.42	F5	0.4	.028	.019	g
8486	12 10.9	+ 6 11	10.0	*K6*	1.5	.043	.044	g
8489[g]	12 11.1	+41 13	5.80	*Mo*	3.08	.0098	.004	f
8498	12 12.0	+70 42	8.2	Go	0.2	.0098	.006	f
12h25[5]	12 12.6	−35 32	6.28	Ao	0.2	.0079	.004	F:
8505	12 13.0	− 3 23	6.05	*F4 F5*	0.32	.025	.016	f
8506	12 13.6	+12 21	8.7	*G1 G1*	0.2	.018	.014	f
8508	12 14.3	+18 17	7.9	F5	1.6	.013	.009	g
8515[g]	12 15.0	−21 37	6.11	G5	2.5	.016	.008	F
8519	12 15.7	+27 37	6.30	F2	0.1	.015	.008	p
8531	12 17.5	+ 5 52	6.46	*F7*	2.8	.051	.042	g
8534	12 18.5	+14 27	8.3	G5	3.5	.027	.022	f
8535[g]	12 19.0	+54 43	7.71	G5	0.8	.0075	.004	g
8540	12 19.5	+43 39	7.84	Fo	0.3	.0028	.001	f
8546	12 20.9	+45 18	8.0	F5	0.8	.013	.009	f
8548	12 21.0	− 3 56	8.1	F5	0.8	.0097	.006	f
12h41[b, d]	12 21.0	−62 32	1.05	B1	0.51	.030	.008	g
8549	12 21.3	− 1 20	8.9	F5	0.3	.023	.020	f
8550	12 21.6	+ 0 23	7.65	A5	3.5	.015	.011	f
8551	12 21.6	− 5 2	7.50	F2	0.5	.0092	.005	p
8553	12 22.3	+27 36	8.6	*K5 K6*	0.5	.033	.027	g
12h45[6]	12 22.7	−61 12	6.51	Go	0.6	.040	.029	F
8562	12 23.3	+37 14	9.0	....	1.5	.021	.017	f
8561	12 23.3	+45 21	6.91	*F8 G2*	0.55	.027	.019	f
8573	12 24.9	−12 50	6.41	*F8*	3.8	.046	.037	g
8576	12 25.5	+ 4 3	7.38	*G7*	2.0	.026	.018	p
8575	12 25.5	+10 17	7.9	*F5*	0.3	.016	.012	g
12h53[6]	12 26.0	−40 57	7.86	Fo	1.2	.0068	.004	F
8586	12 26.7	+27 33	9.2	G5	1.8	.022	.019	f
8598	12 29.5	+ 6 32	8.5	F5	0.1	.0076	.005	f
8601	12 30.0	+ 7 59	7.72	F8	1.5	.027	.021	g
8600[a, g]	12 30.1	+18 56	4.94	*G9 A9s*	1.54	.034	.016	f
8606	12 31.0	+11 57	8.3	*Go Go*	0.0	.022	.017	g
8611	12 32.2	+21 45	7.71	F2	0.9	.0090	.005	g
12h58	12 32.5	−55 23	6.96	A2	1.8	.024	.017	F:
———	12 34.3	−36 44	8.6	K5	0.3	.034	.028	F
8625	12 35.9	+ 9 23	7.12	F2	0.5	.0071	.004	f
8627[b, d]	12 36.1	−12 28	5.28	*F6 F1*	0.10	.028	.014	g
8635[9]	12 37.2	+26 54	9.6	Ko	0.4	.036	.036	g
12h68[b]	12 40.1	−67 34	3.26	B3	0.3	.036	.015	g
8657	12 40.2	− 3 20	6.84	A3	0.4	.016	.010	p
8673	12 44.3	+22 20	9.3	....	2.2	.011	.008	f
8677	12 45.2	+24 41	9.6	G5	0.7	0.0070	0.005	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
12h76g	12h46m3	−32°48′	8.1	G5	2.5	0″.0052	0″.002	F:
8690	12 47.0	+19 42	7.15	*Fo* A2	0.48	.027	.019	f
8693	12 47.9	+15 34	8 1	F5	0.2	.016	.011	f
8695g	12 48.4	+21 48	5.10	*G6*	4.74	.033	.018	g
8698	12 48.5	−28 46	7.58	G5	2.3	.015	.010	F
8700	12 48.9	−27 25	7.18	Go	1.0	.029	.020	F:
12h83	12 51.0	−47 9	6.86	F5	0.1	.019	.012	F
8707	12 51.1	− 4 19	6.85	Ao	1.5	.0078	.004	p
8708	12 51.3	− 0 24	6.56	F5	0.4	.012	.007	g
8706d	12 51.4	+38 52	2.80	*A1s*	2.49	.033	.015	g
8710	12 51.9	+54 39	5.84	A2	1.9	.020	.012	g
8721g	12 53.9	+28 0	7.82	G5	0.0	.0050	.002	f
8735[3]	12 55.7	+18 55	9.5	....	0.4	.018	.015	f
8749	12 57.7	+24 2	7.61	F2	2.3	.016	.011	f
8751	12 57.8	+14 0	8.1	F5	0.5	.015	.010	f
8754	12 58.0	+10 58	9.3	G5	1.3	.017	.014	f
8755	12 58.2	− 5 54	8.3	Fo	0.9	.019	.014	f
8757[7]	12 58.4	−20 3	5.68	*F8*	0.1	.0071	.003	F
8759	12 58.8	− 3 8	6.51	*A7n*	0.0	.0095	.005	g
8738	12 58.9	+83 28	8.6	Ao	0.4	.0053	.003	p
8774	13 1.0	+26 46	10.0	....	0.2	.032	.027	p
8772	13 1.7	+73 33	6.33	A5	2.0	.017	.011	g
8782	13 2.2	+40 55	9.6	....	0.1	.011	.008	g
8786	13 2.3	+ 1 8	7.18	F5	0.5	.023	.016	g
8791	13 3.3	+27 28	8.2	F8	0.4	.0074	.004	g
8796	13 3.5	+16 2	7.7	Ko	1.0	.017	.011	f
8795g	13 3.6	+39 17	7.04	Ko	1.3	.012	.006	p
8801b	13 4.8	− 5 0	4.44	*A2s*	3.78	.011	.005	f
8802	13 5.1	+31 54	9.6	G	1.5	.020	.017	f
13h14	13 6.0	−59 23	4.76	B8	3.2	.025	.014	F
8811	13 6.5	+31 22	9.2	G5	1.4	.0093	.006	g
8814b	13 7.3	+32 37	6.66	*F4*	0.5	.032	.021	g
8824b	13 8.1	−18 18	6.27	*Aon A1n*	0.35	.020	.011	f
13h18	13 8.2	−63 3	6.96	B3	1.8	.028	.019	F:
———	13 11.3	−34 4	7.5	G5	0.3	.021	.014	F
8840	13 11.4	+17 47	8.5	Go	1.5	.016	.012	f
8841	13 11.9	+17 34	6.55	*K3*	3.2	.099	.089	g
8844	13 12.0	−21 1	7.9	Fo	0.7	.0050	.003	F:
8851	13 13.8	+59 17	8.2	G5	4.0	.0094	.006	g
8864	13 15.7	+ 3 28	6.23	Ao	0.7	.010	.005	g
8881	13 17.9	−14 24	7.78	F8	0.2	.010	.006	f
8885	13 18.5	−20 24	6.60	Ko	4.0	.022	.014	F:
8884	13 18.6	− 0 12	8.78	G5	0.0	.012	.008	f
8883	13 18.6	+ 3 14	6.74	G5 G5	0.30	.042	.031	f
8887	13 18.9	+29 45	8.86	*Mo K6*	0.2	0.023	0.016	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
8890	$13^h19^m.2$	+1°55′	7.08	A2	0.5	0″.0062	0″.003	f
8900	13 20.8	−22 43	8.39	F0	0.1	.014	.010	F
8901[9]	13 21.5	+45 1	8.17	K0	0.0	.041	.036	g
8906	13 21.7	−21 51	7.77	F8	0.1	.013	.008	F
13^h40[g]	13 22.2	−43 15	7.20	K0	2.5	.0052	.002	F
8914	12 23.6	+16 14	7.9	F5	0.5	.024	.019	g
8917	13 24.2	+12 0	8.5	*G8*	2.5	.032	.028	f
8918	13 24.4	+22 42	7.98	F8	3.0	.022	.017	g
8919	13 25.2	+60 26	8.8	F8	2.0	.0098	.007	f
13^h41[g]	13 25.2	−38 54	3.96	K0	0.3	.011	.004	F
8926	13 25.5	+8 0	8.5	A5	0.0	.012	.008	g
13^h45[6]	13 25.8	−61 50	6.58	B9	0.5	.0043	.002	F
8934[g]	13 27.9	+37 19	6.90	G5	0.9	.0083	.004	g
8939	13 28.3	+35 25	6.80	*A3n*	0.5	.0056	.003	g
8940	13 28.8	+49 39	8.12	G0	0.2	.027	.022	f
8943	13 29.0	+30 15	8.4	G0	0.6	.011	.008	g
8946	13 29.0	+9 18	8.3	F2	0.5	.012	.008	g
8950	13 29.1	−8 6	7.38	F8	0.4	.018	.012	f
8949	13 29.2	+0 12	7.36	*K1*	1.1	.043	.034	g
8954	13 29.3	−12 42	5.81	*A1n*	0.5	.0088	.004	p
13^h52	13 31.1	−31 54	7.52	F2	0.5	.015	.010	F
8959	13 31.2	+68 17	8.4	*G2 G4*	0.5	.018	.014	f
8966[5]	13 31.3	−25 59	5.49	A2	1.0	.018	.010	F
13^h55	13 32.1	−34 33	7.24	G0	2.0	.049	.040	F:
8971	13 32.3	−10 17	8.71	G5	0.6	.015	.011	f
8972[g]	13 32.3	−7 22	7.11	K2	0.0	.0044	.002	p
8981	13 33.7	+39 41	7.82	*G2*	2.5	.021	.015	g
8979[g]	13 33.7	+51 13	6.59	*M3*	1.5	.0073	.003	f
8976	13 34.1	+70 16	8.0	F5	0.7	.0098	.006	f
13^h63	13 35.3	−54 3	5.40	B9	1.45	.0073	.003	F
8991	13 35.9	+20 28	5.65	*A3s A7s*	2.9	.027	.018	g
13^h64	13 36.6	−33 29	6.75	F5	0.2	.028	.019	f
9000[g]	13 38.1	+4 2	5.62	*K3*	2.4	.014	.006	g
9004	13 39.9	−11 20	9.1	F5	0.5	.013	.010	f
9019	13 41.1	+5 37	7.22	G0	0.4	.024	.017	g
9022	13 41.7	+11 20	8.7	K2	2.8	.015	.010	f
9020[g]	13 41.8	+41 32	8.0	K0	2.2	.0055	.003	f
9025	13 42.5	+17 57	4.51	*F6*	6.6	.11	.096	g
13^h79	13 46.0	−32 30	4.47	B5	1.45	.031	.017	p
13^h80	13 46.3	−31 7	6.20	F8	1.0	.025	.016	F
13^h82^b	13 47.7	−35 11	5.64	F2	0.2	.018	.009	g
9047	13 49.0	+10 38	8.1	F0	3.2	.014	.010	g
9048	13 49.6	+33 20	8.2	F8	0.2	.022	.017	g
9053	13 49.7	−7 33	6.20	*F7 G1*	1.2	.037	.026	g
13^h87	13 49.7	−53 38	6.36	A2	1.3	0.019	0.012	F

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
9051	13ʰ49ᵐ.9	+30° 25′	7.38	F2	2.9	0″.015	0″.010	f
9060	13 51.3	+ 5 47	7.9	F5	0.5	.0081	.005	f
9064	13 51.5	−27 2	8.1	G5	0.8	015	.010	F
9067	13 52.8	+34 56	8.47	G0	0.2	.0093	.006	f
9073	13 53.3	+ 2 43	8.5	G0	0.0	.015	.011	f
9075	13 53.5	−25 4	9.20	F8	0.1	.0083	.006	F
9071	13 53.9	+52 29	8.7	G5	0.1	.043	.041	g
9069	13 55.2	+78 53	7.9	F2	1.8	.022	.017	g
9084	13 56.6	+25 51	9.3	....	0.3	.012	.009	g
13ʰ102	13 57.2	−31 12	6.32	*F5*	2.3	.051	.041	g
9089	13 58.6	+57 42	7.8	F5	0.3	.013	.008	g
13ʰ103[g]	13 59.0	−49 55	7.3	K0	0.1	.0068	.003	F:
9106[6]	14 0.6	−26 6	7.65	F0	0.00	.016	.011	F
14ʰ3	14 1.2	−49 24	6.59	G0	0.3	.033	.023	F
9115	14 2.7	−12 27	7.39	*F7*	0.9	.040	.033	g
9116	14 3.3	+25 12	9.0	G	1.0	.011	.008	g
9118	14 3.6	+21 40	8.2	F8	1.0	.015	.010	f
9121[g]	14 4.0	+37 13	8.2	G0	0.8	.0030	.001	p
14ʰ9	14 4.6	−46 26	7.9	F2	0.7	.022	.016	F:
9136	14 5.6	+27 5	8.4	G5	1.0	.015	.011	f
9144	14 6.1	− 2 52	8.1	F8	0.4	.018	.012	p
9149	14 6.3	−28 25	8.8	F5	0.2	.013	.009	F:
9156	14 7.1	−29 32	8.8	F2	0.3	.023	.019	F:
14ʰ12	14 7.7	−61 14	6.52	Oe5	2.0	.011	.005	F:
9160	14 7.9	+11 32	8.4	A3	0.3	.0040	.002	g
9158	14 8.0	+29 11	7.56	F2	0.2	.0067	.004	g
9159	14 8.3	+44 39	7.67	F2	0.2	.012	.007	g
9163	14 8.4	+ 5 52	7.7	A3	0.1	.0084	.005	f
9165	14 8.6	+13 3	8.1	G0	0.4	.014	.010	g
9170	14 8.9	− 8 4	8.7	F2	0.1	.012	.008	f
9168[a]	14 9.0	+12 28	6.63	K0	2.2	.014	.007	p
9174	14 9.4	+29 35	6.76	F0 A2	0.10	.015	.009	g
9177	14 9.7	+27 10	8.4	F8	0.5	.013	.009	g
9167	14 9.7	+55 48	8.2	*K5 K5*	0.3	.037	.031	g
9173[b, d]	14 9.9	+52 16	4.44	*A7n F1*	2.01	.034	.017	g
9182	14 10.3	+ 3 35	7.04	*F7 F8*	0.1	.040	.032	g
9185[a]	14 10.9	+10 46	8.7	K0	0.5	.014	.010	g
9183	14 11.4	+47 26	8.4	F8	1.0	.0023	.001	p
9203	14 11.8	−27 22	9.2	K5	0.4	.024	.020	F
9192	14 11.9	+20 35	6.36	*F4*	1.7	.041	.031	g
9207[8]	14 12.4	−27 3	8.5	F5	0.1	.011	.007	F
9197	14 12.9	+57 11	7.03	F0	2.7	.015	.010	f
9205	14 13.2	+45 58	9.1	G	0.6	.0084	.006	g
9215[g]	14 13.9	+ 4 21	8.7	G5	0.0	.0059	.003	g
9219	14 14.1	−13 16	8.1	G0	0.5	0.023	0.018	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
14^h27	$14^h14^m.5$	−41° 59′	7.8	F0	0.1	0″.012	0″.008	F
9217	14 14 8	+48 23	9.1	....	1.2	.010	.007	f
9227	14 15.4	+ 9 3	7.9	G5	3.5	.013	.009	f
14^h28[b, g]	14 15.5	−58 0	4 89	G0	1.95	.034	.016	p
9228	14 15.9	+12 37	10.6	....	1.3	.042	.040	f
9220[9]	14 16.1	+69 42	8.19	F2	0.2	.013	.009	g
9237	14 17 3	− 7 19	6.80	*F6*	0.04	.031	.022	g
9238	14 18 5	+51 34	8.4	F5	0.0	.013	.009	g
9241	14 18 6	+48 59	9.2	....	1.0	.027	.022	p
9254	14 19.3	−11 13	6.48	*F0*	1.6	.019	.012	g
9251	14 19.3	+11 42	6 70	F5	0.34	.016	.010	f
9249	14 19.6	+48 3	9.1	G5	0.2	.0082	.005	g
9258[3, 6]	14 19.9	−19 30	6.66	A0	0.9	.0087	.005	g
9261	14 20.6	−23 46	7.9	F8	1.0	.023	.017	F:
9265	14 22.0	+ 4 8	8.5	G0	0.0	.011	.008	f
9264	14 22.0	+16 52	7.7	F8	0.2	.021	.015	f
9269	14 22.9	+21 4	9.9	....	0.1	.014	.011	f
9273	14 23.0	− 1 47	4.97	*F8*	4.00	.032	.020	g
9287	14 25.1	+ 6 44	8.7	G0	0.4	.012	.008	f
9281	14 25.5	+54 57	8.81	K0	3.5	.025	.022	f
9291	14 25 8	−15 11	7.86	*G4*	0.9	.014	.009	p
14^h47[g]	14 27.2	−30 16	6.11	K0	3.4	.012	.006	F
14^h48	14 28 3	−45 43	9.1	F5	0.6	.015	.011	F:
9308	14 28 9	+27 51	9.3	....	1.6	.012	.009	g
9306	14 29.0	+49 38	7.82	*F3*	3.9	.040	.036	f
14^h53	14 29.5	−36 7	7.9	F8	0.2	.018	.013	F
9312	14 29.6	+36 1	7.65	G5	0.8	.021	.015	f
9318	14 30.7	+ 0 40	8.79	F8	0.0	.013	.009	f
14^h57	14 31.0	−37 6	7.37	A0	0.5	.013	.008	F:
9328	14 33.2	−21 54	7.5	A2	0.2	.014	.009	g
9324	14 33 4	+48 39	7.8	F2	0.2	.019	.014	g
14^h63	14 34.4	−64 32	3.41	F0	5.41	.10	.074	g
9333	14 34.6	−25 50	6.94	A2	2.0	.0046	.002	F
9333[10]	14 34.7	−25 49	8.8	A5	1.2	.0035	.002	F:
9329	14 34.7	+52 0	6.79	*F4*	0.3	.0076	.004	g
9334	14 35.1	+ 5 30	7.9	F0	1.0	.0096	.006	g
9344[5]	14 35.8	−29 16	7.16	F0	0.3	.011	.006	F:
14^h69	14 35.9	−30 30	6.47	B9	0.8	.0072	.003	F
9338[b]	14 36.0	+16 51	4.54	A0	0.87	.026	.012	g
9340	14 36.5	+31 43	7.9	F5	0.5	.013	.008	g
9339	14 36.8	+49 8	8.1	F0	4.0	.024	.020	p
9345[g]	14 36.9	+ 9 57	7.77	G5	0.0	.0028	.001	f
9353	14 37.9	+ 7 1	7.5	F8	2.5	.031	.025	g
9355	14 38.0	+ 8 31	7.9	F2	2.9	.022	.017	f
9346	14 38.2	+58 22	7.06	*K0 G7*	1.0	0.034	0.025	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
9350	14h38m3	+51°48′	7.24	F0	0.0	0″.016	0″.010	g
9357	14 39.5	+61 41	6.17	*F4*	2.2	.034	.024	g
9366	14 39.9	+ 8 7	8.3	F5	0.5	.024	.019	g
9375	14 40.2	−25 1	5.03	*F1 F9*	1.88	.037	.023	p
9372[R]	14 40.6	+27 30	2.59	*K0 A3n*	2.42	.020	.007	g
9379	14 41.1	− 6 58	7.9	G0	0.5	.018	.013	g
9387	14 43.0	−16 55	6.97	*G7*	0.8	.031	.022	g
9396	14 43.8	−13 44	5.38	*A4sp A4sp*	0.9	.027	.016	g
9392	14 44.0	+ 6 23	6.72	F8	0.0	.021	.013	g
9389	14 44.0	+24 47	6.05	F5	1.6	.017	.010	g
9397	14 44.4	+10 38	8.5	G5	0.0	.033	.029	g
14h91[R]	14 44.6	−47 0	7.5	K0	1.0	.011	.005	F:
9400	14 44.8	+ 8 24	6.95	F5	0.2	.012	.007	f
9407	14 45.4	+ 0 23	8.5	F5	0.5	.015	.011	f
14h92	14 45.7	−66 0	7.4	G0	0.2	.042	.035	f
9410	14 46.2	+10 8	7.42	K0	2.0	.020	.013	f
9406[a]	14 46.3	+49 7	5.64	*F6 F1*	0.7	.020	.010	g
9418	14 47.8	+45 21	7.78	*G0*	0.1	.019	.013	g
9423	14 47.9	+19 9	8.0	*G0*	1.7	.025	.020	g
9425	14 48.7	+16 6	6.43	*F9*	0.7	.032	.022	g
9358[R]	14 49.6	+86 22	7.02	K0	3.0	.0059	.003	f
9436	14 50.3	+ 0 34	9.0	F8	0.1	.0079	.005	g
9435	14 50.4	+34 30	8.52	G0	1.7	.016	.012	f
9440	14 50.5	−24 16	9.4	G0	0.6	.0076	.005	F:
14h104[R]	14 51.4	−34 59	7.06	K0	1.6	.0073	.003	F:
9443[9,R]	14 51.5	+ 3 19	8.1	K0	0.6	.0063	.003	g
9446	14 51.6	−20 58	5.76	*K5 M2*	2 18	.23	.22	g
9441	14 51.9	+40 3	7.92	F0	0.0	.011	.007	f
9453	14 52.8	−27 15	5.68	A5	0.1	.015	.008	g
9463	14 54.4	+17 56	9.4	....	0.0	.018	.014	f
9476[9]	14 56.0	− 3 42	9.2	K2	3.0	.075	.086	g
9475	14 56.5	−19 53	8.2	F8	1.0	.0088	.005	F
9480[R]	14 56.7	+ 0 14	5.91	*M2*	2.3	.0039	.002	1
14h116	14 58.3	−46 40	4.02	B5	0.10	.020	.009	g
9493	14 59.2	+ 5 52	6.44	F0	0.28	.025	.017	g
9497	15 0.2	− 6 38	7.52	G0	0.5	.025	.018	g
9496	15 0 9	+34 51	8.42	F8	1.0	.0095	.006	f
9504	15 2.0	− 1 54	8.5	F8	0.5	.011	.008	f
9507	15 2.8	+ 9 37	6.69	G5	0.0	.020	.012	f
9514	15 3.5	− 0 36	8.5	G0	0.1	.016	.012	f
9516	15 3.6	− 6 12	8.6	F2	0.5	.0084	.005	g
9517	15 3.9	+ 2 3	7.8	*F1*	4.3	.035	.032	f
9518	15 4.1	+ 5 34	8.5	F2	0.8	.023	.019	f
9523	15 4.2	−22 20	8.0	F5	1.8	.0050	.003	F:
9527	15 6.1	+39 21	7.92	F8	2.5	0.025	0.020	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
9532[3]	15^h $6^m.6$	$-19°25'$	10.5	....	0.4	0″.015	0″.012	F:
9535	15 8.3	+19 40	6.41	*G5 G6*	0.80	.050	.038	g
9552	15 9.7	−27 14	6.78	F2	1.5	.015	.009	f
9546	15 10.3	+56 25	7.40	A2	4.5	.016	.012	f
9557	15 10.8	− 4 31	7.14	F8	0.0	.0072	.004	f
9553	15 11.1	+38 41	7.09	Fop	2.3	.018	.012	g
9554	15 11.2	+37 12	7.9	F5	1.6	.0065	.004	f
9564	15 11.5	− 7 54	7.95	*G4*	1.5	.037	.032	g
15^h27	15 11.5	−47 31	4.36	B8	2.57	.025	.013	g
9569	15 11.9	−26 38	7.8	K2	1.6	.028	.021	F
9579	15 13.3	−23 54	7.17	A2	2.0	.019	.013	F
9586	15 13.8	−23 55	7.21	F2	0.4	.018	.012	g
9580	15 14.0	+10 48	6.71	*F5 G0*	1.4	.037	.028	g
9584	15 14.2	+ 2 9	5.18	*F6*	4.61	.048	.034	g
15^h35	15 15.4	−58 58	4.54	B5 F8	0.1	.020	.010	g
15^h37[6]	15 15.7	−47 33	7.80	G0	0.3	.042	.036	F
15^h38[5, a]	15 15.9	−44 20	3.75	B3	1.7	.024	.009	F:
9596[g]	15 16.0	+ 1 4	5.48	*K4*	4.5	.025	.013	f
9614	15 17.3	−25 40	9.0	G5	0.3	.0091	.006	F:
9636	15 21.4	− 5 18	8.35	F5	0.5	.0094	.006	f
9630	15 21.5	+26 58	8.3	F5	0.0	.010	.007	f
15^h42	15 21.6	−58 0	6.82	A3	0.8	.024	.016	F:
9643[g]	15 22.5	+10 3	7.87	G0	0.7	.0091	.005	g
9648	15 22.7	+ 5 44	9.5	G5	0.5	.017	.014	f
9647	15 22.7	+ 6 27	7.9	F8	0.6	.011	.007	g
9659[5]	15 22.9	−28 31	7.51	F8	0.3	.023	.017	F
9639	15 23.0	+44 21	7.40	G5	1.6	.024	.018	g
9654	15 23.2	+ 3 12	8.1	A5	0.0	.0088	.006	g
9644	15 24.5	+67 24	9.4	....	0.3	.0075	.005	f
15^h48	15 24.8	−57 4	7.6	F0	1.52	.027	.020	F:
15^h49[8]	15 25.0	−33 29	6.90	A2	2.0	.024	.017	F
9674	15 26.2	+47 12	9.4	F8	1.0	.0098	.007	g
9686	15 27.1	+10 0	8.22	F8	1.7	.0076	.005	f
15^h56	15 29.0	−44 37	4.84	B3	2.9	.026	.014	f
9695	15 29.6	+27 3	8.9	G0	0.6	.027	.024	f
9694	15 29.8	+42 9	8.3	G5	1.4	.031	.026	g
9701[b]	15 30.0	+10 53	3.85	*A5n F0*	0.93	.043	.022	g
9702	15 30.9	+35 3	9.5	....	1.5	.017	.014	f
9708	15 31.1	+13 16	7.67	F8	1.7	.013	.008	f
9710	15 31.8	+28 42	10.1	....	0.2	.025	.022	f
9728	15 33.2	− 8 28	5.82	*F6 F6*	0.07	.019	.011	f
9733[6]	15 33.5	−27 19	7.67	A0	0.0	.0075	.004	F
9727	15 33.8	+30 26	8.2	*F6 G1*	0.4	.025	.020	f
9734	15 33.9	−20 41	8.57	G5	2.0	.013	.009	F:
9735	15 34.1	−19 27	6.90	F2	0.2	0.018	0.011	F

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
9731[3]	15h 34m.5	+36° 34′	7.72	....	1.5	0″.0078	0″.005	f
15h70	15 34 5	−39 39	6.66	F5	1.1	.014	.008	F
9696	15 35.0	+80 47	6.47	*G3 Ko*	0.68	.057	.045	g
9737[a]	15 35.6	+36 58	4.69	*B6n*	0.93	.022	.010	g
9742	15 36.0	+19 0	7.62	A2	0.0	.013	.009	g
9747	15 36.9	+ 0 47	7.39	*A7s*	0.0	.0092	.005	f
9751[g]	15 37.1	−15 42	7.04	Go	1.1	.014	.007	p
9752	15 37.1	−21 36	9.3	Go	0.6	.016	.012	F:
9754	15 37.2	−25 6	7.16	Fo	1.9	.027	.020	F
9758[g]	15 38.5	+13 59	6.44	G5	0.5	.0050	.002	f
9763[9]	15 39.0	+ 2 50	5.80	*G5*	6.2	.093	.084	g
9768	15 39.4	−14 52	8.5	F8	1.3	.0099	.007	f
9756	15 39.4	+60 18	8.4	Ko	0.7	.022	.016	g
9778	15 41.6	+15 44	3.74	*A1n*	6.23	.028	.016	f
9795	15 43.9	− 2 56	8.3	Ao	0.2	.0085	.005	f
15h90	15 44.5	−34 45	8.0	F5	0.5	.012	.008	F:
9805	15 45.5	− 8 43	9.2	Go	0.4	.012	.009	f
9794	15 45.5	+59 47	7.86	F5	0.4	.0036	.002	g
9810[9]	15 46.4	− 5 41	8.7	Ko	2.0	.011	.008	g
9802	15 46.6	+44 49	7.57	Ao	0.5	.0086	.005	g
15h92	15 47.2	−59 53	6.04	A3	3.7	.037	.028	F
9823	15 47.6	−25 2	4.66	*B3n*	3.1	.013	.006	F
9836	15 49.6	−26 27	7.28	F8	0.6	.021	.014	F
9831	15 49.6	+17 17	7.7	F2	0.0	.0085	.005	g
15h97	15 50.5	−33 40	4.78	Ao Ao	0.36	.032	.018	F
9842	15 50.7	− 1 52	6.72	*F7*	1.1	.050	.040	g
9850	15 52.1	+12 46	6.94	F2	0.7	.018	.012	g
15h101[5]	15 53.5	−38 7	3.61	B3	4.12	.019	.008	F:
9864	15 53.6	− 2 48	8.0	Go	0.3	.011	.007	f
9861	15 54.0	+41 57	8.9	F5	1.3	.017	.013	p
9865[3]	15 54.6	+22 5	8.7	A2	0.0	.0064	.004	f
9872	15 55.6	+11 58	8.3	Go	2.0	.0090	.006	g
9880	15 56.3	+13 33	7.02	F5	0.5	.016	.010	g
9894	15 56.5	−24 18	8.6	Ao	0.9	.0079	.005	F
9899	15 56.9	−24 44	6.93	Ao	1.8	.011	.006	F
15h112	15 57.4	−32 48	7.06	Fo	0.1	.0059	.003	F:
9904	15 58.4	+14 16	8.0	F2	0.8	.0085	.005	f
9891	15 58.4	+59 13	8.0	A3	1.7	.014	.010	f
9910[1]	15 59.0	−11 11	6.93	*G6 K1*	0.7	.050	.040	g
9907	15 59.1	+29 8	9.5	K2	1.0	.0082	.005	f
15h117	15 59.3	−32 35	7.87	G5	0.7	.040	.034	F
9913[7, b]	15 59.6	−19 32	2.90	*Bon*	5.6	.013	.004	F
9914	15 59.6	−20 13	9.0	G5	2.1	.028	.024	F:
9918[b]	16 0.4	− 6 1	6.36	F5	3.0	.015	.008	f
9853	16 1.0	+83 55	7.06	A3	0.5	0.0058	0.003	f

TABLE 53—*Continued*

ADS	α_{1900}	δ_{1900}	Mag.	Sp.	Δm	h_1	d	Grade
9925	16^h $2^m.6$	+17° 10′	8.5	F5	0.1	0″.0096	0″.006	f
9931	16 3.3	+14 41	7.9	F0	0.5	.0072	.004	g
9930	16 3.6	+29 16	7.68	A3	2.6	.012	.008	p
9935[g]	16 4.8	+45 39	7.35	K0	0.2	.0032	.001	f
16^h8	16 5.4	−56 9	7.7	F8	1.9	.019	.013	F:
9948	16 5.7	−27 18	8.2	A5	1.8	.019	.014	F:
9940	16 5.8	+45 36	8.4	F5	1.1	.015	.010	p
9953	16 6.1	−28 9	5.70	B9	2.5	.031	.021	F
9951[b]	16 6.2	−19 12	4.29	*B2n*	2.5	.013	.005	g
9951[10]	16 6.2	−19 12	6.49	A	0.8	.015	.009	F
9952	16 6.9	+15 23	8.5	G5	0.1	.011	.008	g
9959	16 7.4	+12 10	8.3	*A8s*	1.4	.026	.022	f
9958[g]	16 7.8	+33 36	6.41	K0	4.0	.017	.009	f
16^h16	16 8.0	−38 53	7.66	F0	0.2	.021	.013	F:
9969	16 8.6	+13 48	6.84	*K0 K1*	0.08	.045	.035	g
9966	16 8.7	+26 56	6.37	F2	3.6	.018	.012	g
———[6, 21]	16 9.2	−53 27	6.76	K0	1.5	.023	.014	F
9974	16 9.6	+ 5 47	8.5	F5	0.7	.012	.009	g
9980	16 10.3	+ 4 31	8.5	F0	0.0	.0053	.003	p
16^h21	16 10.7	−64 24	7.03	A0	2.4	.0080	.004	F
9989	16 11.8	+ 1 26	9.5	G0	0.0	.016	.013	f
9992	16 12.9	− 2 2	8.32	A3	0.2	.019	.014	f
9997	16 14.3	+17 38	9.0	G5	0.0	.0079	.005	f
10007	16 15.7	+27 1	9.1	G5	0.0	.018	.015	f
10006	16 15.9	+41 54	7.86	A5 G	0.2	.0056	.003	g
10024	16 16.9	−22 53	7.45	A2	1.7	.0069	.004	F
16^h33[5]	16 17.5	−32 58	6.54	A0	0.53	.015	.009	F
16^h34	16 17.9	−48 55	6.7	G5	0.2	.050	.039	F
10035	16 18.4	−29 28	5.46	G0 G0	0.65	.052	.036	F
10030	16 18.5	+14 3	7.6	F2	2.0	.019	.014	f
10045	16 19.4	−23 14	6.56	B3	1.4	.010	.005	F
16^h38	16 19.4	−47 49	7.3	B8	3.0	.0053	.003	F:
10050[6, g]	16 19.5	−29 41	7.79	G5	1.5	.0088	.004	F:
10049	16 19.6	−23 13	4.76	*B4n*	0.70	.026	.013	g
16^h41	16 19.6	−34 45	8.24	F8	0.4	.0068	.004	F:
10036	16 19.8	+33 35	8.9	....	0.5	.011	.008	g
10038	16 20.3	+47 52	8.10	F8	0.5	.0086	.005	f
10044	16 20.6	+37 15	8.00	K0	0.2	.028	.022	f
10065[9]	16 22.1	+12 16	8.3	....	0.0	.0079	.005	f
10052[g]	16 22.5	+61 56	5.64	*G7*	1.2	.0074	.003	g
10058[g]	16 22.6	+61 44	2.89	*G6 K1*	5.86	.031	.014	g
10074[5, 7, g]	16 23.3	−26 13	1.22	*M1* A3	5.78	.015	.004	F
10070	16 23.8	+26 12	6.68	A3	1.0	.010	.006	g
10077	16 24.7	+10 49	7.7	K0	1.5	.020	.014	f
10092	16 26.4	− 6 48	8.1	A0	0.0	0.0081	0.005	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
10095	16h26m6	− 2° 3′	8.52	F5	0.0	0″.016	0″.012	f
10094	16 26.7	+ 5 39	7.6	*F1*	1.1	.025	.019	f
10059	16 26.8	+80 17	9.0	F5	2.8	.010	.007	f
10096	16 27.3	+19 30	9.8	....	0.2	.013	.010	f
10105	16 28.8	+45 50	5.55	*A0n F9*	2.94	.032	.022	g
10111[k]	16 29.2	+40 19	7.17	G5	0.6	.0083	.004	g
10113	16 29.3	+31 7	7.9	F2	2.8	.0011	.0004	f
10118[g]	16 29.7	+ 8 53	6.90	K0	1.7	.011	.005	f
10120	16 31.0	+47 27	8.4	F5	0.0	.018	.013	g
16h57[6]	16 32.5	−47 35	7.30	F5	0.2	.018	.012	F:
16h59	16 33.3	−47 28	8.0	A0	0.5	.0050	.003	F
16h61[b]	16 33.8	−48 34	5.90	Oe5	3.0	.015	.007	F:
10129	16 33.9	+53 8	5.20	A2	1.02	.018	.010	g
10150	16 35.6	+ 0 3	9.1	....	1.1	.028	.024	f
16h66	16 35.8	−36 53	6.56	G5	0 8	.021	.013	F:
10148	16 36.1	+29 12	9.8	....	0.5	.036	.031	f
10140	16 37.2	+70 0	8.04	F5	0.5	.017	.013	g
10158[2]	16 37.4	+30 18	10.2	....	0.6	.019	.016	g
10168	16 38.1	+13 49	8.4	K0	0.5	.031	.026	f
10167	16 38.1	+21 46	7.06	A0	1.5	.015	.010	f
10173	16 38.1	−27 16	6.38	A0	4.0	.019	.012	F:
10171	16 38.4	+23 51	8.1	G5	0.0	.010	.006	f
10169	16 38.9	+41 23	7.81	F0	0.5	.0068	.004	g
10184	16 40.0	+23 42	6.76	F2	0.3	.0084	.004	f
16h71	16 40.2	−48 10	8.6	A2	0.9	.019	.014	p
10196	16 40.6	− 0 35	7.9	F8	0.0	.0072	.004	f
10194	16 41.1	+28 32	7.09	F5	2.0	.0099	.006	f
10193	16 41.2	+35 55	8.9	G5	0.2	.0082	.005	f
10206	16 42.0	+ 8 32	9.5	G0	1.2	.0079	.005	g
10203	16 42.2	+35 9	7.27	G0	2.7	.018	.013	g
16h73[8]	16 43.0	−49 52	6.55	A5	0.01	.014	.008	F:
16h75[8]	16 44.2	−37 20	6.22	B9	2.02	.011	.006	F
10225	16 45.0	+13 26	5.95	A0	3.89	.021	.013	f
10224	16 45.1	+36 5	7.07	F2	1.8	.012	.007	g
10229	16 46.3	+ 9 35	6.77	*F4*	1.7	.016	.010	g
10227[13]	16 46.3	+46 10	4.86	*A4sp*	4.4	.036	.024	g
10227[3]	16 46.3	+46 9	10.3	....	0.1	.025	.022	g
10230	16 46.4	+ 1 23	5.47	*A2s*	1.9	.010	.005	g
10251	16 48.7	−21 53	8.55	A2	0.3	.0039	.002	F
10249	16 49.4	+21 21	8.1	K0	3.2	.017	.011	p
10257	16 49.6	−21 25	6.83	B8	0.1	.0068	.003	f
10260	16 50.3	− 5 1	8.85	G0	0.0	.0073	.005	f
10265	16 50.8	−22 59	5.60	A0	0.2	.016	.009	g
10266	16 51.2	−19 23	6.14	B8	1.1	.015	.008	F:
16h93[8]	16 52.1	−37 28	6.24	A3	0.0	0.021	0.013	F

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
10277[g]	16h53m5	+15° 18′	7.77	*G9*	1.0	0″.0040	0″.002	f
10285	16 53.8	+ 4 6	8.5	G0	0.1	.016	.012	g
10294[g]	16 54.8	+14 28	8.06	K0	1.0	.0047	.002	f
10276	16 54.8	+57 20	9.1	G0	0.1	.015	.012	g
10276[10]	16 54.8	+57 20	9.8	....	0.2	.0070	.005	g
10291	16 55.7	+51 57	8.9	G5	2.2	.010	.007	g
10279	16 55.9	+65 11	6.44	F0	0.5	.018	.011	g
10312[b]	16 57.2	+ 8 36	6.24	A0	1.2	.018	.010	g
16h107	16 59.4	−38 29	7.4	F2	0.4	.023	.016	F
10331	17 0.8	−13 48	7.35	F8	0.0	.015	.010	g
10336	17 0.8	+10 40	8 1	A5	1 1	.0048	.003	f
———[6]	17 1.1	−41 29	7.16	A3	0.6	.0041	.002	F:
10341[2]	17 1.5	+ 0 47	8.34	G0	1.0	.028	.024	g
10329	17 2.0	+59 43	8.19	*K5*	1.2	.060	.055	g
(7866)[14]	17 2.1	+ 6 56	9.1	K0	0.0	.037	.034	p
10353	17 2.4	−19 40	9.0	F8	0.3	.010	.007	F:
17h4[5]	17 2.9	−46 37	6.98	B2	1.47	.014	.008	F
10355[2, b]	17 3.1	− 0 57	6.02	*A0n*	2.0	.016	.008	g
10345	17 3 2	+54 36	5.06	*F6 F6*	0.03	.073	.053	g
17h8	17 4.3	−58 28	7.00	F2	2.0	.011	.006	F
10340	17 4.4	+69 56	7.89	F5	0.3	.010	.007	g
10374	17 4.6	−15 36	2.63	*A2s*	0.4	.019	.007	g
10376	17 5.0	− 0 37	7.9	A0	3.0	.0062	.004	f
10384[6]	17 5.4	−26 35	7.71	F8	1.0	.0072	.004	F
10388[g]	17 6.0	−26 55	6.84	G5	2.7	.014	.007	F
17h14	17 7.0	−38 10	6.77	F5	2.0	.0072	.004	F:
10395	17 7.5	− 3 56	8.4	F2	0.7	.012	.008	p
10398	17 8.2	+ 7 52	6.83	F0	1.9	.022	.015	g
10403[9]	17 9.2	+17 23	9.1	....	0.0	.012	.009	f
10417[g]	17 9.2	−26 27	4.56	*K3 K1*	0.04	.076	.045	g
10418[d, g]	17 10.1	+14 30	3.31	*M5 F8*	1.91	.022	.007	g
10423[9]	17 10.2	− 9 42	7.06	F5	0.5	.014	.009	g
10427	17 10.5	−19 13	8.1	A5	0.5	.019	.014	f
10429[g]	17 11.4	− 0 20	4.82	*K4*	3.0	.023	.010	f
10436	17 11.4	−26 31	6.90	A0	2.0	.019	.012	F:
17h33[6]	17 11.8	−59 20	7.05	A0	0.1	.0066	.003	F:
10442[5, 7, g]	17 11.9	−24 11	5.15	*K1 F5*	1.51	.010	.004	F:
10425	17 12.2	+56 15	7.9	*F4*	0.3	.010	.007	g
10452	17 12.8	−23 53	9.3	A	1.0	.010	.007	F:
10449[a]	17 13.6	+33 12	4.6	B3	5.4	.022	.010	f
10450	17 13.8	+32 11	8.6	F5	0.7	.0096	.006	f
10467	17 14.0	−21 26	9.34	G0	1.0	.020	.017	F
10465	17 14.1	−17 39	6.04	A0	1.1	.019	.012	g
10459	17 14.7	+32 46	8.9	....	0.5	.010	.007	g
10458	17 14.8	+45 42	8.9	F8	0.2	0.014	0.010	p

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
10448[1]	17ʰ15ᵐ.0	+60°49′	6.73	*A9n*	3.3	0″.036	0″.028	g
10460	17 15.4	+49 24	8.8	K0	0.5	.017	.013	g
10509	17 17.6	−21 37	8.17	F8	0.5	.013	.008	F:
17ʰ47	17 18.1	−30 26	7.8	K0	0.7	.018	.012	F:
17ʰ52[8]	17 19.5	−45 45	5.55	B9	0.3	.017	.009	F
10526	17 20.2	+37 14	4.14	*B9n*	0.95	.021	.010	g
10523	17 20 2	+42 13	9 3	G5	0.0	.013	.010	g
10547	17 20.6	−26 15	7.37	B9	2.3	.024	.018	F
10540	17 21.1	+30 50	8.7	....	0.5	.033	.028	f
10564	17 22.5	−25 26	7.05	F0	0.4	.0074	.004	F
10576	17 23.8	− 9 54	7.81	F2	0.1	.0077	.004	f
10575	17 24.0	+10 34	9.1	K5	0.5	.021	.016	f
—	17 24.0	−37 21	6.75	A0	0.4	.023	.015	F
17ʰ66	17 24.7	−40 58	7.46	B9	0.3	.0054	.003	F:
10567	17 24.9	+56 26	8.8	....	2.5	.014	.010	f
17ʰ70	17 25.9	−30 23	8.1	A5	0.6	.0076	.004	F:
10580	17 26.1	+56 14	9.1	....	0.2	.014	.011	g
10597	17 26.6	+50 58	6.97	F0	0.2	.0089	.005	g
10614	17 27.1	+ 2 54	7.7	B8	1.5	.0084	.005	g
10621	17 28.5	+28 53	8.1	F2	0.0	.011	.007	g
10648	17 30.8	+13 26	7.7	A3	2.2	.0096	.006	g
—	17 32.3	−72 10	6.66	F5	0.4	.053	.043	F
10669	17 32.8	+12 36	8.6	F5	0.5	.010	.007	f
10696	17 34.8	− 0 35	6.66	A0	0.0	.0062	.003	g
10699	17 36.7	+55 48	7.34	F5	0.6	.024	.018	g
10723	17 37.5	+16 0	5.58	*F1*	5.9	.045	.034	g
10722	17 38.1	+41 42	6.97	A2	0.3	.0073	.004	g
10741	17 39.0	+ 5 53	7.9	A0	0.8	.0029	.001	f
10748	17 39.4	+ 3 1	8.6	K0	2.7	.033	.029	f
10728	17 39.6	+63 44	6.83	F5	1.2	.025	.017	g
10765	17 41.1	+31 11	7.52	F8	0.5	.015	.009	f
10775	17 41.3	− 4 26	9.3	*G2*	1.8	.033	.032	f
10769	17 41.3	+17 45	7.9	K0	0.4	.028	.021	g
10781	17 41.5	− 1 10	8.1	*G1*	1.0	.033	.028	f
10773[g]	17 41.8	+30 34	8.9	G5	0.0	.0050	.002	f
10784	17 42.1	+14 49	8.09	G0	0.4	.016	.012	f
10777	17 42.5	+51 58	9.1	K2	0.5	.060	.063	f
10795[b]	17 42.7	+17 44	5.58	A0	2.0	.0096	.004	g
10819[g]	17 44.8	+ 0 33	8.5	G5	0.2	.0047	.002	g
10828	17 45.7	+ 7 16	7.7	F2	0.5	.0088	.005	g
10838[3]	17 45.9	−28 27	10.0	....	0.4	.030	.026	F
10827	17 46.2	+25 19	6.57	A2	1.5	.016	.010	g
17ʰ100	17 46.7	−34 42	6.20	B9	0.1	.010	.005	F
17ʰ101[5,g]	17 46.7	−34 52	5.68	K0	0.6	.0085	.003	F
10832	17 46.9	+35 28	8.1	G0	2.0	0.012	0.008	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
10850[g]	$17^h47^m.5$	$+15°21'$	6.54	K0	0.2	$0''.010$	$0''.004$	g
17^h105	17 48.9	−57 39	7.3	G5	0.6	.041	.033	F
10866	17 49.0	+29 42	8.1	A5	0.0	.0037	.002	f
10874	17 49.8	+36 5	8.5	G5	0.5	.0092	.006	f
10875[g]	17 50.0	+40 1	5.12	*K1*	3.9	.0075	.003	f
17^h110	17 51.2	−36 0	6.88	A0	2.5	.042	.034	F:
10899	17 51.3	+ 3 0	7.9	A2	0.3	.0053	.003	g
10904	17 51.9	+21 30	8.1	*G3*	2.4	.033	.028	g
10912	17 52.0	+ 0 5	6.14	A2	0.2	.0080	.004	g
10905	17 52.0	+18 21	6.59	A2	0.0	.0076	.004	f
10888	17 52.2	+53 58	9.3	....	0.0	.0070	.004	p
10911	17 52.9	+39 27	7.7	G0	1.9	.020	.014	f
10920	17 53.7	+41 16	8.8	F2	2.0	.040	.040	f
10947	17 53.9	− 6 51	8.6	B9	1.0	.013	.009	f
10949	17 54.4	+12 26	8.5	A2	0.4	.0058	.003	p
10961	17 55.6	+29 30	9.1	....	0.5	.015	.011	f
10984	17 55.8	−27 31	7.3	B8	0.6	.011	.006	F:
10971[9]	17 56.1	+21 54	8.8	G0	2.0	.011	.008	f
10987	17 56.5	+ 3 32	8.1	A0	1.1	.0095	.006	f
10995[g]	17 56.5	−27 5	7.4	G0	1.0	.012	.006	F
10990[b]	17 56.7	+ 1 19	4.44	*B9n*	5.6	.018	.008	f
10982	17 56.9	+26 33	8.3	G0	1.0	.017	.012	p
10993	17 57.2	+21 36	4.42	*A1n G3*	0.08	.016	.007	g
11002	17 57.7	+ 1 37	9.1	....	1.0	.016	.013	f
11015[8]	17 58.1	−24 15	8.1	....	1.5	.0082	.005	F
10988	17 58.2	+52 51	7.66	G0	1.0	.028	.022	g
11001	17 58.5	+40 11	7.82	A3	0.0	.013	.009	g
17^h128	17 59.0	−36 35	7.4	B8	1.0	.0059	.003	F:
11012	17 59.2	+26 3	8.5	....	0.0	.0093	.006	f
11010	17 59.6	+44 15	7.22	F2	2.0	.023	.017	g
11023	17 59.9	+39 22	8.7	F5	0.2	.012	.008	f
11065	18 1.0	−19 23	8.6	G4	0.0	.012	.009	g
11056	18 1.1	+12 0	6.46	A0	0.36	.015	.009	f
11016	18 1.3	+65 57	7.64	F5	1.6	.0055	.003	f
11078	18 2.1	−22 17	8.3	B5	0.5	.015	.010	F
11084[g]	18 2.3	−27 53	7.14	G5	1.0	.014	.008	f
11083	18 3.0	+ 9 45	10.4	....	0.1	.015	.013	f
11080	18 3.1	+19 39	7.40	A2	1.3	.0066	.004	g
11074	18 3.3	+40 22	7.02	A0	1.0	.012	.007	f
18^h6[18, g]	18 3.6	−30 45	5.58	K0	2.4	.0098	.004	F
11089	18 3.8	+26 5	5.21	A3 A3	0.08	.021	.012	f
11112	18 3.9	−29 14	8.1	A2	0.2	.0074	.004	F:
11090[b]	18 4.6	+49 42	6.31	A0	4.1	.015	.008	g
11110	18 4.7	+ 6 8	7.6	F5	0.5	.0099	.006	g
11127	18 5.3	−19 52	6.33	A2	0.4	0.019	0.012	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
11123	18h 5m.7	+16° 27′	6.14	*F3* Ao	1.1	0″.0090	0″.004	g
18h16	18 6.5	−57 53	7.7	G5	0.2	.023	.017	F
11166	18 7.9	−15 38	7.06	F5	2.0	.016	.010	f
11160	18 7.9	+ 5 48	8.0	B9	3.0	.016	.011	g
11155	18 8.2	+27 37	8.2	A2	0.1	.010	.007	f
18h23[6]	18 8.8	−46 3	7.21	A2	1.0	.022	.015	F:
11170	18 9.3	+38 34	8.6	....	0.0	.011	.007	f
11174	18 9.5	+41 22	7.9	F5	1.2	.015	.010	f
11180[9]	18 10.2	+43 13	9.3	....	0.5	.0083	.006	p
11214	18 10.7	−25 2	8.55	Ao	1.2	.0075	.005	F
11208	18 11.1	+19 44	7.80	F2	0.8	.013	.008	f
11240	18 12.7	−18 39	6.38	Bo	2.0	.031	.020	g
11233	18 13.9	+51 17	8.7	G5	1.6	.018	.014	f
18h37[b, g]	18 14.0	−61 32	4.25	K2	4.0	.021	.008	F:
11257	18 14.5	− 5 0	7.80	Go	1.3	.025	.019	g
11262	18 14.7	− 8 2	6.51	*F2*	2.5	.037	.028	g
11247[9]	18 14.8	+43 48	8.7	....	0.7	.012	.008	g
11268	18 15.1	−19 42	7.89	Fo	1.6	.013	.009	F
18h42[6]	18 16.1	−42 50	8.02	F5	0.2	.014	.009	F:
11273	18 16.5	+22 45	6.69	B9	3.3	.018	.012	p
11275	18 16.9	+27 29	7.13	*G4*	5.0	.034	.028	f
11287	18 17.6	+21 28	6.64	B8	1.9	.0069	.003	f
11318[g]	18 19.3	− 6 39	7.25	Go	1.3	.0084	.004	f
11325[5, 7, g]	18 19.4	−20 36	4.96	*K1* Ao	2.52	.014	.006	F
11330	18 19.7	−21 6	8.4	A2	0.1	.017	.012	F:
11324	18 19.8	− 1 38	6.11	F5	0.2	.018	.011	g
11339	18 21.0	+ 0 44	6.90	A3	0.2	.0074	.004	g
11334	18 21.0	+27 20	6.20	Ao	1.0	.013	.007	g
11354	18 21.5	−26 41	6.23	A5	0.2	.016	.009	F
11353[a]	18 22.1	+ 0 8	5.33	Ao G	2.10	.015	.007	g
11311[a]	18 22.2	+71 17	4.24	Aop	1.8	.0089	.003	g
11316	18 22.3	+68 52	9.3	Ko	2.0	.015	.011	f
11343	18 22.5	+46 50	7.61	Ao	1.9	.0064	.004	f
11336[b]	18 22.5	+58 45	4.85	A2	2.65	.026	.013	g
11344[13, g]	18 22.7	+48 42	7.24	G5	0.7	.0067	.003	f
11378[g]	18 22.8	−25 6	7.40	Go	0.2	.013	.007	F:
11372	18 23.4	+19 14	7.6	F5	0.5	.012	.007	f
11373	18 23.7	+24 38	6.82	B9	1.9	.011	.006	f
11396	18 24.9	+ 1 7	8.04	Ao	1.6	.0024	.001	f
11433[3]	18 26.0	−19 12	8.8	....	0.4	.0033	.002	F
11430	18 26.8	− 7 54	8.0	Ao	0.7	.018	.013	f
11432	18 27.2	+ 6 43	7.34	F5	0.8	.010	.006	g
11424	18 27.4	+32 11	7.30	Ao	0.6	.012	.007	f
18h64[5]	18 27.8	−34 54	6.94	Fo	0.19	.012	.007	F
11461	18 29.2	−21 45	9.3	Go	0.8	0.022	0.019	F:

TABLE 53—*Continued*

ADS	$\alpha 1900$	$\delta 1900$	Mag.	Sp.	Δm	h_1	d	Grade
11454	$18^h29^m.4$	$+17°39'$	7.02	F5	0.8	$0''.012$	$0''.007$	f
11484	18 31.3	+11 39	7.37	A2	0.1	.0084	.005	g
11479[g]	18 31.4	+23 31	5.76	*G8*	0.3	.013	.006	g
11483	18 31.4	+16 54	6.17	Go	0.4	.035	.024	g
11468[g]	18 31.7	+52 16	5.47	*G5*	0.0	.013	.006	g
11504	18 32 9	+33 23	5.46	*B8s*	4.48	.0095	.005	f
11500	18 33.0	+41 12	6.86	Ao	0.0	.0065	.003	f
11524[22]	18 33.6	+ 8 44	var	*M6e* G	......	.0073	....	g
11530[g]	18 34.1	+16 27	7.60	G5	0.0	.0091	.005	g
11529	18 34.5	+28 37	8.0	Fo	1.0	.0064	.004	f
11534	18 34.9	+35 58	7.20	A5	1.1	.018	.012	g
11546	18 35.0	+20 51	7.54	B8	1.2	.010	.006	f
11558	18 36.6	+52 14	6.86	*A1n Go*	0.2	.0079	.004	f
11574	18 36.9	+24 44	7.79	A5	0.0	.0092	.006	g
11579[g]	18 37.4	+30 11	6.88	G5	0.5	.017	.009	g
11608	18 38.0	−19 59	8.23	G5	0.1	.021	.016	F
11568	18 38.5	+67 2	8.1	G5	0.5	.023	.017	g
11622	18 38.6	−25 54	7.7	F8	0.6	.016	.010	F:
11617	18 38.9	+ 2 31	7.7	Go	0.5	.016	.011	g
11621	18 39.8	+35 27	8.4	A	0.5	.017	.013	f
11623	18 40.0	+31 35	9.1	....	0.5	.024	.020	f
11642	18 40.3	−10 36	6.84	F2	1.0	.010	.006	p
11640[b]	18 40.5	+ 5 23	5.72	Ao	0.0	.014	.006	g
11635	18 41.0	+39 34	4.68	*A2n A4n*	0.96	.027	.015	g
11635[10]	18 41.1	+39 30	4.50	*A3n*	0.23	.038	.023	g
11667	18 41.3	− 1 4	5.68	*A6s*	1.43	.017	.009	f
11632	18 41.8	+59 29	8.41	*M4 M5*	0.9	.20	.20	g
11669	18 42.2	+34 25	6.91	A5	1.4	.0096	.005	f
11584	18 42.3	+77 35	6.94	Fo	0.2	.0076	.004	g
18^h83	18 43.1	−33 42	7.80	F2	0.1	.0086	.005	F:
11661[g]	18 43.1	+60 56	6.23	*G7*	2.8	.013	.006	p
11685[g]	18 43.4	+31 17	7.10	G5	2.3	.011	.005	f
11715	18 44.6	+21 3	6.93	B5	1.5	.013	.007	p
11722	18 45.0	+10 34	8.03	A3	0.4	.012	.008	g
11698[a]	18 45.0	+49 19	7.18	*F1* A	2.2	.017	.010	g
11697	18 45.5	+59 13	8.1	F8	0.5	.0054	.003	p
11735	18 45.6	+11 24	7.06	Ao	2.0	.012	.007	f
11750[g]	18 46.0	+10 52	6.63	K2	1.05	.011	.005	f
11766	18 47.2	+10 39	8.3	Ao	1.2	.0093	.006	g
11791[2]	18 48.4	− 5 40	8.9	G5	0.6	.021	.017	g
11828	18 49.7	−28 15	7.86	Fo	0.5	.019	.014	F
11816	18 50.2	+20 29	6.83	Ao	1.9	.0072	.004	p
11811	18 50.5	+37 15	7.7	Go	0.5	.019	.014	g
11842	18 50.8	+ 3 19	6.94	A2	0.0	.010	.006	g
11853	18 51.2	+ 4 4	4.10	A5 A5	0.87	0.034	0.019	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
11869	18h 53m.0	+25° 57′	7.34	A0	0.1	0″.0077	0″.004	g
18h99	18 54.6	−63 56	7.31	A0	0.0	.010	.006	F
11900	18 54.9	+36 17	8.0	A3	0.6	.014	.010	f
11916[g]	18 55.4	+12 45	7.14	*K4 A0n*	1.4	.015	.007	f
11915	18 55.4	+14 46	8.1	F2	1.8	.026	.022	f
11914[g]	18 55.5	+29 28	8.6	G0	0.0	.0070	.004	f
18h103	18 55.8	−45 51	7.80	B9	0.7	.010	.006	F:
11949[6]	18 56.2	−28 47	8.36	G0	0.5	.015	.011	F
11953	18 56.8	− 6 51	9.4	A5	0.1	.0050	.003	g
11870[b, d]	18 56.9	+75 39	6.18	A0	1.1	.0078	.003	g
11971[3]	18 57.6	− 0 51	9.25	*G6*	2.1	.028	.026	g
11956	18 57.6	+19 2	7.46	G5	0.2	.013	.008	g
11989	18 58.4	−21 41	6.87	G0	0.1	.037	.028	g
11977[a]	18 58.9	+31 15	8.2	*A7s G8*	1.6	.046	.039	g
11999	18 59.6	+14 38	8.0	F5	0.4	.020	.015	g
11979	18 59.8	+52 7	6.42	*G8*	2.7	.021	.013	p
11998[9]	18 59.8	+26 33	9.0	F8	0.0	.012	.008	g
12002	19 0.1	+35 36	7.7	A3	0.0	.0064	.003	p
12010[b]	19 0.5	+23 11	6.94	B3	1.7	.022	.013	f
11997	19 0.6	+51 26	8.1	F8	0.3	.023	.018	g
12026[b]	19 0.8	+13 43	3.02	*B9n*	9.0	.042	.020	f
12029	19 0.9	+ 6 24	6.88	*F4 G4*	1.9	.033	.025	g
12037	19 1.5	+ 7 0	6.72	F2	0.7	.0088	.005	g
12032	19 1.6	+27 8	8.0	F5	0.0	.0043	.002	g
12040	19 2.2	+30 17	8.2	*G2 G7*	1.2	.017	.012	g
12054	19 2.6	+ 8 2	9.07	G0	0.2	.0084	.006	g
12015	19 3.0	+69 18	8.8	K0	0.0	.0092	.006	f
12057	19 3.2	+24 22	8.7	A2	0.0	.0037	.002	f
12061	19 3.6	+32 21	5.04	*A7n*	4.1	.041	.028	g
12071	19 4.1	+29 38	8.0	A0	0.5	.0085	.005	f
12079	19 4.2	+26 56	8.0	A0	0.0	.0067	.004	g
12075[b]	19 4.4	+38 46	7.48	A3	1.1	.0063	.003	p
12095	19 4.5	+11 42	8.0	A0	2.3	.0045	.002	f
12089	19 5.4	+48 55	8.0	K2	4.5	.064	.062	f
12101	19 5.4	+34 26	6.54	*G1*	1.3	.054	.044	g
12126[a, g]	19 5.6	− 7 35	6.76	G0	0.4	.0082	.004	g
12129	19 6.2	+ 7 58	7.39	A5	2.8	.037	.032	f
12104	19 6.3	+55 10	7.31	A3	0.9	.0093	.005	g
12147	19 7.1	+ 2 27	6.75	B9	0.8	.0056	.003	f
12168	19 7.8	−14 37	7.81	F5	2.3	.024	.018	f
12160	19 8.1	+16 41	6.44	*B9s*	1.3	.0032	.001	p
12144	19 8.4	+54 19	8.5	....	0.5	.0074	.005	g
12189	19 8.4	−27 29	8.0	A0	0.5	.017	.012	F:
12113	19 8.6	+71 55	7.00	F8	0.3	.0074	.004	f
12172	19 8.7	+18 58	8.3	F0	0.7	0.0095	0.006	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
12169	$19^h\ 9^m.5$	+49°39′	5.97	*G3 G5*	0.22	0″.073	0″.058	g
12201	19 9.9	+18 54	7.69	*F5*	1.5	.017	.012	g
12228	19 11.2	+19 51	8.1	G5	1.2	.014	.010	g
12236	19 11.5	+15 59	7.04	F0	1.2	.0041	.002	p
12239	19 11.9	+27 17	6.26	B9	0.1	.0045	.002	f
12254	19 11.9	− 3 33	8.9	A2	2.0	.013	010	g
12246	19 12.2	+28 7	8.1	A2	1.3	.010	.007	g
12289[g]	19 13.5	+ 0 54	5.32	*K0*	4.0	.010	.004	p
12287	19 13.5	+22 51	5.40	*B4n*	3.8	.014	.007	f
12288	19 13.6	+ 6 9	8.7	G5	0.0	.011	.007	f
12322	19 15.2	+ 3 52	7.9	K0	0.6	.013	.008	p
12296	19 15.9	+63 2	6.91	F5	1.1	.016	.010	g
12336	19 16.6	+18 57	6.77	F5	1.7	.030	.021	g
12348	19 16.8	− 7 47	8.7	F2	0.0	.0088	.006	f
12355	19 17.7	+22 19	7.7	B5	1.6	.011	.007	f
19^h21	19 18.7	−32 26	7.9	F0	1.1	.016	.011	F
12374	19 19.2	+41 54	9.0	....	1.0	.0067	.004	g
12366	19 19.2	+52 11	6.86	A5	0.0	.0073	.004	f
12414	19 20.2	+ 2 15	8.7	F8	0.6	.012	.009	g
19^h26	19 21.2	−44 5	7.6	F8	1.2	.016	.011	F
12429	19 21.3	+14 24	7.9	F5	2.9	.020	.015	p
12453	19 21.6	−29 42	7.88	G0	0.7	.016	.011	F
12449	19 22.2	+12 40	9.1	F0	0.5	.0096	.007	f
12451	19 22.4	+20 57	7.7	B8	0.12	.0074	.004	p
12469	19 22.6	−12 21	7.36	*G7*	0.3	.035	.028	g
12472	19 22.8	− 9 45	8.1	F0	0.1	.024	.019	f
12464	19 23.1	+20 28	8.7	B9	1.5	.012	.009	p
19^h32	19 23.4	−60 29	7.04	A2	0.4	.012	.007	F:
12477	19 23.6	+17 26	8.3	K0	2.0	.024	.019	f
12506[7,g]	19 23.7	−27 11	5.53	*K3*	2.8	.023	.011	F
12495	19 24.0	+20 7	8.7	F8	1.3	.018	.014	f
12515	19 24.9	+12 12	7.67	A3	1.2	.0086	.005	g
12538	19 26.1	− 2 19	6.73	B8	1.7	.0082	.004	f
12540[a,g]	19 26.7	+27 45	3.10	*K0 B8n*	2.12	.028	.009	p
12557	19 27.2	+17 34	8.2	*G2*	3.0	.034	.030	f
12561[10]	19 27.8	+36 30	8.0	G0	0.5	.012	.008	p
12584	19 28.0	+ 5 33	8.7	F8	1.2	.013	.009	f
12552	19 28.2	+56 26	6.78	A0	0.5	.0048	.002	p
12567	19 28.4	+47 16	7.18	A3	0.5	.010	.006	g
12524	19 28.7	+73 9	7.65	F2	0.0	.0058	.003	f
12594	19 29.0	+20 12	7.20	*A5*	1.5	.0094	.005	p
12580	19 29.5	+52 46	8.5	A0	0.5	.021	.016	f
12623[g]	19 30.2	+17 55	7.20	G5	1.2	.0081	.004	g
12618	19 30.5	+42 8	8.3	G0	2.5	.033	.029	f
12654[b]	19 30.6	−25 6	4.66	B9	3.0	0.0052	0.002	F:

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
12631[a]	$19^h30^m.8$	+23° 15′	8 1	A0	0.0	0″.0074	0″.004	g
12664	19 31.3	−10 39	8.5	*K5*	1.6	.050	.046	g
12638	19 31 4	+33 59	8.0	A2	2.7	.013	.009	p
12648	19 31.5	+23 3	9.1	....	0.1	.0070	.005	g
12626	19 32 1	+61 49	7.9	G0	0.8	.013	.008	g
12662	19 32.2	+39 48	7.90	A3	2.0	.012	.008	f
12679	19 32.3	+ 8 6	8.3	A3	1.7	.011	.008	g
12667	19 32.6	+35 26	8.6	A	0.1	.0080	.005	g
19^h45[R]	19 32.9	−39 58	7.42	G5	2.5	.013	.007	F
12697	19 33.1	+22 35	9.2	G0	0.5	.016	.012	p
12708	19 33.2	+ 0 7	7.38	*A3n*	2.1	.011	.007	f
12687	19 33.2	+40 47	8.0	A0	1.8	.0048	002	p
12715	19 33.3	−10 23	6.60	A5	1.9	.019	.013	g
12706	19 33.4	+16 1	8.9	A0	0.8	.0055	.003	f
12608	19 34.3	+78 3	7.06	F0	0.7	.0089	.005	g
19^h51[6]	19 34.4	−53 11	7.29	A3	1.1	.023	.016	F
12752	19 35.1	+22 2	7.22	F2	0.0	.012	.007	g
12798	19 37.8	+27 9	6.74	B8	0.5	.0062	.003	g
12808	19 37.9	+11 35	5.32	*F3* A3	1.2	.0018	.0006	g
12807	19 38.0	+17 11	8.7	G0	1.2	.0099	.007	f
12789	19 38.7	+60 16	6.21	A2	2.3	.035	.026	f
12815	19 39.1	+50 18	5.56	*G3 G2*	0.11	.090	.072	g
12803	19 39.4	+62 25	7.30	F5	0.5	.013	.008	g
12831	19 39.5	+40 29	6.72	A0	1.5	.0075	.004	f
12850[a, R]	19 39.8	+26 54	6.56	*G4* A0	3.5	.0090	.004	f
12864	19 40.2	+10 32	7.36	B3	0.0	.0029	.001	p
12851	19 40.3	+38 5	6.97	B8	0.3	.0073	.004	g
19^h56[R]	19 40.4	−65 9	7.25	G0	0.1	.011	.006	f
12887	19 40.6	−21 46	7.44	A5	2.5	.0024	.001	F:
12872	19 40.8	+20 39	8.2	K0	1.4	.018	.013	f
12882	19 40.9	+ 4 0	7.04	G0	4.3	.037	.030	f
12886	19 40.9	− 3 32	9.2	A0	0.0	.0056	.003	p
12909	19 41.2	−20 8	7.9	G0	0.7	.020	.014	f
———	19 41.5	−55 40	8.3	G5	0.6	.028	.023	F
12917	19 41.6	−22 4	7.9	G0	1.0	.016	.011	F
12911[R]	19 41.7	− 8 24	7.9	K0	0.4	.0064	.003	g
12889	19 41.8	+33 22	7.64	*K5 K5*	0.2	.058	.050	g
12893	19 42.0	+35 50	6.00	A0	0.55	.028	.018	f
12924	19 42.2	− 2 3	8.9	A2	0.0	.0081	.005	f
19^h61	19 42.3	−59 27	5.54	A2	1.7	.014	.007	F
12898	19 42.4	+43 15	9.0	F	0.5	.0091	.006	f
12913	19 42.6	+33 30	5.03	*F3 K6*	3.36	.058	.042	g
12966	19 44.0	+ 5 30	8.4	A0	0.4	.0044	.002	f
12962	19 44.0	+11 34	5.70	*F2* A2	0.8	.0097	.005	g
12961[a]	19 44.1	+14 49	7.64	F5	0.3	0.016	0.011	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
12965	19h 44m.6	+36° 55′	7.6	A2	0.3	0″.0090	0″.005	f
13028	19 46.6	−10 36	7.14	F2	0.4	.015	.009	g
13043	19 47.8	+23 1	8.4	....	2.8	.018	.014	f
13059	19 48.3	+14 10	9.0	....	0.1	.0061	.004	g
13007[g]	19 48.5	+70 1	3.99	*G3 F6*	3.02	.026	.012	g
13095	19 49.3	−20 36	9.6	A2	0.6	.0065	.004	F
13082	19 49.5	+15 2	7.04	F8	1.4	.028	.020	g
13104	19 49.9	− 7 0	6.45	F2	0 5	.016	.010	g
13110	19 50.4	+ 6 9	3.90	*G8*	8.0	.071	.049	f
13109	19 50.7	+24 57	9.0	....	0.2	.0098	.007	g
13147[9]	19 51.8	+ 1 39	8.5	A0	0.3	.0042	.002	f
———[3]	19 52.3	−55 39	9.7	....	0.1	.025	.020	F:
13156	19 52.4	+ 4 57	8.55	F8	0.1	.012	.008	g
13149[g]	19 52.6	+34 49	4.03	*K0*	9.5	.059	.035	f
13161	19 52.9	+13 52	8.1	A3	0.0	.0039	.002	f
13148	19 53.0	+52 10	4.80	*A0n*	2.50	.015	.008	g
13169[a]	19 53.1	+ 4 40	8.7	G5	0.0	.018	.014	g
13178	19 53.1	− 2 30	7.04	F5	0.6	.016	.010	g
13172	19 53.2	+ 5 32	8.7	A3	0.3	.0066	.004	f
———	19 54.5	−38 52	7.60	F2	0.5	.024	.018	F
13186	19 54.6	+41 59	6.48	A2	1.8	.0061	.003	g
13196	19 54.7	+33 0	7.12	*F2*	0.7	.0049	.002	g
13198	19 54.9	+37 50	6.28	B5	1.1	.0092	.004	f
13212	19 55.4	+31 50	8.4	G5	0.4	.011	.007	g
13204	19 55.4	+35 16	7.92	B9	0.5	.0086	.005	p
13209	19 55.9	+47 5	7.61	K0	0.0	.016	.010	f
13248	19 56.1	+ 1 4	9.09	F8	0.6	.012	.009	g
13236	19 56.2	+29 38	7.26	A0	4.0	.015	.010	f
13256[b]	19 56.6	+10 28	6.89	*F4*	0.2	.018	.010	f
13251	19 56.7	+29 33	8.1	G5	4.4	.037	.032	p
13277	19 57.8	+24 39	5.32	*F0*	0.4	.0094	.004	g
13269	19 58.0	+47 59	8.3	G5	0.0	.012	.008	f
13270	19 58.2	+48 4	9.3	....	0.0	.0070	.005	f
13307	19 58.9	+15 11	8.74	A0	0.3	.017	.013	f
13323	19 59.6	+16 43	8.7	F0	0.7	.0068	.004	f
13334	19 59.8	− 4 36	7.65	F5	1.0	.013	.008	g
13330	19 59.8	+ 8 58	7.9	B9	0.2	.0079	.005	f
13312[b, d]	19 59.8	+35 45	6.69	Oe	0.6	.0052	.002	g
13329	20 0.2	+30 15	8.16	A3	0.2	.0029	.001	p
13357	20 0.5	−28 39	7.5	F2	0.6	.010	.006	F:
13384	20 2.0	+12 39	7.23	F2	1.3	.0069	.004	f
13393	20 2.9	+20 49	8.1	G0	1.4	.014	.010	f
13403	20 3.0	+ 9 6	6.38	F5	2.1	.024	.015	f
13422	20 3.5	−19 42	8.7	A2	2.5	.0084	.006	F:
13371	20 3.5	+63 36	6.18	*A3s*	3.1	0.032	0.023	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
13392	20h 4m.2	+63° 25′	8 8	G0	0.0	0″.018	0″.014	f
13434	20 5.1	+16 31	7 7	*G9*	1.5	.035	.028	g
13442	20 5.5	+20 37	6.32	*F1 G5*	2.13	.024	.016	g
13473[g]	20 7.0	+36 45	8.0	G5	2.6	.0078	.004	f
20h13[g]	20 7.3	−34 25	6.74	G0	1.5	.0056	.002	F
13449	20 7.3	+61 47	6.57	A0	0.2	.0044	.002	g
13506	20 7.5	+ 0 34	6.23	A0	0.3	.016	.009	g
13471	20 7.5	+49 31	7.83	F5	0.3	.024	.018	f
13511	20 7.6	− 3 18	6.91	A0	2.5	.016	.010	f
13517	20 8.3	+28 50	9.0	....	0.1	.0063	.004	f
13543	20 9.4	+23 56	6.48	A0	3.1	.017	.011	f
13553	20 9.7	+21 55	7.08	A0	0.0	.010	.006	f
20h19	20 10.6	−32 55	7.46	G5	0.4	.012	.006	F
13560	20 11.0	+52 49	7.02	*F5 K2*	2.1	.040	.032	g
13611	20 12.3	+43 21	8.10	F5	0.0	.0065	.004	f
13524	20 12.3	+77 25	4.40	B9	3.69	.015	.008	g
13636	20 13.2	+39 23	7.65	A0	0.5	.0088	.005	f
13649	20 13.4	+26 4	8.4	F8	0.3	.012	.008	g
13648[g]	20 13.4	+28 50	6.38	K0	4.5	.017	.009	f
20h22	20 14.0	−55 7	8.44	G5	1.6	.032	.028	F
13660	20 14.1	+32 53	8.0	B9	0.5	.0043	.002	f
13665	20 14.1	+28 54	8.8	....	0.8	.010	.007	g
13752[g]	20 14.1	−28 9	8.9	G0	0.2	.0060	.003	F:
13655	20 14.3	+45 19	8.7	A5	0.3	.0091	.006	f
13672[b]	20 14.6	+40 25	5.82	*B8n*	2.2	.0095	.004	f
13688[3, g]	20 14.7	+14 3	10.5	G	0.7	.0030	.002	f
13681	20 14.9	+34 48	8.6	....	0.0	.0029	.001	p
13686	20 15.2	+37 58	7.7	B1	0.0	.0030	.001	g
13692[d]	20 15.9	+55 5	5.71	*A7s F3*	1.4	.019	.010	f
13739	20 16.4	+ 0 15	8.78	F8	2.2	.0091	.006	p
13728	20 16.6	+39 5	6.07	*A0n*	2.0	.0067	.003	g
20h32	20 17.1	−42 45	5.68	A3	1.2	.019	.011	F
13750	20 17.2	+23 26	8.8	F8	0.1	.012	.008	g
13744	20 17.7	+44 17	8.9	....	0.8	.026	.022	f
13767	20 18.0	+13 1	8.4	F2	1.5	.0099	.007	g
13777	20 18.7	+20 33	8.0	F2	0.2	.0052	.003	g
13786	20 19.5	+42 40	6.33	K0	3.7	.022	.014	f
13830	20 21.0	+35 27	8.7	F8	1.3	.0088	.006	f
13813	20 21.5	+63 53	8.8	B9	0.6	.0066	.004	p
13847	20 21.9	+39 46	7.67	G5	1.1	.027	.020	f
13850	20 23.0	+59 16	6.48	A0	0.2	.0089	.005	g
13887	20 23.2	−18 9	4.96	*F1*	2.5	.029	.017	g
13894	20 24.2	+ 6 50	8.5	G0	0.2	.016	.012	g
13920	20 25.5	+10 34	5.92	A0	2.0	.0072	.003	f
13946[3]	20 26.4	+10 55	7.26	A0	0.5	0.0058	0.003	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h	d	Grade
13944	20h 26m5	+15° 28′	6.80	A2	0.0	0″.0004	0″.005	g
13964	20 27.7	+25 28	6.29	A2 G	1.2	.016	.009	g
13914	20 28 0	+72 25	8.7	....	0.2	.0064	.004	g
13986	20 28.2	+13 38	8.3	....	0.3	.011	.007	g
13997	20 28.5	+ 5 6	7.66	A2	0.4	.0092	.005	f
14001	20 29.0	+ 4 13	9.5	....	0.5	.010	.007	g
20h51	20 29.6	−45 54	7.48	A3	0.9	.0070	.004	F
14004	20 29.6	+27 47	8.4	A0	0.9	.016	.012	f
14054	20 31.4	−13 5	7.63	*F1*	1.0	.024	.018	f
14043	20 31.8	+42 32	9.5	....	2.9	.017	.014	f
14099	20 33.7	−15 18	5.30	B5	1.3	.010	.005	g
14077	20 34 0	+42 13	9.1	....	1.3	.022	.019	g
14115	20 34.1	−29 13	8.61	F8	0.6	.012	.008	F:
14126	20 35 9	+40 13	5.93	B8	0.3	.0073	.003	g
14148	20 36.2	+21 34	7.13	B9	0.2	.0092	.005	p
14149b	20 36.5	+29 27	6.09	A0	5.0	.019	.011	f
14158g	20 37.0	+31 57	5.77	*G7* A	2.2	.0088	.004	f
14188	20 37 7	−19 50	8.78	F0	0.3	.020	.016	F
14206	20 38.9	+16 36	8.56	F8	0.2	.016	.011	f
14218	20 39.0	−21 15	7.6	K0	2.2	.013	.008	F:
14196	20 39.8	+57 2	6.87	A3	0.8	.011	.006	f
14233	20 40.2	+11 57	6.70	*A7n*	0.7	.015	.009	g
14260	20 41.4	+23 54	8.8	A7	2.0	.011	.008	f
14280	20 41 4	−26 47	7.12	A2	1.5	.022	.016	f
14270	20 41.6	+15 33	7.04	*G9 G8*	0.7	.043	.034	g
14259g	20 41.6	+30 21	4.34	*G7*	4.90	.038	.020	g
14279g	20 42 0	+15 46	4.12	*K1 F5*	0.98	.063	.033	g
14286	20 42.7	+25 3	8.16	G5	0.2	.013	.009	g
14293	20 42.8	+ 5 38	5.59	A0	3.6	.010	.005	f
20h72	20 43.3	−62 48	5.84	A2	0.00	.016	.009	g
14296	20 43.5	+36 7	4.47	*B3n*	1.0	.017	.008	g
14295	20 43 6	+42 3	7.06	B9	1.1	.026	.019	g
14359	20 46.1	+ 6 1	7.9	K0	0.1	.012	.008	f
14355	20 46.5	+30 32	6.75	*F2*	3.3	.015	.009	f
14368	20 46.9	+ 5 22	8.4	F5	0.1	.011	.007	g
14377	20 47.2	−16 33	8.0	G5	1.3	.013	.009	p
14370	20 48.0	+51 3	6.82	F0	0.9	.013	.008	f
14397	20 48.9	+28 46	7.7	A0	0.6	.0044	.002	p
14430g	20 50.7	+ 4 9	6.04	*G6*	1.5	.0076	.003	f
14421	20 50.7	+32 19	7.38	G0	0.1	.018	.012	g
14420	20 50.8	+36 42	7.24	A0	3.3	.012	.008	g
14412	20 51.3	+58 56	6.82	F2 A2	0.3	.041	.032	g
14515	20 55.2	+19 40	8.4	F0	0.5	.0054	.003	p
14504	20 55.3	+50 4	5.48	*B8n*	1.3	.0075	.003	g
14528	20 55.4	− 8 44	8.2	*G7*	3.5	0.047	0.045	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
14538[g]	20ʰ56ᵐ3	+20°33′	9.0	G5	0.0	0″0030	0″001	g
14556	20 57.3	+ 6 48	6.64	*F3*	0.0	.010	.005	f
14573	20 58.0	+ 1 8	6.50	F5	0.7	.022	.015	g
14558	20 58.0	+38 51	7.50	F0	0.6	.012	.008	g
14569	20 58.5	+37 15	7.7	G5	0.0	.019	.013	f
14592[g]	20 58.7	− 6 13	5.63	*G4* A3	1.42	.0068	.003	p
14590[g]	20 59.2	+28 42	6.97	K0	4.5	.016	.009	f
14585[b]	20 59.3	+45 27	6.23	*B5n*	1.3	.0058	.002	f
14575	20 59.4	+56 16	5.74	B9	1.0	.0088	.004	g
14600	20 59.6	+ 7 22	7.9	F2	2.0	.0044	.002	p
14602[3]	20 59.7	+ 3 9	9.0	....	0.0	.021	.017	g
14615	21 1.5	+52 0	7.38	A0	1.5	.0079	.004	f
14638	21 1.6	−14 19	7.24	*K1*	4.0	.037	.029	f
14636	21 2.4	+38 15	5.12	*K6 M0*	0.71	.37	.36	g
14666	21 3.0	+ 4 45	6.68	A2	1.5	.0092	.005	g
14639	21 3.0	+53 58	8.8	....	4.0	.019	.015	f
21^h7	21 3.1	−54 59	7.18	G0	0.2	.034	.026	g
14635[2]	21 3.2	+60 5	9.0	....	1.0	.0096	.007	g
14634	21 3.3	+61 46	7.58	A0	0.5	.020	.014	f
14682[b]	21 4.4	+29 48	5.57	A0	2.14	.016	.008	g
21^h8	21 4.8	−60 27	7.5	F0	0.2	.013	.008	p
14702	21 5.5	+ 9 44	4.76	cF1	6.2	.020	.011	f
14715	21 6.1	+ 9 8	7.70	A3	0.2	.0067	.004	p
14725	21 6.3	− 3 31	7.56	K0	3.5	.027	.020	f
14711	21 6.5	+43 35	8.1	A0	1.7	.0094	.006	f
14736	21 6.8	−15 24	7.24	*G2 F9*	0.4	.045	.037	g
14738	21 7.4	+23 45	8.0	*F9*	2.0	.013	.009	g
14748	21 8.2	+27 56	8.5	F8	0.4	.0065	.004	f
14759	21 8.5	+ 6 49	7.40	F0	2.3	.0075	.004	p
14770[6]	21 8.9	−26 19	8.07	A3	0.2	.0024	.001	F:
14749	21 9.3	+59 35	5.65	B2	1.0	.0054	.002	g
14775	21 9.5	− 1 15	7.31	A0	0.2	.0099	.006	g
14766	21 9.8	+46 30	8.7	F0	0.1	.0099	.007	f
14778[g]	21 10.5	+40 44	7.17	*G5*	0.40	.0097	.005	g
14784	21 11.3	+57 53	7.02	A0	0.0	.013	.008	g
14804	21 11.4	− 8 4	8.0	F2	0.0	.011	.007	g
14783[g]	21 11.7	+63 59	6.41	G0	0.0	.017	.009	g
14798	21 12.2	+55 6	8.06	A0	0.3	.0070	.004	f
21^h15	21 12.7	−53 52	4.60	A5	2.50	.067	.048	g
14822	21 13.0	+35 21	7.87	A2	0.5	.012	.008	f
14821	21 13.2	+39 36	9.0	A0	0.1	.0047	.003	g
14828	21 13.7	+39 19	7.58	F0	1.9	.013	.008	p
14847	21 14.0	−26 46	6.50	*G4*	2.5	.059	.049	g
14852	21 14.2	−27 44	7.3	G5	0.2	.016	.010	f
14856	21 14.8	+ 9 6	6.91	A2	1.1	0.013	0.008	f

TABLE 53—*Continued*

ADS	α_{1900}	δ_{1900}	Mag.	Sp.	Δm	h_1	d	Grade
14880	$21^h15^m.9$	+ 2°42′	7.9	F8	1.8	0″.027	0″.022	g
14894[g]	21 16.4	+ 2 28	7.62	K0	0.5	.0070	.003	g
14864[g]	21 16 5	+58 13	5.79	K0 A0	4.3	.011	.005	f
14889	21 16.6	+32 2	6.44	G5	0.7	.028	.018	g
14878	21 16.7	+52 33	6.93	*F6 G5*	0.2	.021	.014	g
14876	21 16.7	+52 54	7.8	A0	1.5	.0080	.005	f
21^h225	21 18.0	−41 26	5.86	A0p	1.0	.012	.006	F:
14942	21 19.3	+ 3 17	9.2	G5	0.1	.013	.010	g
14926[2]	21 19.4	+57 8	8.1	G5	1.2	.024	.019	g
14954	21 20.2	+ 8 57	7.7	A2	1.5	.0080	.005	f
14958	21 20.6	+10 55	8.3	F8	1.0	.0051	.003	f
14916	21 21.5	+79 56	7.31	*F4*	0.7	.019	.013	g
14977	21 22.0	+13 16	7.08	A2	1.7	.016	.011	f
14986	21 22.5	+30 24	9 3	....	3.3	.020	.018	f
14921	21 23.0	+82 5	7.83	F8	0.1	.0037	.002	f
15007	21 24.0	+10 39	6.70	F2	0.0	.025	.017	g
15026	21 24.9	− 5 56	9.4	F8	0.3	.0099	.007	p
15032	21 27.4	+70 7	3.32	*B1s A4n*	4.50	.011	.004	p
15060	21 27.6	+33 23	7.7	A5	0.0	.0057	.003	p
15076	21 28.4	+20 16	7.11	*F4 F7*	0.7	.034	.026	g
15086[1]	21 28.9	− 3 54	9.2	K0	2.1	.042	.042	f
15144[g]	21 32.6	+ 6 16	8.9	K2	0.3	.0071	.004	f
15173	21 34.8	+35 56	8.2	F0	1.5	.015	.010	f
15182[2]	21 35.1	+29 42	8.6	A5	0.4	.011	.008	g
21^h45[g]	21 35.6	−83 11	5.38	G0 A3	2.2	.022	.011	F:
15178	21 35.6	+55 20	7.91	A0	1.5	.0058	.003	f
15208[g]	21 36.3	+42 49	5.35	*M1*	5.1	.017	.007	p
15215[g]	21 36.6	+28 53	8.0	G	1.0	.0079	.004	g
15243	21 37.4	−20 52	8.4	A2	2.7	.017	.013	F:
15251	21 38.5	+40 36	7.54	F0	0.0	.0077	.004	g
15270	21 39.6	+28 18	4.45	*F6 F3*	1.35	.087	.063	g
15300	21 41.0	+11 25	8.5	G0	0.3	.012	.008	f
15313	21 41.8	+ 0 24	7.68	F2	0.2	.0091	.005	g
15229	21 41.9	+82 28	8.7	G5	0.5	.034	.031	g
15350	21 46.1	+68 52	8.7	A5	0.8	.0071	.004	f
15378	21 46.6	+28 42	9.3	K0	1.0	.019	.015	p
15392	21 47.0	+ 8 36	7.9	*A9s*	2.8	.037	.034	g
15401	21 47.6	+27 19	8.7	A3	0.0	.011	.008	f
15389	21 48.7	+71 18	8.7	G0	2.0	.014	.010	f
15407	21 49.1	+65 17	6.41	*A8s F0*	0.2	.016	.010	g
15417	21 49.5	+62 38	7.7	B3	0.1	.014	.009	f
15435[g]	21 50.5	+43 35	7.9	G5	0.1	.0039	.002	g
15447	21 50.6	+10 25	7.92	G5	0.3	.020	.015	g
15452	21 50.7	+ 6 47	8.3	F8	1.1	.0066	.004	p
15454[2]	21 51.4	+38 21	8.8	G0	0.0	0.0093	0.006	f

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
15461	$21^h 51^m 5$	+23° 52′	8.0	A0	2.0	0″.017	0″.012	f
15487[9]	21 52.9	+32 12	8.2	A3	0.2	.0062	.003	p
15494	21 52.9	− 3 58	7.8	F0	0.4	.0053	.003	f
15491	21 53.0	+19 46	8.5	F2	2.5	.015	.012	g
15493	21 53.1	+ 5 29	6.65	A2	0.36	.022	.014	f
15536[5]	21 55.1	−28 56	5.42	B8	1.4	.020	.011	F
15562	21 57.0	−17 27	6.49	A2	0.02	.010	.006	f
15542	21 57.5	+67 29	7.92	F5	2.5	.015	.010	f
15596	21 59.5	+13 10	7.04	F5	0.3	.019	.013	f
15599[g]	21 59.7	+15 23	7.97	G5	0.5	.0039	.002	f
15610	22 0.6	+29 23	8.7	G5	1.0	.013	.009	g
15614[g]	22 0.8	+ 4 23	7.7	G5	0.6	.0060	.003	p
15600	22 0.9	+64 8	4.40	A3 *F7*	1.90	.058	.038	g
15571[c]	22 1.9	+82 23	6.49	*F5 G5*	0.25	.047	.032	g
15647[6]	22 2.0	−28 33	7.88	G0	1.2	.035	.028	F:
15639[g]	22 2.0	+ 0 5	7.58	G0	0.1	.011	.006	g
15645	22 2.7	+35 36	7.72	A3	1.8	.010	.006	f
15618	22 2.7	+72 42	9.2	....	0.2	.0065	.004	f
15650	22 2.9	+25 9	8.26	A2	0.2	.0042	.002	g
15663	22 3.6	−18 58	8.5	F8	0.3	.017	.013	F
15666	22 3.8	−19 29	8.9	F5	0.3	.0082	.005	F:
15673	22 4.7	+22 3	8.1	F5	0.5	.0063	.004	f
15685[g]	22 5.1	+ 7 28	8.0	G0	1.1	.0097	.005	f
15670[3]	22 5.3	+58 48	7.4	A0	0.0	.0078	.004	g
15691	22 5.5	+13 16	8.2	G0	3.9	.028	.024	f
$22^h 7$	22 5.8	−34 57	6.72	F2	0.7	.0085	.004	f
$22^h 8$	22 6.1	−38 48	6.71	F2	0.0	.017	.011	g
15701	22 6.7	+41 41	8.9	....	0.0	.0097	.006	p
15707	22 7.0	+39 41	7.9	A2	0.2	.011	.007	g
15712	22 7.9	+62 55	8.0	B5	0.0	.0027	.001	f
15738	22 8.1	+29 43	8.3	F5	0.5	.0095	.006	f
15753[g]	22 8.8	−21 34	5.45	G5	2.0	.021	.011	F
15748	22 9.4	+59 43	8.9	G0	0.0	.0094	.006	f
15767	22 9.5	+ 7 29	6.60	A0	1.5	.0079	.004	g
15769	22 10.0	+29 4	7.10	F5	0.5	.013	.008	g
15778	22 10.8	+48 51	7.52	F8	2.1	.049	.044	g
$22^h 14$	22 11.0	−63 19	7.09	F5	0.5	.017	.011	F
15791	22 11.4	+30 55	8.9	*F3 F6*	1.7	.020	.016	p
15828	22 14.6	+37 15	6.11	F0	2.2	.022	.013	p
15843	22 15.6	+29 0	8.7	F5	0.3	.012	.008	g
15846	22 16.0	+45 54	8.9	....	0.1	.012	.009	f
15854	22 16.5	+47 47	9.2	....	1.0	.011	.007	p
15867	22 17.1	+41 18	8.0	A0	0.7	.0049	.003	f
15877	22 17.2	− 7 58	9.3	G0	0.1	.011	.008	g
15870	22 18.0	+66 28	7.26	A5	2.5	0.023	0.017	p

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h	d	Grade
15896	$22^h 18^m.8$	+20° 20′	6.13	*F4*	3.2	0″.034	0″.025	g
15881[d,g]	22 18.8	+66 12	6.74	*G6 A3s*	1.0	.0058	.002	p
15902	22 18.9	− 5 20	5.85	A0	0.2	.013	.007	g
15905	22 19.4	+ 3 19	8.6	F0	0.6	.019	.015	f
———	22 20.8	−46 8	8.2	F2	0.0	.025	.020	F
15934	22 21.1	−17 15	5.70	*G2 G1*	0.22	.099	.083	g
15935	22 21.5	+ 3 53	5.85	*F5*	6.7	.026	.017	p
15950[g]	22 22.3	+14 38	8.6	G5	0.0	.0033	.001	f
15956	22 22.7	+ 4 1	9.0	F8	0.0	.0066	.004	g
15962	22 23.2	+11 45	7.19	K0	3.0	.023	.016	g
15967	22 23.4	+16 46	7.52	*G8*	1.7	.018	.012	f
15966	22 23.4	+23 1	8.3	*G8 K2*	0.5	.030	.025	g
15971	22 23.7	− 0 32	3.75	*F2 F1*	0.17	.064	.040	g
15994	22 25.3	− 8 37	7.33	F0	1.0	.020	.014	f
15999[g]	22 25.4	−19 42	9.03	G0	1.5	.0054	.003	F:
16010	22 26.4	+39 36	9.1	....	0.5	.0058	.004	f
16061[6]	22 28 9	−23 7	8.50	G0	3.0	.012	.008	F:
16069	22 29.5	+ 3 42	7.8	A0	1.1	.0054	.003	f
16057	22 30.1	+69 24	6.02	F2 A5	0.5	.012	.006	g
16112	22 32.6	+36 41	9.4	....	0.5	.0063	.004	g
16111	22 33.6	+72 22	7.46	F5	0.0	.012	.008	f
16149[3]	22 34.2	−28 52	6.84	F5	0.8	.016	.010	F
16145	22 34.3	−13 8	8.1	*G8*	0.0	.050	.045	g
16157[g]	22 34.9	+ 0 41	8.4	G5	0.5	.0082	.004	g
16153	22 35.1	+40 51	8.5	A2	0.0	.0062	.004	f
16183	22 36.4	− 3 4	8.7	G5	1.2	.011	.007	f
16185	22 37.1	+20 54	8.3	G0	1.0	.022	.018	g
16190	22 37.3	− 1 19	8.4	F5	0.0	.0049	.003	g
16208	22 37.8	− 8 50	6.49	A2	0.8	.0083	.004	g
16199[g]	22 37.9	+29 11	8.9	G5	0.1	.0039	.002	g
16191	22 38.6	+72 12	8.0	A3	1.2	.0089	.006	f
16214	22 38.8	+46 39	6.42	B9	0.4	.0057	.003	f
16228[g]	22 39.5	+38 57	6.12	*K5 K1*	2.3	.012	.005	f
16235[3]	22 39.9	− 3 11	9.4	....	0.5	.018	.015	g
16242	22 40.5	+10 40	9.5	*K6*	1.1	.038	.035	f
16270	22 42.7	− 4 45	6.75	*G3 G3*	0.5	.044	.034	g
16278	22 43.8	+30 34	7.26	B9	0.8	.010	.006	g
———	22 44.4	−33 20	6.35	A5	0.2	.030	.020	F
16292	22 45.0	+30 46	7.98	F0	0.0	.018	.013	f
16293	22 45.1	+39 59	7.51	F8	0.0	.021	.015	f
16291	22 45.6	+68 2	6.39	*F4 F5*	0.11	.032	.023	g
16298	22 46.1	+66 0	6.97	B9	1.7	.0029	.001	f
22^h55^b	22 47.0	−33 24	4.52	A0	4.2	.028	.015	g
16324	22 47.0	−22 14	8.1	F0	0.2	.0076	.004	F
16317[g]	22 47.4	+61 10	5.80	*G3*	1.3	0.019	0.010	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h	d	Grade
16345	22^h49^m2	+44°13′	5.62	*F1*	2.0	0".031	0".020	g
22^h57[g]	22 49.4	−49 0	6.98	Go	0.4	.015	.008	F
16365[g]	22 50.0	− 5 31	5.87	*G7*	2.0	.014	.006	f
22^h60[g]	22 50.4	−33 4	4.33	Ko	6.2	.029	.014	F:
16389	22 51.8	+11 19	6.46	A3	2.1	.014	.008	f
16394	22 52.6	+62 19	7.76	B5	0.0	.0042	.002	p
16428	22 54.2	+11 12	5.79	Fo	1.5	.023	.014	g
16407	22 54.7	+77 58	7.68	G5	1.2	.013	.008	p
16435	22 55.1	+41 17	8.6	F5	0.1	.012	.009	g
22^h63[g]	22 57.0	−36 57	6.50	Ko	3.4	.0068	.003	F:
16465	22 57.3	−19 5	6.62	Ao	2.0	.011	.006	F
16467[b]	22 58.0	+42 13	5.08	A2	2.9	.016	.007	g
16496	23 0.2	+32 50	7.28	Ao	0.2	.0098	.006	f
23^h3	23 1.2	−44 4	4.35	F5	3.7	.044	.027	g
16514	23 2.4	+60 55	6.74	F5	3.9	.014	.009	f
16519	23 2.6	+32 18	5.97	A2	1.2	.018	.011	g
16524	23 2.8	+12 8	7.8	Fo	2.2	.011	.007	f
16518	23 3.0	+60 18	7.61	A2	0.8	.0077	.004	f
16527	23 3.1	+39 15	7.47	F2	2.0	.012	.007	f
16539	23 3.8	+10 25	7.51	F5	0.5	.020	.014	g
16538[b, g]	23 4.7	+74 51	4.56	*G1*	2.3	.031	.014	g
16557	23 5.3	+47 25	6.47	*Go* G5	2.0	.040	.030	f
16561	23 5.5	+31 57	6.89	B9	0.8	.0063	.003	f
16562	23 5.7	+48 28	7.22	Go	2.9	.026	.020	f
16567[3]	23 5.8	+25 59	10.3	....	0.5	.023	.020	f
16579[g]	23 6.8	−12 29	7.04	Ko	0.0	.0083	.004	f
16591	23 7.5	+ 2 9	8.2	G5	0.1	.014	.009	f
16592	23 7.5	−22 29	8.7	Go	0.8	.021	.016	F:
16604	23 8.3	− 9 27	8.2	Fo	1.0	.0090	.006	f
16602	23 8.3	+21 32	8.8	F2	0.1	.017	.013	g
16599	23 8.4	+39 28	7.40	*A8n*	1.7	.031	.025	f
16608[g]	23 8.6	−13 56	7.26	K2	3.3	.011	.005	f
16610	23 9.0	+38 59	9.2	....	0.1	.0058	.004	f
16617	23 9.4	+ 7 7	9.1	....	0.8	.018	.014	g
16623	23 9.9	−27 12	8.5	F8	0.5	.013	.009	F
16633[3]	23 10.6	− 9 37	9.06	*K6*	0.1	.022	.016	g
16642	23 11.4	− 2 7	7.7	G5	0.3	.021	.015	f
16638	23 11.6	+63 34	8.0	....	0.1	.011	.007	g
16644[23]	23 11.9	−14 22	8.3	*A8sp*	0.2	.018	.014	f
16649	23 12.4	− 2 3	8.2	*Go*	1.0	.030	.026	g
16672[b]	23 13.8	−14 0	5.27	*G4 K3*	2.18	.043	.025	g
16664	23 13.8	+24 41	8.9	F5	0.1	.0076	.005	f
16666[g]	23 14.5	+67 34	4.90	*G7*	2.6	.034	.018	g
16686	23 15.8	+21 25	7.8	Fo	0.7	.0079	.005	f
16713	23 17.8	+20 1	6.58	*Go K6*	3.0	0.030	0.022	g

TABLE 53—*Continued*

ADS	α_{1900}	δ_{1900}	Mag.	Sp.	Δm	h_1	d	Grade
16730[g]	23h19m.2	+ 3°10′	6.65	K2	2.0	0″.017	0″.008	f
16733	23 19.2	+ 5 31	9.7	....	0.0	.016	.012	p
16735	23 19.4	+13 56	8.0	F8	3.0	.019	.015	f
16736	23 20.3	+76 31	8.8	A5	0.3	.020	.017	f
16767	23 22.5	−27 14	8.3	F5	0.5	.0034	.002	F:
16766	23 22.6	+16 5	9.3	G0	0.1	.012	.009	f
16766[24]	23 22.6	+16 5	8.2	G0	1.5	.020	.015	g
16769	23 22.9	+10 35	8.47	G0	2.3	.039	.037	g
16775	23 23.8	+73 34	7.17	F0	1.1	.016	.010	g
16795[10]	23 25.2	+58 0	7.06	A0	1.5	.0054	.003	f
16803	23 25.6	+ 4 41	7.12	A3	1.0	.012	.007	f
16812	23 26.4	+15 40	7.79	F8	1.2	.022	.016	f
16818	23 27.2	+43 16	8.1	A0	1.7	.013	.009	f
16836[g]	23 29.0	+30 46	5.21	*K4*	0.00	.016	.006	g
23h40	23 29.8	−58 3	7.4	F2	0.2	.012	.007	F
16850	23 30.4	−28 2	6.68	F8	1.7	.034	.025	F
23h44[g]	23 31.8	−32 25	6.51	K0	3.2	.0068	.003	F
16877	23 32.6	+43 53	5.86	B9	0.90	.0076	.004	g
16882	23 33.0	+12 20	8.8	A2	1.0	.0090	.006	f
16886	23 33.4	+54 41	8.6	A0	0.3	.011	.007	g
16893	23 33.9	+70 5	8.24	A3	0.7	.0099	.006	f
23h48	23 34.1	−47 12	6.28	A3	0.57	.014	.008	g
16904	23 34.4	+45 10	7.77	A2	0.1	.0076	.004	g
16924	23 35.8	+ 7 26	9.5	F8	0.7	.0069	.005	f
16929	23 36.2	− 9 11	9.2	G5	1.5	.023	.020	g
16928	23 36.3	+32 0	7.25	A0	0.4	.011	.006	g
16937	23 37.0	+19 45	7.60	F8	0.6	.012	.008	f
16951[2]	23 38.0	+11 17	8.8	G5	0.0	.017	.013	g
16958	23 38.8	+ 6 42	8.00	F2	0.0	.013	.009	g
16957[b, g]	23 39.0	+28 49	4.98	*G7*	3.1	.022	.010	g
23h56	23 39.5	−45 48	8.7	F5	0.5	.017	.013	F:
16979	23 40.8	−19 14	5.45	A5	1.50	.040	.027	g
17006[b]	23 42.6	+46 16	5.84	B3	2.5	.0060	.002	f
17009[3]	23 42.8	+16 31	7.8	G	0.1	.014	.009	f
17020	23 43.8	+64 19	6.38	A0	0.70	.011	.006	g
17022	23 44.0	+61 40	5.61	A2p	2.5	.0034	.001	p
17030	23 44.8	+27 7	6.94	F0	0.5	.0065	.003	g
23h63[6]	23 45.3	−52 16	7.73	G5	0.2	.042	.035	F
17050	23 46.5	+41 32	7.09	A5	0.3	.0050	.002	f
17052[9]	23 46.6	− 7 10	8.9	G0	0.0	.020	.016	f
17060	23 46.9	+12 19	9.1	....	0.3	.0098	.007	g
17054	23 46.9	+37 20	7.05	*F5 F5*	0.00	.030	.022	g
17062[b]	23 47.5	+74 59	6.55	*K5*	6.5	.068	.050	f
17078	23 47.8	+38 8	8.6	....	0.8	.032	.027	f
17079	23 47.9	+11 22	6.80	F0 F0	0.67	0.035	0.026	g

TABLE 53—*Continued*

ADS	α1900	δ1900	Mag.	Sp.	Δm	h_1	d	Grade
17090[5]	23ʰ49ᵐ.2	−27°36′	6.27	A2	0.39	0″.011	0″.006	F
17092	23 49.3	+ 1 55	8.2	A2	2.0	.0056	.003	p
17105	23 51.1	+24 47	8.9	G0	0.1	.0099	.007	g
17107	23 51.2	−10 3	8.5	*G3*	0.5	.038	.034	g
17111	23 51.7	+ 4 10	6.87	F0	0.5	.017	.011	g
17118	23 52.4	+72 18	7.8	B5	0.5	.0056	.003	g
17126	23 52.8	+56 51	8.4	B9	0.0	.0049	.003	f
17131	23 53.0	+23 48	8.2	G5	1.4	.020	.015	f
17137[g]	23 53.6	− 4 7	5.07	*G6*	5.3	.015	.007	f
17140	23 53.9	+55 12	4.93	B2	2.10	.014	.006	f
17149	23 54.4	+33 11	5.83	*G0 G1*	0.00	.058	.043	g
17153	23 54.5	− 8 22	8.1	G0	1.1	.021	.016	g
17177	23 57.1	−27 42	7.9	F8	0.3	.0051	.003	F
17180	23 57.3	+10 13	8.67	G0	0.7	.0076	.005	g
1[g]	23 57.5	+65 33	5.77	*G5 A1n*	1.45	.019	.010	f
4	23 57.5	− 9 3	8.7	G0	0.0	.0089	.006	g
9	23 57.6	+ 1 34	7.7	G0	2.5	.027	.022	g
17	23 58.2	+52 59	9.2	....	0.3	.0042	.002	g
32[g]	23 59.5	+33 42	7.15	K0	0.00	.0057	.002	g
34	23 59.6	+37 37	9.5	....	0.3	.020	.016	f
40	23 59.7	+29 32	8.2	F5	2.3	.011	.007	g
41	23 59.8	+45 7	7.62	*F6 G5*	1.1	.018	.013	f
43	23 59.8	+82 9	10.2	....	0.5	0.011	0.009	f

NOTES TO TABLE 53

a. Brighter component spectroscopic binary; two spectra visible.
b. Brighter component spectroscopic binary; one spectrum visible.
c. Fainter component spectroscopic binary; two spectra visible.
d. Fainter component spectroscopic binary; one spectrum visible.
g. Treated as a giant in calculation of *d*.

1. Possibly optical.
2. Data published in *Lick Obs. Bull.*, No. 451, 1933, replace those in *A.J.*, 39, No. 930, 1929.
3. Pair BC.
4. Also ADS 440.
5. Magnitude data published by Finsen replaced by those in *Harvard Ann.*, Vol. 56 or 64.
6. Magnitude published by Finsen replaced by that in Boss's *General Catalogue*.
7. Spectral class published by Finsen replaced by Mount Wilson class.
8. Pair AB and C.
9. Data published in *Lick Obs. Bull.*, No. 451, 1933, or *A.J.*, 39, No. 930, 1929, replaced by those in *Lick Obs. Bull.*, No. 485, 1937.
10. Pair CD.

NOTES TO TABLE 53—*Continued*

11. *Henry Draper Catalogue* gives α $2^h32^m.7$.

12. This star is BD+38°667, HD 19923; ADS erroneously identifies it as BD+38°666, HD 19909, 8.6A3.

13. Pair AC.

14. Burnham's *General Catalogue* number.

15. Pair A and BC.

16. See Table 51, n. 18.

17. Published in *A.J.*, **39**, No. 930, as βGC 2977. Aitken states that star was misidentified by Burnham and gives position entered here.

18. Magnitude published by Finsen replaced by photometric magnitude in *Henry Draper Catalogue*.

19. Error in value of *w* published by Finsen has been corrected.

20. Pair Cc.

21. Values of *s* and *ds/dt* published by Finsen have been corrected.

22. Variable, X Ophiuchi.

23. Abnormally faint A star; *d* (computed by main-sequence formula) doubtful.

24. Pair A′B′.

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